3-D Whole-Core Transport Calculation with 3D/2D Rotational Plane Slicing Method

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1. Introduction

Various research groups challenged to develop efficient and fast 3-D whole-core transport calculation techniques for nuclear reactor analysis [1]. Monte Carlo method always has been available for this purpose but computational cost is a barrier up to these days. Use of the method of characteristics (MOC) is very popular due to its capability of heterogeneous geometry treatment and widely used for 2-D core calculation, but direct extension of MOC to 3-D core is not so attractive due to huge calculational cost.

2-D/1-D fusion method [2,3] was very successful for 3-D calculation of current generation reactor types (highly heterogeneous in radial direction but piece-wise homogeneous in axial direction).

In this paper, 2-D MOC concept is extended to 3-D core calculation with little modification of an existing 2-D MOC code. The key idea is to suppose 3-D geometry as a set of many 2-D planes like a phone-directory book. Dividing 3-D structure into a large number of 2-D planes and solving each plane with a simple 2-D S_N transport method would give the solution of a 3-D structure.

This method was developed independently at KAIST [4] but it is found that this concept is similar with that of 'plane tracing' in the MCCG-3D code [5]. Besides the difference in details in implementation, the characteristic difference is in the 2-D S_N transport solver that uses a hybrid approach of linear characteristics (LC) method in axial direction and diamond difference (DD) method in radial direction.

The method developed was tested on the 3-D C5G7 OECD/NEA benchmark problem and compared with the 2-D/1-D fusion method. Results show that the proposed method is worth investigating further.

2. Method Basics

The 3-D structure can be interpreted a set of 2-D plane structures. Usually a nuclear reactor core is very heterogeneous along radial direction but piece-wise homogeneous in axial direction.

Main concept of this method is as follows: (i) Slicing the 3-D core along a set of characteristic planes as in 2-D MOC calculation. Then we obtain each 2-D plane that is with rectangular mesh cells with heterogeneity. (ii) Solving this 2-D plane problem with a well-known 2-D S_N transport method is very straightforward. (iii) Summarizing (accumulating) these 2-D plane results become 3-D whole-core solution.

2.1 2-D Plane Slicing of 3-D Core

Based on this idea, we slice the 3-D core vertically along the characteristic plane. For example, consider four fuel pin problem as in Fig. 1(a).

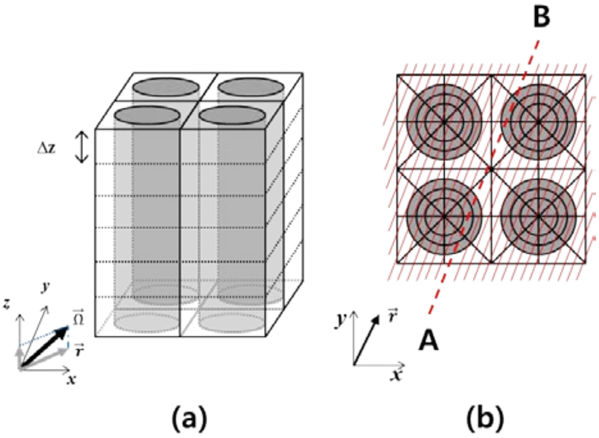

Fig. 1. Four-pin problem and its top view.

Fig. 1(b) is the top view of this problem. Each cell is divided into fine mesh cells with few circles and radial lines. The red lines drawn vertically become characteristic planes along the discrete angle. The 'plane' AB is chosen for the slicing plane. Fig. 2 is the cross section along this plane. This sliced sheet has rectangular mesh cells with various aspect ratios and material heterogeneity as shown.

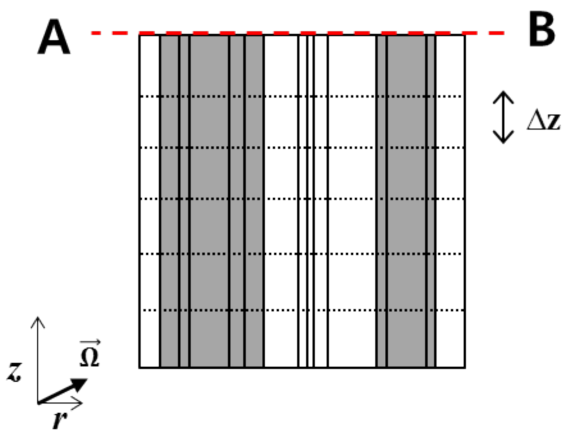

Fig. 2. Cross section along the slicing plane AB.

