Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 29-30, 2014

A Proposal of Estimation Methodology to Improve Calculation Efficiency of Sampling-based Method in Nuclear Data Sensitivity and Uncertainty Analysis

Myung Sub SONG[a], Song Hyun KIM[a], Jae Man NOH[b], and Jong Kyung KIM[a,*]
[a]*Department of Nuclear Engineering, Hanyang University, Seoul, Korea,*
[b]*Korea Atomic Energy Research Institute, 150 Deokjin, Yuseong, Daejeon 305-353, Korea*
**Corresponding Author: jkkim1@hanyang.ac.kr*

1. Introduction

Recently, the response uncertainty caused by the nuclear data uncertainties has received attention in reactor core analysis [1]. To enhance the reliability of reactor core design and evaluation of reactor characteristics, the nuclear data uncertainty should properly considered. Sampling-based method [2] is one of the response uncertainty analysis method caused by the nuclear data uncertainty. The uncertainty with the sampling-based method is evaluated by repeating transport calculations with a number of cross section data sampled from the covariance uncertainty data. In the transport calculation with the sampling-based method, the transport equation is not modified; therefore, all uncertainties of the responses such as k_{eff}, reaction rates, flux and power distribution can be directly obtained all at one time without code modification. However, a major drawback with the sampling-based method is that it requires expensive computational load for statistically reliable results (inside confidence level 0.95) in the uncertainty analysis [3]. The purpose of this study is to develop a method for improving the computational efficiency and obtaining highly reliable uncertainty result in using the sampling-based method with Monte Carlo simulation. The proposed method is a method to reduce the convergence time of the response uncertainty by using the multiple sets of sampled group cross sections in a single Monte Carlo simulation. The proposed method was verified by estimating GODIVA [4] benchmark problem and the results were compared with that of conventional sampling-based method.

2. Methods

For the sensitivity and uncertainty analysis with the sampling-based method, at first, each response is calculated with a single set of sampled cross sections. Then, the standard deviation (uncertainty of response) of the distributed responses is statistically estimated. Hence, one response corresponded with a set of sampled group cross sections is individually produced in each transport calculation [5]. As the result, a large number of simulations are required to get the reliable uncertainty analysis result of the response. In order to reduce the high computational load in using the sampling-based method, a method based on central limit theorem is proposed in this study.

2.1. Proposed Method for S/U Analysis

Responses evaluated by independent Monte Carlo simulations can be expressed as shown Eq. (1).

$$R_N(\alpha_1), R_N(\alpha_2), \dots R_N(\alpha_t) \sim D_N(\mu, s'^2) \tag{1}$$

where α_t is a set of sampled group cross sections, $R_N(\alpha_t)$ is the response corresponded with α_t at N sample group, and $D_N(\mu, s'^2)$ is the distribution of the responses having mean μ and standard deviation s' at N sample group. The average response of sample group N is defined as $\overline{R_N} = \{R_N(\alpha_1) + R_N(\alpha_2) + \cdots + R_N(\alpha_t)\}/t$. From the central limit theorem, the average responses $\overline{R_N}$ (or, sample groups) has Gaussian probability distribution for t > 30 as shown in Eq. (2) [6]. Also, the average of the sample group averages can be expressed as Eq. (3).

$$\{\overline{R_1},\ \overline{R_2},\ \cdots, \overline{R_N}\} \sim N\left(\mu, \frac{s^2}{t}\right) \tag{2}$$

$$\overline{R} = \{\overline{R_1} + \overline{R_2} + \cdots + \overline{R_N}\}/\mathrm{N} \tag{3}$$

where s is the standard deviation of $\overline{R_N}$ responses. Using the property, estimating the sample variance from $N\left(\mu, \frac{s^2}{t}\right)$ is practicable. Generally, variance of the population is estimated from the variance of the sample distribution $D(\mu, s'^2)$. In this study, for the uncertainty estimation of the response, Eq. (2) is used to improve calculation efficiency as shown in Figure 1.

