# Static Instability Analysis of the Natural Circulation Flow in a Passive Containment Cooling System

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# 1. Introduction

When a severe accident occurs in a nuclear power plant, the containment pressure can increase up to 4 bar and, then, it can threaten the containment's integrity. To avoid such over-pressure in the containment, a passive containment cooling systems (PCCS) has been developed instead of existing active cooling systems [1].

The PCCS can cool down the containment by using a natural circulation flow and, thus, flow instabilities may easily occur [2]. It should be confirmed that both static and dynamic flow instabilities do not occur due to the system characteristics. In this study, mathematical models for the single- and two-phase natural circulation flows in a PCCS are developed. Using the flow models, static instability [3] of the natural circulation flow is investigated.

#### 2. Mathematical Modeling of the PCCS Flow

Fig. 1 shows the schematic diagram of a PCCS [1, 2]. The PCCS is located at the top of the containment. The PCCS consists of four heat exchanger assemblies with water tanks. Each heat exchanger assembly consists of several heat exchanger tubes and pipes.



Fig. 1. Overall the PCCS schematic diagram

Fig. 2 shows the side view of six heat exchanger tubes (A heat exchanger assembly of the PCCS has 2520 tubes). Water from the water tank flows through the PCCS heat exchanger tubes, removing the containment's heat, and returns to the water tank by gravity. For a mathematical modeling, the natural circulation loop is divided into 12 regions according to the flow area and vertical orientation as shown in Fig. 2. Some numbers are not shown for simplicity in the side view. It is assumed that the PCCS is insulated except the heat exchanger (numbered 7~8). Heat transfer from the containment's air to the PCCS occurs via the PCCS heat exchanger. According the flow conditions, singleor two-phase natural circulation flow occur in the PCCS.



Fig. 2. Simplified PCCS natural circulation loop.

# 2.1 Single-Phase Flow in the PCCS

Let's assume that flow inside the PCCS heat exchangers remain in a single-phase liquid flow. In other words, the temperature of circulating water doesn't increase up to a saturation temperature.

Then, assuming a steady state, a momentum equation is applied to each segment of the natural circulation loop. Integrating all the momentum equations yields a loop momentum balance.

$$\rho_{l,l2} gH_{l,l2} = \rho_{l,2} gH_{l,2} + \rho_{2,3} gH_{2,3} + \dots + \rho_{l0,l1} gH_{l0,l1} + \rho_{l1,l2} gH_{l1,l2} + \frac{1}{2} \sum (f \frac{L}{D} + K) \frac{\dot{m}}{\rho A}.$$
(1)

where K is a form loss coefficient and f is a friction coefficient.

Because the loop is assumed to be insulated except at the PCCS heat exchanger, Eq. (1) can be simplified into:  $(a - \overline{a})gH + (a - a)gH$ 

$$(\rho_{i} - \overline{\rho}_{p})gH_{7,8} + (\rho_{i} - \rho_{e})gH_{10,11}$$

$$= \frac{1}{2}\sum (f\frac{L}{D} + K)\frac{\dot{m}}{\rho A},$$
(2)

where  $\rho_i$  and  $\rho_e$  is density of inlet and exit of the heat exchanger, and  $\bar{\rho}_p$  is the average density in the heat exchanger.

The density of water can be represented as a function of temperature by Boussinesq approximation:

$$\rho(T) = \rho_0 \left[ 1 - \beta(T - T_0) \right], \qquad (3)$$

where  $\beta$  is a thermal expansion coefficient of the water.  $\rho_0$  and  $T_0$  are reference density and temperature. By inserting Eq. (3) into Eq. (2), the loop momentum equation can be written as follows:

$$\rho_0 \beta g[(\bar{T}_p - T_i)H_{7,8} + (T_e - T_i)H_{10,11}] = \frac{1}{2} \sum (f\frac{L}{D} + K)\frac{\dot{m}^2}{\rho A^2}.$$
(4)

where  $T_i$  and  $T_e$  are the coolant temperature at the inlet and exit of the PCCS heat exchanger, respectively.  $T_p$  is the average coolant temperature in the PCCS heat exchanger

