Rhodium SPND's Error Reduction using Extended Kalman Filter combined with Time Dependent Neutron Diffusion Equation

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1. Introduction

The self-powered neutron detectors (SPND's) are commonly used to get core neutron flux distributions in nuclear reactors. Fixed Rhodium SPND's are used extensively at KHNP's to monitor on-line neutron flux and on-demand surveillance of core power distributions.

The Rhodium SPND is accurate in steady-state conditions but responds slowly to changes in neutron flux.[1] The slow response time of Rhodium SPND precludes its direct use for control and protection purposes specially when nuclear power plant is used for load following.

To shorten the response time of Rhodium SPND, there were some acceleration methods but they could not reflect neutron flux distribution in reactor core. On the other hands, some methods for core power distribution monitoring could not consider the slow response time of Rhodium SPND and noise effect.[3]

In this paper, time dependent neutron diffusion equation is directly used to estimate reactor power distribution and extended Kalman filter method is used to correct neutron flux with Rhodium SPND's and to shorten the response time of them.

2. Methods and Results

To use extended Kalman filter in time dependent neutron diffusion equation, the equation should be reformed and the reformed equations are inserted in steps of Kalman filter calculation.

In this study, basic Kalman filter method and time dependent neutron diffusion equation are combined. The feasibility of Kalman filter to shorten the response of SPND's and to correct detector errors, is approached with this combination.

2.1 Extended Kalman Filter Method

Kalman filter is used for noise reduction of monitored signals. The neutron flux at time step t+1 is developed with variables at time step t as follows.[2]

$$\boldsymbol{\varphi}_{t+1} = f(\boldsymbol{\varphi}_t, \mathbf{u}_t, \mathbf{w}_t),$$
$$\mathbf{z}_{t+1} = h(\boldsymbol{\varphi}_{t+1}, \mathbf{v}_{t+1}).$$

$\boldsymbol{\varphi}_{t+1}$ and $\boldsymbol{\varphi}_t$ are neutron flux of time step t+1 and t, $\mathbf{u}_t$ is user control effect and $\mathbf{w}_t$ is calculation error. $\mathbf{z}_{t+1}$ is detected signal value at time step t+1 and $\mathbf{v}_t$ is measurement error. f and h are conversion functions. Two kind of estimated neutron flux are $\hat{\boldsymbol{\varphi}}_t^-$ and $\hat{\boldsymbol{\varphi}}_t$, and the definition of them are as follows.

$$\hat{\boldsymbol{\varphi}}_t^- = f(\hat{\boldsymbol{\varphi}}_{t-1}, \mathbf{u}_{t-1}, 0),$$
$$\hat{\boldsymbol{\varphi}}_t = \hat{\boldsymbol{\varphi}}_t^- + \mathbf{K}_t\left(\mathbf{z}_t - h\left(\hat{\boldsymbol{\varphi}}_t^-, 0\right)\right).$$

$\mathbf{K}_t$ is Kalman coefficient to be optimized. The object of this method is to get $\hat{\boldsymbol{\varphi}}_t$ close to $\boldsymbol{\varphi}_t$ (real neutron flux). There are two steps to calculate optimized Kalman coefficient and neutron flux.

Step 1: Get prio-flux and prio-covariance

$$\hat{\boldsymbol{\varphi}}_{t+1}^- = f(\hat{\boldsymbol{\varphi}}_t, \mathbf{u}_t, 0)$$
$$\mathbf{P}_t^- = \mathbf{A}_{t-1}\mathbf{P}_{t-1}\mathbf{A}_{t-1}^T + \mathbf{W}_{t-1}\mathbf{Q}_{t-1}\mathbf{W}_{t-1}^T$$

$\mathbf{e}_t^- = \hat{\boldsymbol{\varphi}}_t^- - \boldsymbol{\varphi}_t \Rightarrow \mathbf{P}_t^- = E[\mathbf{e}_t^- \mathbf{e}_t^{-T}]$, prio-covariance, $\mathbf{A}_{[i,j]} = \dfrac{\partial f_{[i]}\left(\hat{\boldsymbol{\varphi}}_t, \mathbf{u}_t, 0\right)}{\partial \boldsymbol{\varphi}_{[j]}}$, $\mathbf{W}_{t-1} = \dfrac{\partial f\left(\hat{\boldsymbol{\varphi}}_{t-1}, \mathbf{u}_{t-1}, 0\right)}{\partial \mathbf{w}_{t-1}}$, and $\mathbf{Q}_{t-1} = E[\mathbf{w}_{t-1}\mathbf{w}_{t-1}^T]$.

