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Investigation of Some Variations of p-CMFD Acceleration Method in Continuous-Energy Monte Carlo Calculation

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1. Introduction

To overcome slow source convergence in the whole-core transport calculation, many acceleration methods have been proposed over the past decades in the case of deterministic calculation. Coarse-Mesh Finite Difference (CMFD) method is a well-known acceleration method which was originally proposed with MOC calculation [1].

As an alternative to the CMFD method, partial current-based CMFD (p-CMFD) which is more physically based than CMFD was proposed [2,3]. It shows significant improvement over the CMFD method in acceleration in terms of convergence and stability [4,5]. Also, the p-CMFD method effectively accelerates source convergence of fission source distributions in continuous-energy MC calculation [6,7]. The readers are advised to refer to Ref. 7 for its comparison with other acceleration methods.

In Ref. 2, a variant of the p-CMFD method was also suggested. The variant includes an additional surface flux term to express partial current in the 'closure' relation (see Eq. (1) below). Recently, this variant of the p-CMFD method is studied in acceleration of Monte Carlo calculation [8].

In this paper, some variations of the p-CMFD method are investigated and applied to accelerate source convergence of continuous-energy MC calculation, and their performances are assessed.

2. Variations of p-CMFD

2.1 Variation 1

To describe the variant of p-CMFD acceleration method [2], let us consider coarse-mesh cell i and i+1. At the right interface i+1/2 of cell i, the outgoing and incoming partial currents are related with cell-average scalar fluxes and surface scalar flux at the interface as:

$$\bar{J}^{+}_{G,i+1/2} = \frac{\bar{\phi}_{G,i+1/2}}{4} - \frac{\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right)}{2} + \hat{D}^{+}_{G,i+1/2}\bar{\phi}_{G,i}\,, \tag{1a}$$

$$\bar{J}^{-}_{G,i+1/2} = \frac{\bar{\phi}_{G,i+1/2}}{4} + \frac{\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right)}{2} + \hat{D}^{-}_{G,i+1/2}\bar{\phi}_{G,i+1}\,, \tag{1b}$$

where $\tilde{D}_{G,i+1/2}$ is arbitrary but usually chosen as coupling coefficient determined in the finite difference method, and the two correction factors $\hat{D}^{+}_{G,i+1/2}$ and $\hat{D}^{-}_{G,i+1/2}$ are defined to preserve respective partial currents as:

$$\hat{D}^{+}_{G,i+1/2} = \frac{2\bar{J}^{+}_{G,i+1/2} + \tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right) - \frac{\bar{\phi}_{G,i+1/2}}{2}}{2\bar{\phi}_{G,i}}\,, \tag{2a}$$

$$\hat{D}^{-}_{G,i+1/2} = \frac{2\bar{J}^{-}_{G,i+1/2} - \tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right) - \frac{\bar{\phi}_{G,i+1/2}}{2}}{2\bar{\phi}_{G,i+1}}\,, \tag{2b}$$

where the partial currents, cell-average fluxes, and surface-average flux are obtained from the high-order transport calculation. Although there is the additional surface flux term in Eq. (1) compared to the original p-CMFD, the net current in the variant is same with that of the original p-CMFD as:

$$\bar{J}_{G,i+1/2} = -\tilde{D}_{G,i+1/2}\left(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}\right) - \left(\hat{D}^{-}_{G,i+1/2}\bar{\phi}_{G,i+1} - \hat{D}^{+}_{G,i+1/2}\bar{\phi}_{G,i}\right). \tag{3}$$

When Fourier convergence analysis in deterministic calculation setting was performed for Variation 1, the results became similar to those of CMFD, thus it was not pursued further. When the high-order equation is solved by MC calculation, tally of the surface flux is ill-posed and thus implementation of Variation 1 would be problematic. Therefore, we consider in this study the following variation.