The temperatures in Eq. (4) can be replaced with mass flow rate by using the energy equation. The tubeside temperature increment  $dT_p$  along the heat exchanger [4] can be represented as:

$$\dot{m} C_p dT_p = U_{l\varphi} \pi D(T_c - T_p) dz, \qquad (5)$$

where  $T_c$  is containment temperature and  $U_{1\emptyset}$  is overall heat transfer coefficient. It is assumed that overall heat transfer coefficient of single-phase  $(U_{1\emptyset})$  and heat capacity  $(C_p)$  are uniform along the heat exchangers and the containment temperature  $(T_c)$  is uniform. Integrating Eq. (5) along the heat exchanger, the coolant temperature distribution,  $T_p(z)$ , can be represented as follows:

$$T_{p}(z) = T_{c} + (T_{i} - T_{c})e^{-(\eta/i\eta)z},$$
(6)

where  $\eta = U_{l_{\varphi}} \pi D / C_{\rho}$  and  $T_i$  is the heat exchanger inlet temperature, which is assumed to be the PCCS tank temperature in a steady-state. Then, the average coolant temperature in the heat exchanger can be obtained:

$$\overline{T}_{p} = T_{c} - (T_{c} - T_{l}) \frac{\dot{m}}{\eta H_{7,8}} (l - e^{-(\eta/m)H_{7,8}})$$
(7)

Inserting Eq. (7) into Eq. (4) yields the following equation, which involves one unknown, i.e., the loop mass flow rate.

$$\rho_{0}\beta g(T_{c}-T_{i})[\{1-\frac{\dot{m}}{\eta H_{7,8}}(1-e^{-(\eta/\dot{m})H_{7,8}})\}H_{7,8}+(1-e^{-(\eta/\dot{m})H_{7,8}})H_{10,11}]$$
$$=\sum (f\frac{L}{D}+K)\frac{\dot{m}^{2}}{\rho A^{2}},$$
(8)

where  $\Delta T_{I_{\varphi}} = T_c - T_i$ .

To find the mass flow rate that satisfies Eq. (8), let's define a function  $f(\dot{m})$  as follows:

$$f(\dot{m}) = \sum (f\frac{L}{D} + K)\frac{\dot{m}^2}{\bar{\rho}A^2} - \rho_0 \beta g \Delta T_{l_{\varphi}} [\{I - \frac{\dot{m}}{\eta H_{7,8}} \times (I - e^{-(\eta/\dot{m})H_{7,8}})\}H_{7,8} + (I - e^{-(\eta/\dot{m})H_{7,8}})H_{10,11}], \qquad (9)$$

Figure 3 shows a typical graph of f(m) when the inlet temperature is 303 K. It is clearly shown that only one solution for f(m) = 0 exists. This implies the static

instability does not exist in the case of single-phase flows in the PCCS.



Fig. 3. Function  $f(\dot{m})$  versus mass flow rate in the case of a single-phase flow.

## 2.2 Two-Phase Flow in the PCCS

When the coolant temperature in the PCCS tank increases close to the saturation temperature, a boiling may occur inside the PCCS heat exchangers. Then, the governing equations in Section 2.1 should be modified appropriately.

To consider the boiling flow, it is convenient to divide heat exchanger  $(H_{7,8})$  into single-phase region  $(L_{10})$  and two-phase region  $(L_{20})$ . Then, a steady-state momentum equation is written by inserting two-phase flow terms into Eq. (2).

$$(\rho_{i} - \overline{\rho}_{l\phi})gL_{l\phi} + (\rho_{i} - \overline{\rho}_{2\phi})gL_{2\phi} + (\rho_{i} - \rho_{2\phi})gH_{l0,ll} = \frac{l}{2}\sum (f\frac{H}{D} + K)\frac{\dot{m}}{\rho A} .$$
 (10)

Single-phase density can be obtained by inserting Eq. (6) into (3).

$$\rho_{I_{\varphi}}(z) = \rho_{i} [I - \beta \Delta T_{I_{\varphi}} (I - e^{-(\eta / in) H_{7,8}})].$$
(11)

Thereafter, the two-phase density must be obtained.

$$\rho_{2\varphi}(z) = \frac{1}{v_f + x(z)v_{fe}},$$
(12)

where x(z) is the local thermodynamic quality,

$$x(z) = \frac{h(z) - h_f}{h_c},\tag{13}$$

and h(z) is local enthalpy that can be obtained by using the energy balance in a small segment along the PCCS heat exchanger:

$$\dot{m} dh = U_{2\alpha} \pi D \Delta T_{2\alpha} dz, \qquad (14)$$

where  $\Delta T_{2\varphi} = T_c - T_b$  and  $T_b$  is a temperature at the boiling point. It is assumed that overall heat transfer coefficient of two-phase flow  $(U_{2\varphi})$  is also uniformed along the heat exchanger. Then,

$$h(z) = h_i + \frac{U_{2\varphi} \pi D \Delta T_{2\varphi}}{\dot{m}} z.$$
(15)