Step 2: Get Kalman Coefficient and corrected values

$$\mathbf{K}_t = \mathbf{P}_t^- \mathbf{H}_t^T (\mathbf{H}_t \mathbf{P}_t^- \mathbf{H}_t^T + \mathbf{V}_t \mathbf{R}_t \mathbf{V}_t^T)^{-1}$$
$$\hat{\boldsymbol{\varphi}}_t = \hat{\boldsymbol{\varphi}}_t^- + \mathbf{K}_t\left(\mathbf{z}_t - h\left(\hat{\boldsymbol{\varphi}}_t^-, 0\right)\right)$$
$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{H}_t)\mathbf{P}_t^-$$

$\mathbf{H}_t = \dfrac{\partial h\left(\hat{\boldsymbol{\varphi}}_{t-1}, \mathbf{u}_{t-1}\right)}{\partial \boldsymbol{\varphi}_{t-1}}$, $\mathbf{R}_{t-1} = E[\mathbf{v}_{t-1}\mathbf{v}_{t-1}^T]$, and $\mathbf{e}_t = \hat{\boldsymbol{\varphi}}_t - \boldsymbol{\varphi}_t \Rightarrow \mathbf{P}_t = E[\mathbf{e}_t \mathbf{e}_t^T]$, corrected covariance.

Now, conversion functions f and h should be set to use above two steps.

2.2 Time Dependent Neutron Diffusion Eq.

To follow the power and neutron flux of a reactor with real time, time dependent neutron diffusion equation is used. Reactor power transient occurs within a minute and delayed precursor is very important in this time scale.

The multi-group neutron fluxes are described as follows in m-th node.

$$\frac{1}{v_g}\frac{d\varphi_g^m(t)}{dt} = -\sum_{u=x,y,z}\frac{a_u^m}{V^m}\left[J_{gu}^{m+}(t) - J_{gu}^{m-}(t)\right] + \frac{1}{k_{eff}}\sum_{i=1}^{N_F}\chi_{gi}\left(1-\sum_{k=1}^{6}\beta_{ik}^m\right)\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{i,m}(t)\varphi_{g'}^m(t) + \sum_{k=1}^{6}\chi_{dgk}\lambda_k C_k^m(t) + \sum_{g'=1}^{2}\Sigma_{g'g}^m(t)\varphi_{g'}^m(t) - \Sigma_{rg}^m(t)\varphi_g^m(t) - \sigma_{Xe,g}\varphi_g^m(t)X^m(t).$$

Where $\varphi_g^m(t)$ is g group neutron flux at m-th node. a_u^m and V^m are area orthogonal to neutron current and volume of m-th node, respectively. And balance equation for delayed precursors is as follows.

$$\frac{dC_k^m(t)}{dt} = \frac{1}{k_{eff}}\sum_{i=1}^{N_F}\beta_{ik}^m\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{i,m}(t)\varphi_{g'}^m(t) - \lambda_k C_k^m(t)\ .$$

Using θ method with implicit case, the neutron equation can be reformed such simple matrix equation.

$$\mathbf{R}_{t+1}\boldsymbol{\varphi}_{k,t+1} = \mathbf{T}_t\,\boldsymbol{\varphi}_t + \mathbf{c}_t \Rightarrow \boldsymbol{\varphi}_{k,t+1} = \left(\mathbf{R}_{t+1}\right)^{-1}\left(\mathbf{T}_t\,\boldsymbol{\varphi}_t + \mathbf{c}_t\right),$$
$$\mathbf{c}_t = \sum_{k=1}^{6}\chi_{dgk}\lambda_k e^{-\lambda_k\Delta t}\mathbf{C}_{k,t}\ ,$$
$$\mathbf{C}_{k,t+1} = e^{-\lambda_k\Delta t}\mathbf{C}_{k,t} + \mathbf{F}_k^1\boldsymbol{\varphi}_{t+1} - \mathbf{F}_k^0\boldsymbol{\varphi}_t\ .$$

The first of above equations is conversion function *f* in step 1 of Kalman filter calculation. It is very hard to calculate inverse matrix of $\mathbf{R}_{t+1}$, so neutron flux at time step t+1 is usually solved with iteration method. Since $\mathbf{R}_{t+1}^{-1}$ is not calculated, matrix $\mathbf{A}_t$ cannot be built as explicit form. To get prio-covariance $\mathbf{P}_t^-$, error propagation is considered. With step 1 of Kalman filter calculation prio- neutron flux can be set as follows.

$$\hat{\boldsymbol{\varphi}}_{t+1}^- = \left(\mathbf{R}_{t+1}\right)^{-1}\left(\mathbf{T}_t\,\hat{\boldsymbol{\varphi}}_t + \hat{\mathbf{c}}_t\right).$$