2.2 Variation 2

To get around the surface flux, the following relation from P_1 approximation is used:

$$\bar{J}^{tot}_{G,i+1/2} \equiv \bar{J}^{+}_{G,i+1/2} + \bar{J}^{-}_{G,i+1/2} \simeq \frac{\bar{\phi}_{G,i+1/2}}{2}\,. \tag{4}$$

By substituting Eq. (4) into Eq. (2), the two correction factors are then expressed as:

$$\hat{D}^{+}_{G,i+1/2} = \frac{2\bar{J}^{+}_{G,i+1/2} + \tilde{D}_{G,i+1/2}(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}) - \bar{J}^{tot}_{G,i+1/2}}{2\bar{\phi}_{G,i}}\,, \tag{5a}$$

$$\hat{D}^{-}_{G,i+1/2} = \frac{2\bar{J}^{-}_{G,i+1/2} - \tilde{D}_{G,i+1/2}(\bar{\phi}_{G,i+1}-\bar{\phi}_{G,i}) - \bar{J}^{tot}_{G,i+1/2}}{2\bar{\phi}_{G,i+1}}\,, \tag{5b}$$

Eq. (5) turns out that the formulation is very close to $\hat{D}_{G,i+1/2}$ in the CMFD method. The numerators in Eq. (5) are exactly the same with that of the CMFD method, while the denominators are a little bit different ($2\bar{\phi}_{G,i} \to \bar{\phi}_{G,i}+\bar{\phi}_{G,i+1}$ and $2\bar{\phi}_{G,i+1} \to \bar{\phi}_{G,i}+\bar{\phi}_{G,i+1}$, and thus $\hat{D}^{+}_{G,i+1/2} = \hat{D}^{-}_{G,i+1/2}$ in the CMFD method).

2.3 'Generalization' of Variation 2

By introducing an arbitrary factor α into Eq. (5), Variation 2 may be generalized as:

$$\hat{D}^{+}_{G,i+1/2} = \frac{2\bar{J}^{+}_{G,i+1/2} + \tilde{D}_{G,i+1/2}(\bar{\phi}_{G,i+1} - \bar{\phi}_{G,i}) - \alpha \bar{J}^{tot}_{G,i+1/2}}{2\bar{\phi}_{G,i}} \quad (6a)$$

$$\hat{D}^{-}_{G,i+1/2} = \frac{2\bar{J}^{-}_{G,i+1/2} - \tilde{D}_{G,i+1/2}(\bar{\phi}_{G,i+1} - \bar{\phi}_{G,i}) - \alpha \bar{J}^{tot}_{G,i+1/2}}{2\bar{\phi}_{G,i+1}} \quad (6b)$$

Note that when $\alpha = 0$, Eq. (6) becomes the correction factors of the original p-CMFD method. On the other hand, when $\alpha = 1$ (Variation 2), it becomes close to $\hat{D}_{G,i+1/2}$ of the CMFD method as noted above. By varying α, a number variations can be produced. In numerical results, these variations will be compared in the context of acceleration of source convergence of continuous-energy MC calculation.

3. Numerical Results

A 1-D test problem shown in Fig. 1 is solved by continuous-energy MC calculation using McSLAB [9] and source convergence is accelerated by several variants ($\alpha = -100$, -1, 0, 0.5, 0.7, and 1) of one-group p-CMFD method with two coarse-mesh cells per assembly. For each MC cycle, 2×10^5 histories are used and initial fission source distribution is set as uniform. MC tallies for p-CMFD parameters are accumulated after first 10 cycles to stabilize fission source fluctuations [7].

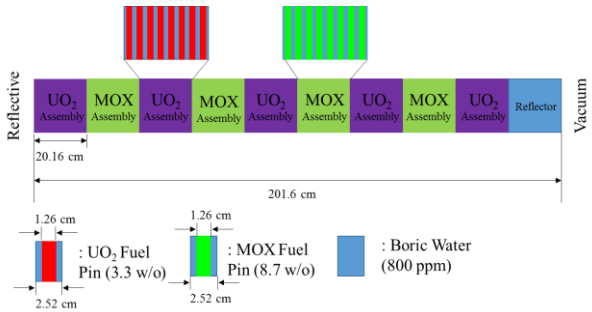

Fig. 1. 1-D thermal reactor test problem

To compare effectiveness of the variant methods, the sample means and sample standard deviations of Shannon entropy [10] for each cycle are obtained by 30 independent batch runs, as shown in Figs. 2 and 3. The reference Shannon entropy is obtained from the 30 independent batch runs of conventional MC calculation consisting of 200 inactive cycles and 50 active cycles.