Slicing is successively done for every characteristic plane of equi-azimuthal angles. Then we have a set of 2-D sliced sheets.

2.2 2-D S_N Transport Solver

Once the formulation of the sheets is done, next step is to perform 2-D S_N transport calculation for each plane.

Since DD scheme can produce negative flux for optically thick region, this may lead to negative partial current at the interface which may harm convergence of the acceleration (p-CMFD) method. To avoid this situation, small mesh size must be maintained in both directions. In this research, instead of using DD scheme, we formulate another scheme which uses linear characteristic (LC) scheme in axial direction and DD scheme in radial direction (LC/DD). Since LC scheme maintains accuracy and rarely produces negative flux, this approach allows us to use larger axial cell size.

Neutron transport equation in rectangular mesh can be written as

$$\sin\theta\frac{d}{dr}\psi+\cos\theta\frac{d}{dz}\psi+\sigma_t\psi=Q+T(z-L_z/2), \quad (1)$$

where θ is a discrete polar angle, and Q is average source term. T is a gradient of the source in z direction. L_R and L_Z are cell size in radial and axial direction, respectively. We assumed the source term has linear distribution only in axial direction since the cell size is usually larger in axial direction.

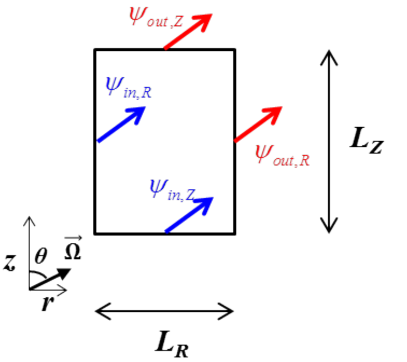

Fig. 3. Rectangular cell description

For axial direction, by moving the leakage term in radial direction to the right hand side, Eq. (1) can be rewritten as,

$$\cos\theta\frac{d}{dz}\psi+\sigma_t\psi=Q+T(z-L_z/2)-\sin\theta\frac{d}{dr}\psi. \quad (2)$$

If we integrate Eq. (2) in radial direction assuming $\psi_{in,R}$, $\psi_{out,R}$ are constant along axial direction, we obtain,

$$\cos\theta\frac{d}{dz}\psi+\sigma_t\psi=$$
$$Q+T(z-L_z/2)-\frac{\sin\theta}{L_R}(\psi_{out,R}-\psi_{in,R}). \quad (3)$$

By solving Eq. (3), the outgoing angular flux in axial direction can be expressed as

$$\psi_{out,Z}=\psi_{in,Z}e^{-\sigma_t L_Z/\cos\theta}+\frac{Q}{\sigma_t}(1-e^{-\sigma_t L_Z/\cos\theta})$$
$$+\frac{TL_z}{\sigma_t}[1-(1/2+1/(\sigma_t L_Z/\cos\theta))(1-e^{-\sigma_t L_Z/\cos\theta})] \quad (4)$$
$$-\frac{\sin\theta}{\sigma_t L_R}(\psi_{out,R}-\psi_{in,R})(1-e^{-\sigma_t L_Z/\cos\theta}).$$

Similarly, outgoing angular flux in radial direction can be written, using DD relation, as

$$\psi_{out,R}=(1+\frac{\sigma_t L_R}{2\sin\theta})^{-1}[(\frac{\sigma_t L_R}{2\sin\theta}-1)\psi_{in,R}+Q\frac{L_R}{\sin\theta}$$
$$-\frac{\cos\theta}{L_Z}\frac{L_R}{\sin\theta}(\psi_{out,Z}-\psi_{in,Z})]. \quad (5)$$

Eq. (4) and Eq. (5) are 2 by 2 matrix equation with two unknowns of outgoing angular fluxes and can be solved easily. Once the matrix equation is solved, the cell average flux is obtained from neutron balance equation:

$$\bar{\psi}=\frac{L_Z\sin\theta(\psi_{out,R}-\psi_{in,R})+L_R\cos\theta(\psi_{out,Z}-\psi_{in,Z})-QL_ZL_R}{\sigma_t L_Z L_R}. \quad (6)$$

3. Numerical Results

The Rotational Plane Slicing (RPS) method described above was tested on OECD/NEA 3D C5G7 benchmark problem [1]. This problem has three cases (Unrodded, Rodded A, Rodded B) according to the control rod position. We compared the results of the RPS method with 2D/1D fusion method solution.

Fuel pin-cell is divided with 4 circles (0.33, 0.43, 0.55, 0.6 cm) and 32 radial lines (12.5 degree interval) as described in Fig. 4. Reflector cell is divided with 8 by 8 mesh grid.