And prio-error $\mathbf{e}_{t+1}^-$ and corrected error $\mathbf{e}_{t+1}$ are rebuilt with corrected error $\mathbf{e}_t$.

$$\begin{aligned}\mathbf{e}_{t+1}^- &= \hat{\boldsymbol{\varphi}}_{t+1}^- - \boldsymbol{\varphi}_{t+1}\\ &= \left(\mathbf{R}_{t+1}\right)^{-1}\left(\mathbf{T}_t\,\hat{\boldsymbol{\varphi}}_t + \hat{\mathbf{c}}_t\right) - \left(\mathbf{R}_{t+1}\right)^{-1}\left(\mathbf{T}_t\,\boldsymbol{\varphi}_t + \mathbf{c}_t\right)\\ &= \left(\mathbf{R}_{t+1}\right)^{-1}\mathbf{T}_t\,\mathbf{e}_t + \left(\mathbf{R}_{t+1}\right)^{-1}\left(\hat{\mathbf{c}}_t - \mathbf{c}_t\right)\\ &\Rightarrow \mathbf{R}_{t+1}\mathbf{e}_{t+1}^- = \mathbf{T}_t\,\mathbf{e}_t + \left(\hat{\mathbf{c}}_t - \mathbf{c}_t\right)\end{aligned}$$

$$\begin{aligned}\mathbf{e}_{t+1} &= \hat{\boldsymbol{\varphi}}_{t+1} - \boldsymbol{\varphi}_{t+1}\\ &= \hat{\boldsymbol{\varphi}}_{t+1}^- + \mathbf{K}_{t+1}\left(\mathbf{z}_{t+1} - h\left(\hat{\boldsymbol{\varphi}}_{t+1}^-, 0\right)\right) - \boldsymbol{\varphi}_{t+1}\\ &= \mathbf{e}_{t+1}^- + \mathbf{K}_{t+1}\left(\mathbf{z}_{t+1} - h\left(\hat{\boldsymbol{\varphi}}_{t+1}^-, 0\right)\right)\end{aligned}$$

The last term of prio-error is delayed precursor error and this term can be described with previous delayed precursor error and corrected error.

The covariance terms are calculated from errors directly. They should be made from numerous cases and errors. As time step gets increased, the number of cases is increased and this system can deal with numerous cases and errors in normal operation condition, so that it can work like real covariance.

2.3 Final Algorithm

The calculation steps are finalized as follows.

Step 0: Guessing initial errors $\mathbf{e}_0$, $\mathbf{w}_0$ and $\mathbf{v}_0$ and calculate initial neutron flux $\hat{\boldsymbol{\varphi}}_0$.

Step 1: Get prio-flux and prio-covariance

$$\mathbf{R}_{t+1}\hat{\boldsymbol{\varphi}}_{k,t+1}^- = \mathbf{T}_t\,\hat{\boldsymbol{\varphi}}_t + \hat{\mathbf{c}}_t$$
$$\mathbf{R}_{t+1}\mathbf{e}_{t+1}^- = \mathbf{T}_t\,\mathbf{e}_t + \left(\hat{\mathbf{c}}_t - \mathbf{c}_t\right)$$
$$\mathbf{P}_t^- = E[\mathbf{e}_t^-\mathbf{e}_t^{-T}]$$

Step 2: Get Kalman Coefficient and corrected values

$$\mathbf{K}_t = \mathbf{P}_t^-\mathbf{H}_t^T(\mathbf{H}_t\mathbf{P}_t^-\mathbf{H}_t^T + \mathbf{V}_t\mathbf{R}_t\mathbf{V}_t^T)^{-1}$$
$$\hat{\boldsymbol{\varphi}}_t = \hat{\boldsymbol{\varphi}}_t^- + \mathbf{K}_t\left(\mathbf{z}_t - h\left(\hat{\boldsymbol{\varphi}}_t^-, 0\right)\right)$$
$$\mathbf{P}_t = (\mathbf{I} - \mathbf{K}_t\mathbf{H}_t)\mathbf{P}_t^-$$
$$\mathbf{e}_{t+1} = \mathbf{e}_{t+1}^- + \mathbf{K}_{t+1}\left(\mathbf{z}_{t+1} - h\left(\hat{\boldsymbol{\varphi}}_{t+1}^-, 0\right)\right)$$

In step 2, there is an inversion of matrix and each term of this inversion is made of diagonal matrix so it can be solved direct inversion. For this, only diagonal terms are taken from product of prio-error vector. Real covariance of prio-error should be diagonal matrix when diffusion equation is sufficiently exact and signal noise of each detected variable (temperature, pressure, etc.) is the dominant cause of error.

