

Generation of Optimal Basis Functions for Reconstruction of Power Distribution

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1. Introduction

This paper describes the Group Method of Data Handling (GMDH) algorithm to reconstruct 20-node axial core power shapes from five-level in-core detector powers. Conventional methods in this field are parameter identification framework by adopting fixed basis functions and the determination of parameters weighted to each terms of known functions like Fourier series or spline fitting. This study proposes GMDH to find not only the best functional form but also the optimal parameters those describe the power distribution most accurately. A total of 1,060 cases of axially 1-dimensional core power distributions of 20-nodes are generated by 3-dimensional core analysis code covering BOL to EOL core burnup histories to validate the method. Axially five-point box powers at in-core detectors are considered as measurements. The reconstructed axial power shapes using GMDH method are compared to the reference power shapes. The results show that the proposed method is very robust and accurate compared with spline fitting method.

2. Methods and Results

The on-line reactor core monitoring/protection system performs important safety functions. It receives measurement data from in-core and/or ex-core detectors and analyzing them on real-time, providing important core parameters to operators. The application of the fifth order Fourier series method has long history of implementation [1].

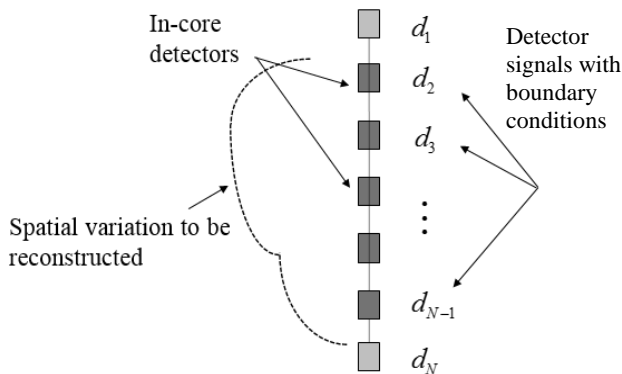


Figure 1. One-dimensional spatial detector system

The cubic spline synthesis is being used with improved accuracy in OPR1000/APR1400 power plants in Korea.[2] In these framework, the detector signals are transformed into the preset Fourier series or cubic spline

form with weighting coefficients by evaluating the matrix product of a pre-set parameter matrix and the vector of the detector signals. The axial power distribution is then constructed by forming the sum, at each axial node, of the Fourier or cubic spline functions times their respective coefficients. The accuracy of current deterministic fitting methods highly depends on the number of detector signals and the functional forms used also appears to be inaccurate for certain axial shapes, especially saddle power shapes Figure 1 shows a general axial detector string used to reconstruct the continuous axial power distribution.

2.1 Fitting Method

The axial power shape is given by the weighted sum of basis functions and the weights as in Eq. 1.

$$P(z) = \sum_{j=1}^N a_j x_j(z) \quad (1)$$

where $P(z)$: power at core height z , a_j : amplitude coefficient, $x_j(z)$: basis function at a specific location.

The key element of this framework is that amplitude coefficient vector \mathbf{a} to inter-connect pre-set basis function and detector measurements defined as follows.

$$\mathbf{a} = \mathbf{H}^{-1} \mathbf{d} \quad (2)$$

where \mathbf{a} : vector of basis function amplitudes, \mathbf{H}^{-1} : pre-calculated matrix for a selected basis function node set, \mathbf{d} : vector of detector responses and boundary point powers. This kind of problem is to identify the structure and parameters of the detection system.

2.2 GMDH Algorithm

The GMDH method is based on the evolutionary algorithm selecting the optimal representation of polynomial support functions that describes the optimal functional form of given measurements according to a specified criterion.[4] The algorithm starts with the construction of polynomial support functions of non-linear bases but linear-in-parameters known as the Kolmogorov-Gabor polynomial [5],

$$P(z) = a_0 + \sum_{i=1}^N a_i x_i + \sum_{i=1}^N \sum_{j=1}^N a_{ij} x_i x_j + \sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N a_{ijk} x_i x_j x_k \dots \quad (3)$$

where $P(z)$: power at core height z , a_j : polynomial coefficient, $x_j(z)$: detector measurements defined at specific location and N : number of detections. The multilayered iterative algorithm is applied to find the

structure of polynomials. The optimal solution for the estimation $\hat{\mathbf{a}} = [\hat{a}_0, \hat{a}_1, \hat{a}_2, \dots]^T$ is obtained by the conventional least squares method.

The identified polynomial of $P(10)$ for the 20-points reconstruction becomes

$$\begin{aligned}
 P(10) = & -14.0257 + 0.1016 x_1 + 0.0050 x_1 x_2 \\
 & - 0.0008 x_1 x_3 + 0.0037 x_1 x_4 + 0.0061 x_2 x_3 \quad (4) \\
 & + 0.0003 x_2 x_4 + 0.0023 x_2 x_5 - 0.0056 x_3 \\
 & + 0.0041 x_3 x_4 + 0.0007 x_3 x_5 + 0.0667 x_4 \\
 & + 0.0014 x_4 x_5 + 0.1327 x_5.
 \end{aligned}$$

Eq. (4) means that the 10th node power can be reconstructed by the in-core detector measurements of $x_1 \sim x_5$. The powers of $P(1) \sim P(20)$ can be given in a similar way. Eq. (4) is a single set of GMDH function to reconstruct $P(10)$ for all of the 1,060 power distributions. Figs. 2~4 shows the reconstruction results for various core states. GMDH results give nearly perfect and robust reconstruction of the power shapes.