Using Eqs. (12), (13), and (15), the coolant density in the two-phase flow region is represented as:

$$\rho_{2\varphi}(z) = \frac{1}{v_f + (h(z) - h(L_{1\phi}))\frac{v_{fg}}{h_{fg}} + \frac{U_{2\varphi}\pi D\Delta T_{2\varphi}}{\dot{m}} z \frac{v_{fg}}{h_{fg}}}.$$
 (16)

Then, average density of single-phase flow and twophase flow can be obtained each from Eq. (11) and Eq. (16).

$$\overline{\rho}_{l\varphi} = \rho_i [l - \beta \Delta T_{l\varphi} (l - \frac{\dot{m}}{\eta L_{l\varphi}} e^{-(\eta/\dot{m})L_{l\varphi}})]$$
(17)

$$\overline{\rho}_{2\varphi} = \frac{\dot{m}h_{fg}}{U_{2\varphi}\pi D \Delta T_{2\varphi}v_{fg}L_{2\varphi}} \ln(v_f + \frac{h(H_{7.8}) - h(L_{1\varphi})}{h_{fg}}v_{fg}), \quad (18)$$

where  $L_{I_{\varphi}} = \frac{\dot{m}(h_i - h_f)}{U_{1_{\varphi}}\pi D\Delta T_{1_{\varphi}}}$ ,  $L_{2_{\varphi}} = H_{7,8} - L_{I_{\varphi}}$ .

Using Eqs. (10) through (18), all terms in Eq. (10) are finally presented as a function of mass flow rate. The resulting function  $f(\dot{m})$  is presented as

$$\begin{split} f(\dot{m}) &= \sum (f \frac{L}{D} + K) \frac{\dot{m}^2}{\rho_i A^2} + (f \frac{L_{i\varphi}}{D})_{i\varphi} \frac{\dot{m}^2}{\bar{\rho}_{i\varphi} A^2} + (f \frac{L_{2\varphi}}{D})_{2\varphi} \frac{\dot{m}^2}{\bar{\rho}_{2\varphi} A^2} \\ &+ \sum (f \frac{L}{D} + K) \frac{\dot{m}^2}{\rho_{2\varphi} A^2} + \rho_i \beta \Delta T_{i\varphi} (1 - \frac{\dot{m}}{\eta L_{i\varphi}} e^{-(\eta / \dot{m}) L_{i\varphi}}) g L_{i\varphi} \\ &+ (\rho_i - \frac{\dot{m} h_{jg}}{U_{2\varphi} \pi D \Delta T_{2\varphi} v_{fg} L_{2\varphi}} ln(v_f + \frac{h(H_{2,g}) - h(L_{i\varphi})}{h_{fg}} v_{fg})) g L_{2\varphi} \\ &. (19) \end{split}$$

$$+ \left(\rho_{i} - \frac{l}{v_{f} + (h(L_{1\phi}) - h_{f})\frac{v_{fg}}{h_{fg}} + \frac{U_{2\phi}\pi D\Delta T_{2\phi}}{\dot{m}}L_{2\phi}\frac{v_{fg}}{h_{fg}}}gH_{10,11}\right)$$



Fig. 4. Function f(m) versus mass flow rate in the case of a two-phase flow.

A graph of  $f(\dot{m})$  versus mass flow rate is presented in Fig. 4 when the inlet temperature is 346 K. It is clearly shown that only one solution for  $f(\dot{m}) = 0$  exists. This implies the static instability does not exist in the case of two-phase flows in the PCCS.

## 3. Conclusions

In this study, mathematical models for the single- and two-phase natural circulation flows in a PCCS are developed. Using the models, both single- and twophase natural circulation flows were investigated in terms of static instability. It is shown that, for both cases, there is only one steady-state mass flow rate which satisfies the integrated momentum equation and the pressure drop along the natural circulation loop increases monotonically according to the mass flow rate. Therefore, it can be said that static instability doesn't exist in the PCCS natural circulation loop.

Additional research is needed to investigate the dynamic instability of the PCCS natural circulation flow.

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