2.4 Feasibility Test

To check the feasibility of this algorithm KHNP (YGN 3) reactor core is used. There are 44 assemblies with 5 fixed Rhodium SPND's so total number of detector is 220 and 133 assemblies without detector. Full core steady state neutron distribution is calculated with FDM for reference case. Calculated neutron flux is used as detector signal. Random measurement error is added to detector signal within $\mathbf{v}_t$ domain.

Initial condition of this reactor is power increasing from 50% to 100% instantly. 50% values of reference neutron fluxes of every nodes are used as initial flux vector $\hat{\boldsymbol{\varphi}}_0$. Normal transient equation cannot increase neutron flux automatically to increase reactor power without neutron cross-section changing.

The time interval of transient equation is 5 seconds because computers of plant monitoring system in nuclear power plant are not oriented to this computing but also processing many other tasks.

Calculation error and measurement error may affect the convergence speed and region. Calculation error is determined by calculation method and precision of cross sections. It is examined by sensitivity study that calculation error under 20% hardly affects the convergence speed and region. So any fast method such as FDM can be effective tool for a monitoring system.

For measurement error, 1%, 5%, 10% error bounds are tested. The more error makes the large convergence area but all convergence areas are under measurement errors. With this test it is shown that extended Kalman filter can works for reducing measurement error.

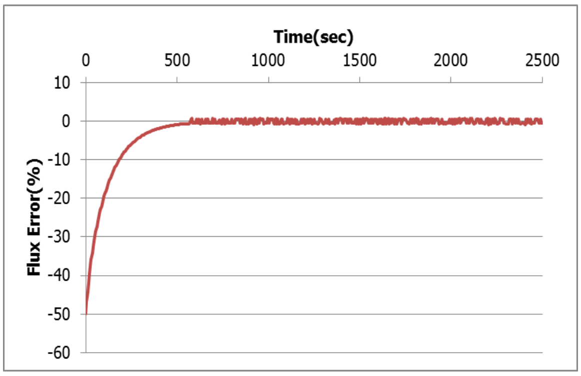

Fig. 1. Maximum Thermal Flux Error with $\mathbf{v}_0$ = 1%.

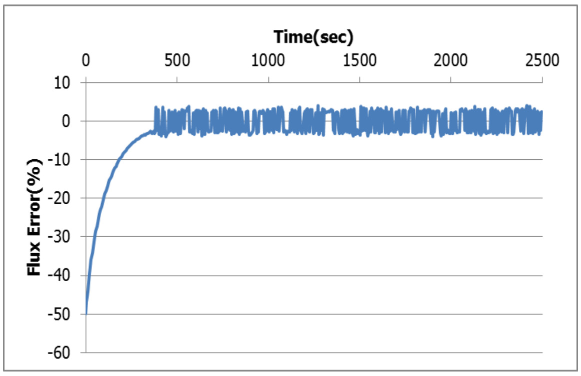

Fig. 2. Maximum Thermal Flux Error with $\mathbf{v}_0$ = 5%.

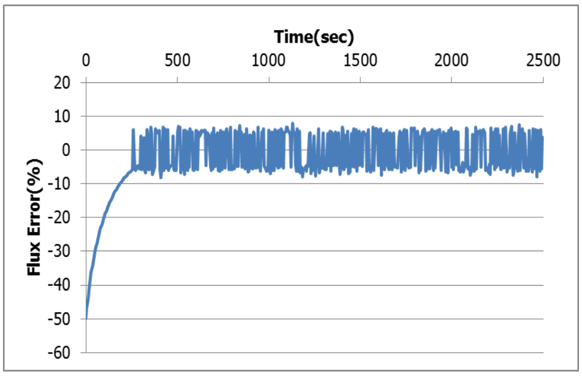

Fig. 3. Maximum Thermal Flux Error with $\mathbf{v}_0$ = 10%

3. Conclusions

Extended Kalman filter is effective tool to reduce measurement error of Rhodium SPND's and even simple FDM to solve time dependent neutron diffusion equation can be an effective measure. This method reduces random errors of detectors and can follow reactor power level without cross-section change. It means monitoring system may not calculate cross-section at every time steps and computing time will be shorten.

To minimize delay of Rhodium SPND's conversion function *h* should be evaluated in next study. Neutron and Rh-103 reaction has several decay chains and half-lives over 40 seconds causing delay of detection. Time dependent neutron diffusion equation will be combined with decay chains.

Power level and distribution change corresponding movement of control rod will be tested with more complicated reference code as well as xenon effect. With these efforts, final result is expected to be used as a powerful monitoring tool of nuclear reactor core.

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