A Proposal on the Advanced Sampling Based Sensitivity and Uncertainty Analysis Method for the Eigenvalue Uncertainty Analysis

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1. Introduction

Sensitivity and uncertainty (S/U) analysis is a research field to analyze the uncertainties of the responses which is caused by the uncertainty of input parameters. For the uncertainty analysis, there are two main approaches which are deterministic and statistical methods. The deterministic approach is a deterministic method which uses perturbation theory for the uncertainty analysis. TSUNAMI [1], McCARD [2], and MUSAD [3] codes are based on the deterministic approach. In using the perturbation theory, the uncertainty of the response can be estimated by a single transport simulation, and therefore it requires small computational load. However, it has a disadvantage that the computation methodology must be modified whenever estimating different response type such as multiplication factor, flux, or power distribution. Hence, it is suitable for analyzing few responses with lots of perturbed parameters. Statistical approach is a sampling based method which uses randomly sampled cross sections from covariance data for analyzing the uncertainty of the response. XSUSA [4] is a code based on the statistical approach. The cross sections are only modified with the sampling based method; thus, general transport codes can be directly utilized for the S/U analysis without any code modifications. However, to calculate the uncertainty distribution from the result, code simulation should be enough repeated with randomly sampled cross sections. Therefore, this inefficiency is known as a disadvantage of the stochastic method. In this study, an advanced sampling method of the cross sections is proposed and verified to increase the estimation efficiency of the sampling based method.

2. Methods and Results

2.1 Proposal of Analysis Method

Let $\Sigma^{\#N}$ is nth set of sampled group cross sections from the covariance data. From the cross section set, a response $R(\Sigma^{\#N})$ can be calculated by a transport calculation. If N sets of the cross sections are calculated by transport code, N outputs are produced as the follows:

$$\Sigma^{\#} = [\Sigma^{\#1}, \Sigma^{\#2}, \Sigma^{\#3}, ..., \Sigma^{\#N}] \quad (1)$$

$$R^{\#} = [R^{\#1}(\Sigma^{\#1}), R^{\#2}(\Sigma^{\#2}), ..., R^{\#N}(\Sigma^{\#N})] \quad (2)$$

Usually, the standard deviation of the responses in Eq. (2) is evaluated for the S/U analysis. In this study, a sampling method of the cross sections is proposed, which the averaged cross section (Σ^{*v}) is used instead of a single set of the randomly sampled cross sections. The proposed scheme is shown in Fig. 1. First, the n cross sections, which are randomly sampled by covariance data, are averaged to Σ^{*v}. Then, the transport calculations are performed with the Σ^{*v} cross sections. If the response using Σ^{*v} cross section gives an average value of the responses $R^{\#u}(\Sigma^{\#u})$, the distribution of the responses R^{*} follows the central limit theory [5]; therefore, the original response uncertainty of $R^{\#}$ can be estimated from the distribution of the responses R^{*}.

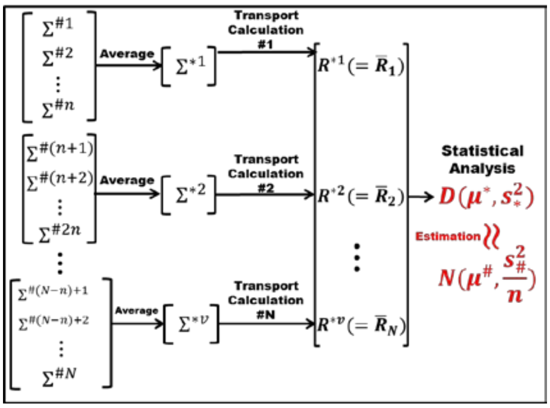

Fig. 1. Proposed Scheme for the Sampling Based S/U Analysis

To use the proposed strategy, it must be verified that the R^{*v} gives an average value of [$R^{\#1}$, $R^{\#2}$, … , $R^{\#n}$]. In Section 2.2, the verification of the proposed method is performed.

2.2 Validation of Proposed Method

To use the method proposed in Section 2.1, the response R^{*v} must be equal to the average of [$R^{\#1}$, $R^{\#2}$, … , $R^{\#n}$] responses. For the verification, it is started from the fundamental mode transport equation. The

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fundamental mode transport equation for the unperturbed reactor is expressed as the following equation:

$$H_0\psi_0 - \frac{1}{k_0}G_0\psi_0 = 0 \quad (3)$$

$$H_0^+\psi_0^+ - \frac{1}{k_0}G_0^+\psi_0^+ = 0 \quad (4)$$

And, the transport operators are given as follows;

$$H_0\psi_0 = \hat{\Omega}\cdot\nabla\psi_0 + \Sigma_t\psi_0 - \int dE'\int d\hat{\Omega}\Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi_0'$$

$$H_0^+\psi_0^+ = -\hat{\Omega}\cdot\nabla\psi_0^+ + \Sigma_t\psi_0^+ - \int dE'\int d\hat{\Omega}\Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi_0^{+\prime}$$

$$G_0\psi_0 = \chi(E)\int dE'\nu\Sigma_f(\vec{r},E')\int d\hat{\Omega}'\psi_0(\vec{r},E',\hat{\Omega}')$$

$$G_0^+\psi_0^+ = \nu\Sigma_f(\vec{r},E)\int dE'\chi(E')\int d\hat{\Omega}'\psi_0(\vec{r},E',\hat{\Omega}')$$