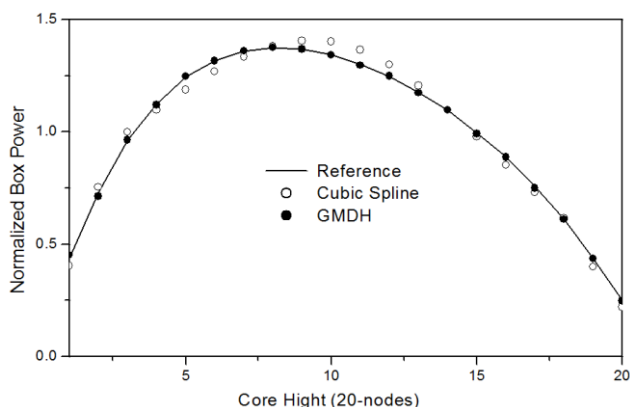


Fig. 2. BOL power shape reconstruction

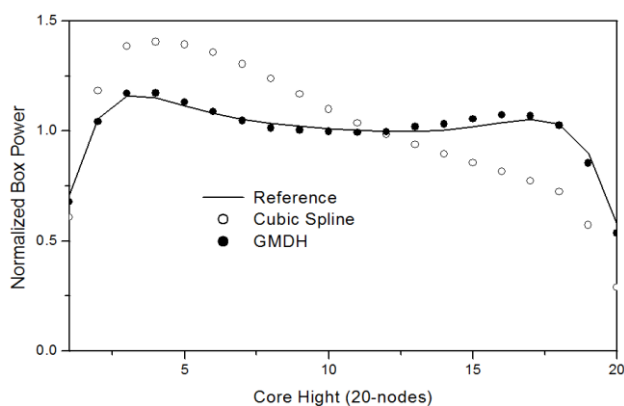


Fig. 3. Saddle power shape reconstruction

The average RMS error of GMDH polynomial is 0.0428 and that of cubic spline function is 0.1489. The maximum RMS error of GMDH polynomial is 0.5685 far smaller than that of cubic spline function with the value of 1.8894. Figure 5 describes the RMS errors for cubic spline and GMDH basis function fittings.

3. Conclusions

It is shown that the GMDH analysis can give optimal basis functions for core power shape reconstruction. The in-core measurements are the 5 detector snapshots and the 20-node power distribution is successfully reconstructed. The effectiveness of the method is demonstrated by comparing the results of spline fitting for BOL, saddle and top-skewed power shapes.

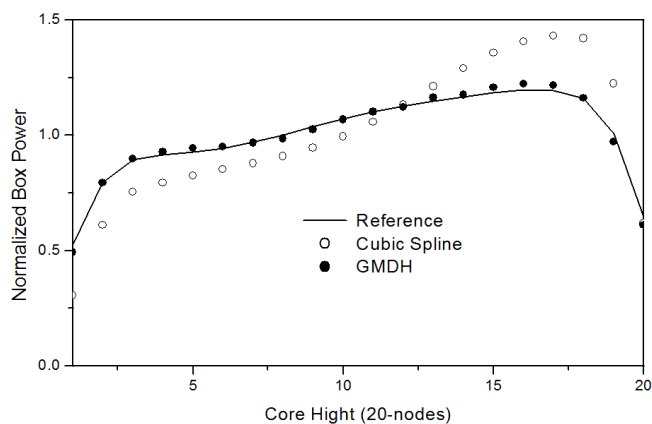


Fig. 4. Top-skewed power shape reconstruction

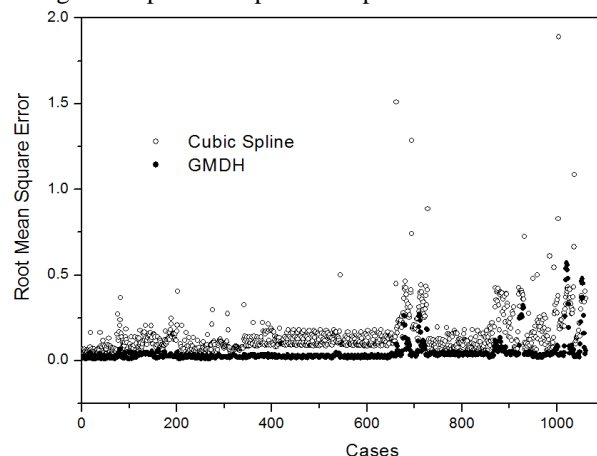


Fig. 5. Root mean square errors

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