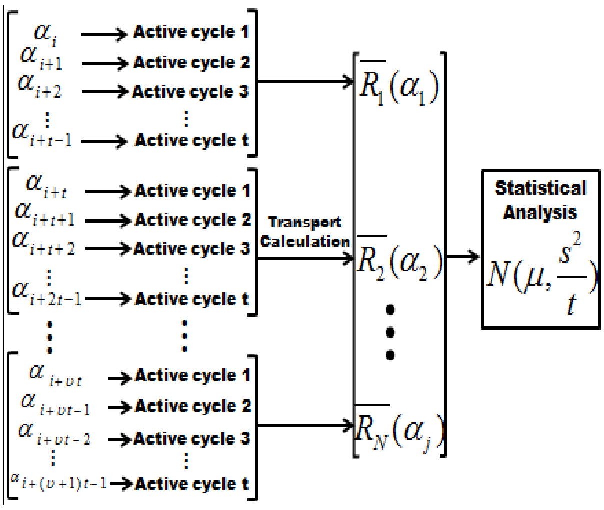

Fig. 1. Overview of Proposed Procedure for Sensitivity and Uncertainty Analysis

In the proposed method, multiple sets of sampled group cross sections (α_i) are used for a single Monte Carlo transport calculation. At first, a set of sampled group cross sections is used, and then, the cross section set is replaced to another set of sampled group cross sections. This procedure is repeated until the end of the active cycle *t*. As the result of a calculation, an average response ($\overline{R_1}(\alpha_j)$) of transport calculation #1 is obtained. After repeating the transport calculations with *t* cross section samples to *N* times, *N* numbers of the average responses are obtained [$\overline{R_1}(\alpha_j)$, $\overline{R_2}(\alpha_j)$, …, $\overline{R_N}(\alpha_j)$]. According to Eq. (2), the distribution of the average responses is approximately Gaussian having mean μ and variance s^2/t. Finally, the variance of the population (uncertainty of response) is calculated by multiplying *t* to s^2/t. Through this proposed procedure, if a transport calculation repeated *N* times with sample size *t*, the total number of the calculations is reduced to *N/t* for estimating $D(\mu, s^2)$ by comparison with the previous sampling-based method required *N* times transport calculations for the estimation.

2.2. Overall Algorithm for Criticality S/U Analysis

The overall algorithm of the proposed sampling-based method is shown in Figure 2. NJOY code [7] is used to generate 44-energy-group material cross sections library (MATXS format) and 44-energy-group covariance data from ENDF-VII.1 cross section library. Cross sections of all the nuclear reactions listed in the multi-group covariance data are automatically sampled by developed program in this study. The cross section sampling was pursued with the method in the previous study [8]. Monte Carlo transport calculations are performed by McCARD [9] with ENDF/B-VII.1 cross section library. In order to compare calculation efficiency between previous and new method proposed in this study, criticality uncertainty analysis for GODIVA benchmark problem was performed with the each method.

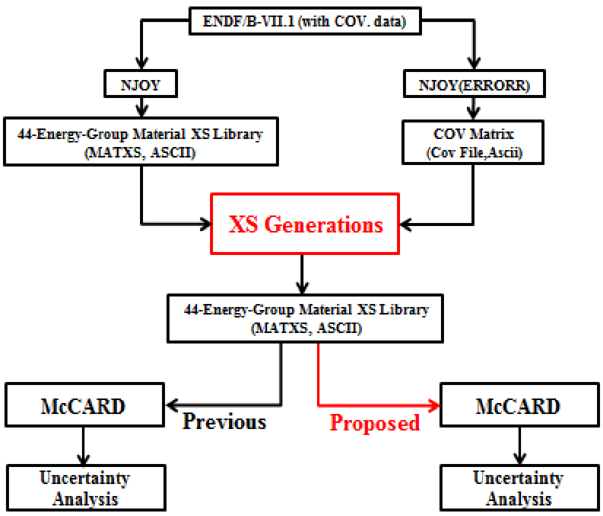

Fig. 2. Flowchart for Criticality Uncertainty analysis with Sampling-based Method

3. Results and discussion

After performing criticality calculations for each case, σ_{keff}/k_{eff} values are evaluated. The numerical results of the calculations are given as Tables I and II. All cross sections are independently sampled, and thus, there are no correlations in each other. Results with *N* = 1000 in Table I are assumed to be true values since those have the smallest confidence interval among the all cases. Comparing the results in the table, the averages of k_{eff} evaluated with proposed method are more close to the true values than those of previous method with regard to each calculation time. Convergence speed of the σ_{keff}/k_{eff} values evaluated by proposed method is higher than that of the σ_{keff}/k_{eff} estimated by previous method. As shown in Table II, the uncertainty analysis results for *N/t*=100 case seems to be converged to the true values (inside of the 95% confidence interval boundary). Overall, these analyses show that proposed sampling-based method considerably improves calculation efficiency.