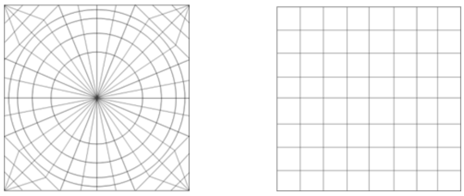

Fig. 4. Cell division for C5G7 pin cell/reflector cell.

Axial cell size is 1.785cm resulting in 36 mesh cells in axial direction. Four polar angles and eight azimuthal angles are used for one octant. Since every cell at the same x-y position has to have the same mesh structure, reflector cells above the fuel region are also has the same geometric structure with fuel pin cell.

Effective multiplication factors for each case are listed in Table I. The RPS method gave fairly accurate results in terms of multiplication factor.

Table I: k_{eff} values for each case

Case	Unrodded	Rodded A	Rodded B
MCNP (Ref.)	1.14308 (±6pcm)*	1.12806 (±6pcm)*	1.07777 (±6pcm)*
RPS Method	1.14291 (-17pcm)	1.12801 (-5pcm)	1.07766 (-11pcm)
2D/1D Fusion Method**	1.14301 (-6pcm)	1.12819 (11pcm)	1.07785 (7pcm)

*98% confidence interval of the reference MCNP solution
** Results from [6]

The assembly power is compared and listed in Table II. The RPS method gives more accurate values for every case, but if we compare the power only in the third assembly slice (near top reflector), RPS method is little bit worse in Rodded A case.

Table II: Percent error of assembly power

Case		Unrodded	Rodded A	Rodded B
MCNP (Ref.)	Max. Pin	2.481 (±0.14)	2.253 (±0.14)	1.835 (±0.19)
	Inner UO2	491.15 (±0.29)	461.18 (±0.28)	395.43 (±0.26)
	MOX	212.70 (±0.21)	221.71 (±0.22)	236.62 (±0.23)
	Outer UO2	139.46 (±0.15)	151.39 (±0.16)	187.34 (±0.18)
RPS	Max. Pin	0.002	-0.074	0.012
	Inner UO2	-0.013	-0.005	0.006
	MOX	-0.002	-0.001	-0.007
	Outer UO2	0.053	0.019	0.003
2D/1D	Max. Pin	-0.31	-0.38	-0.41
	Inner UO2	-0.27	-0.28	-0.36
	MOX	0.20	0.19	0.19
	Outer UO2	0.34	0.29	0.34

Table III. Percent error of power in 3rd assembly slice

Case		Unrodded	Rodded A	Rodded B
MCNP (Ref.)	Max. Pin	0.491 (±0.30)	0.304 (±0.47)	0.217 (±0.56)
	Inner UO2	97.90 (±0.13)	56.26 (±0.09)	41.12 (±0.08)
	MOX	42.88 (±0.10)	39.23 (±0.09)	29.42 (±0.08)
	Outer UO2	27.79 (±0.07)	28.21 (±0.07)	30.68 (±0.07)
RPS	Max. Pin	-0.249	0.230	-0.606
	Inner UO2	-0.038	-0.103	-0.214
	MOX	-0.094	-0.192	-0.156
	Outer UO2	-0.088	-0.101	0.026
2D/1D	Max. Pin	-0.69	-0.02	-1.41
	Inner UO2	-0.41	-0.35	-0.77
	MOX	-0.03	0.12	0.20
	Outer UO2	0.12	0.27	0.39

The results of 2D/1D fusion method are given from a separate code which is developed independently [6]. Therefore, the comparison of the computing time has little meaning and not given here. It is obvious that the RPS method requires more computational effort than the 2D/1D fusion method. Approximately 12 hours are taken for the RPS method (36 axial planes, 4 polar angles) with 12 cores, whereas around 3 hours are taken for the 2D/1D fusion method (12 axial planes, 2 polar angles) in the same computer system (Intel Xeon X5670@2.93GHz ×2).

4. Conclusions

A new approach to 3-D whole-core transport calculation is described and tested. By slicing 3-D structure along characteristic planes and solving each 2-D plane problem, we can get 3-D solution. The numerical test results indicate that the new method is comparable with the 2D/1D fusion method and outperforms other existing methods. But more fair comparison should be done in similar discretization level. As a concluding remark, there are rooms for further research to improve the efficiency.

Acknowledgment

This work was supported in part by Nuclear R&D Program through the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No. NRF-2014M2A8A2021222).

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