Figs. 2 and 3 show the results as α varies from -100 (a large negative value) to 0.7. As α decreases from 0 to -100, slower convergence but smaller sample standard deviations of Shannon entropy are observed. When α is set to -100, the sample mean and sample standard devations of Shannon entropy become close to those of the conventional power iteration. On the other hand, as α increases from 0 to 0.7, faster convergence but larger sample standard deviations of Shannon entropy are observed. If $\alpha = 0$, it is the original p-CMFD and it shows "best" performance in terms of convergence speed and standard deviation.

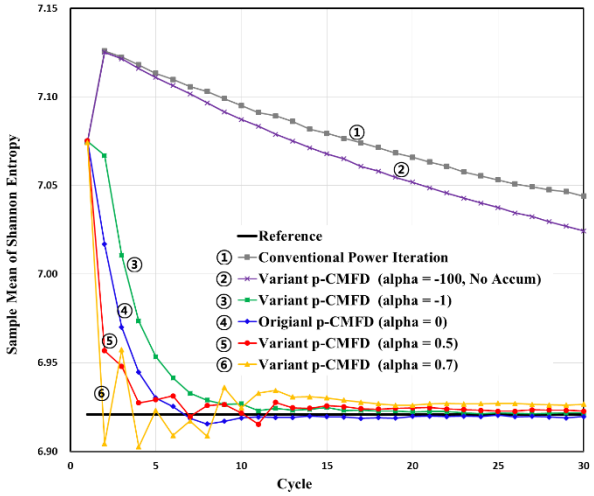

Fig. 2. The sample means of Shannon entropy for each acceleration method.

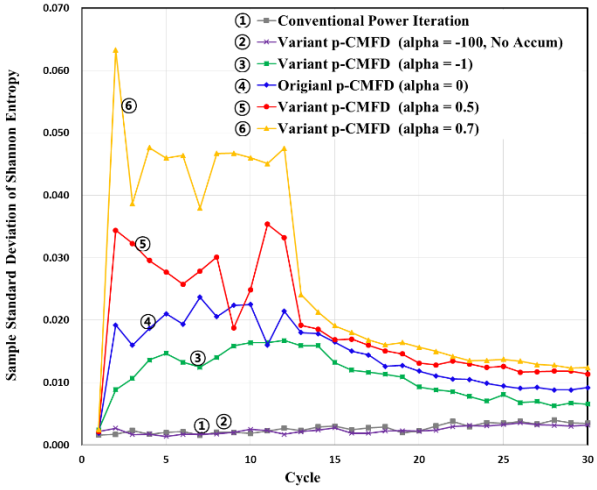

Fig. 3. The sample standard deviations of Shannon entropy for each acceleration method

Note that for $\alpha = 1$, the high-order and low-order iteration becomes unstable and fails to converge perhaps due to negative scalar flux from low-order calculation (not shown in the figures).

4. Summary and Conclusions

In this paper, the variant of p-CMFD method [2] is revisited and modified by the P_1 approximation to replace the surface flux with total current for robust tally in MC calculation. This modified variant of p-CMFD method is generalized by multiplying an arbitrary factor α to the total current.

For a 1-D continuous-energy thermal reactor problem, variants of p-CMFD method are compared for several α's. Note that for the $\alpha = 1$ case (of which Fourier

convergence analysis in deterministic calculation shows almost same characteristics with that of CMFD method), the variant p-CMFD method fails to converge due to negative scalar flux from low-order calculation.

In conclusion, compared to several variants of p-CMFD, the original p-CMFD method ($\alpha = 0$ case) is an 'optimizer' for continuous-energy MC calculation in terms of acceleration speed and source fluctuation.

As a further remark, preliminary results on 2-D problems support the same conclusions obtained in the 1-D problem above.

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