Table I. U/A Results of k_{eff} by Previous Sampling-based Method with *N* Transport Calculation

N	Average k_{eff}	σ_{keff}/k_{eff} (%)	Calculation Time (h)	*Relative Difference (%)
10	1.00674	1.676	0.419	6.21
20	1.01296	1.841	0.844	16.66
30	1.00725	1.203	1.264	23.76
40	1.00653	1.382	1.693	12.42
50	1.00701	1.467	2.120	7.03
1000	1.00934	1.578	42.731	-

* Relative Difference is calculated regarding 1.578 (*N=1000*) as true value.

Table II. U/A Results of k_{eff} by Proposed Sampling-based Method with *N/t* Transport Calculation

N/t	Average k_{eff}	σ_{keff}/k_{eff} (%)	Calculation Time (h)	*Relative Difference (%)
10	1.00718	1.840	0.420	16.6
20	1.00801	1.426	0.846	9.63
30	1.00766	1.648	1.261	4.44
40	1.00819	1.623	1.689	2.81
50	1.00793	1.599	2.121	1.33
100	1.00887	1.557	4.254	1.33

* Relative Difference is calculated regarding 1.578 (*N=1000*) as true value.

4. Conclusions

In this study, sampling-based method based on central limit theorem is proposed to improve calculation efficiency by reducing the number of repetitive Monte Carlo transport calculation required to obtain reliable uncertainty analysis results. Each set of sampled group cross sections is assigned to each active cycle group in a single Monte Carlo simulation. The criticality uncertainty for the GODIVA problem is evaluated by the proposed and previous method. The results show that the proposed sampling-based method can efficiently decrease the number of Monte Carlo simulation required for evaluate uncertainty of k_{eff}. It is expected that the proposed method will improve computational efficiency of uncertainty analysis with sampling-based method.

Acknowledgement

This work was supported in part by NHDD Project coordinated by Korea Atomic Energy Research Institute (NRF-2012M2A8A2025679), Energy Efficiency & Resources of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by Korea government Ministry of Knowledge Economy (2012T100100477), and Innovative Technology Center for Radiation Safety (iTRS).

REFERENCES

[1] G. Aliberti et al., "Nuclear Data Sensitivity, Uncertainty and Target Accuracy Assessment for Future Nuclear Systems," Annals of Nuclear Energy, vol. 33, 700-733, 2006.

[2] Helton, J. C. and F. J. Davis, "Illustration of sampling based methods for uncertainty and sensitivity analysis," Risk Analysis, 22.3, 591-622, 2002.

[3] Cacuci, Dang G and Mihaela lonescu-Bujor, "A comparative review of sensitivity and uncertainty analysis of large-scale systems. II: statistical methods," Nuclear Science and Engineering, 147.3, 204-217, 2004.

[4] BRIGGS, J. Blair, et al., "International Handbook of Evaluated Criticality Safety Benchmark Experiments," Report NEA/DOC (95), 2004.

[5] Helton, J. C, et al., "Survey of sampling-based methods for uncertainty and sensitivity analysis," Reliability Engineering & System Safety, 91.10, 1175-1209, 2006.

[6] Billingsley, Patrick, "Convergence of probability measures," John Wiley & Sons, Vol. 493, 2009.

[7] R. E. MacFarlane and D. W. Muir, "The NJOY Nuclear Data Processing System: Version 91," Unknown 1, 1994.

[8] D. L. Smith and N. Otuka, "Experimental Nuclear Reaction Data Uncertainties: Basic Concepts and Documentation," Nuclear Data Sheets, Vol. 113, 3006-3053, 2012.

[9] Shim, Hyung Jin, et al., "McCARD: Monte Carlo code for advanced reactor design and analysis," Nuclear Engineering and Technology, 44.2, 161-176, 2012