For the analysis of the sampling based S/U, first, the perturbed operator for the sampled cross section is defined to Eq. (5).

$$\Sigma_r^{\#u} = \Sigma_r + f_r^u(\xi,\mathrm{cov}) \quad (5)$$

where $f_r^u(\xi,\mathrm{cov})$ is a sampled value with Gaussian random variables ξ and covariance data (cov), r is the reaction type. Using the perturbation operator in Eq. (5), the transport operators can be expressed as follows:

$$H^{\#u}\psi = \hat{\Omega}\cdot\nabla\psi + \Sigma_t^{\#}\psi - \int dE'\int d\hat{\Omega}\Sigma_s^{\#u}(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi'$$

$$G^{\#u}\psi_0 = \chi(E)\int dE'\nu\Sigma_f^{\#u}(\vec{r},E')\int d\hat{\Omega}'\psi(\vec{r},E',\hat{\Omega}')$$

Eq. (3) can be expressed to the perturbed state transport equation with sampled cross section of Eq. (5) as follows:

$$H^{\#u}\psi - \frac{1}{k^{\#u}}G^{\#u}\psi = 0 \quad (6)$$

where, $H^{\#u} = H_0 + \delta H^{\#u}$, $G^{\#u} = G_0 + \delta G^{\#u}$, $k^{\#u} = k_0 + \delta k^{\#u}$, and $\psi^{\#u} = \psi_0 + \delta\psi^{\#u}$. Substituting these expressions into Eq. (6), it can be expressed as the following equation:

$$(H_0+\delta H^{\#u})(\psi_0+\delta\psi^{\#u}) - \frac{1}{k_0+\delta k^{\#u}}(G_0+\delta G^{\#u})(\psi_0+\delta\psi^{\#u}) = 0 \quad (7)$$

δ^2 is very smaller than the other terms; hence, it can be assumed to be 0. Also, $1/(k_0+\delta k^{\#u})$ term can be approximated as follows:

$$\frac{1}{k_0+\delta k^{\#u}} = \frac{1}{k_0(1+\delta k^{\#u}/k_0)} \simeq \frac{1}{k_0}(1-\frac{\delta k^{\#u}}{k_0}) \quad (8)$$

Then, Eq. (7) with Eq. (8) can be rewritten to Eq. (9).

$$-\frac{\delta k^{\#u}}{k_0^2}G_0\psi_0 \simeq (H_0 - \frac{1}{k_0}\mathrm{G}_0)\psi_0 + (\delta H^{\#u} - \frac{1}{k_0}\delta\mathrm{G}^{\#u})\psi_0 + (H_0 - \frac{1}{k_0}\mathrm{G}_0)\delta\psi_0 \quad (9)$$

The first term of right hand side in Eq. (9) is 0 with Eq. (3). ψ_0^+ is multiplied to the both side of Eq. (9) and integration over the independent variable of Eq. (9) is pursued. Then, Eq. (9) become as the following equation:

$$-\frac{\delta k^{\#u}}{k_0^2}<\psi_0^+G_0\psi_0> \simeq <\psi_0^+(\delta H^{\#u} + \frac{1}{k_0}\delta\mathrm{G}^{\#u})\psi_0> + <(H_0+\frac{1}{k_0}\mathrm{G}_0)\delta\psi_0> \quad (10)$$

If the property of the adjoint is applied to the Eq. (10), Eq. (10) is expressed to follows:

$$\frac{\delta k^{\#u}}{k_0^2} \simeq \frac{<\psi_0^+(\frac{1}{k_0}\delta\mathrm{G}^{\#u} - \delta H^{\#u})\psi_0>}{<\psi_0^+G_0\psi_0>} \quad (11)$$

From here, the averaged cross section, which is the proposed method in this study, is applied. The cross section is expressed as given in Eq. (12).

$$\Sigma_r^{*v} = \Sigma_r + \frac{1}{n}\sum_{i=1}^{n} f_r^i(\xi,\mathrm{cov}) \quad (12)$$

As the same way from Eq. (5) to Eq. (11), Eq. (13) can be derived with Eq. (12).

$$\delta k^{*v} \simeq \frac{k_0^2<\psi_0^+(\frac{1}{k_0}\delta\mathrm{G}^{*v} - \delta H^{*v})\psi_0>}{<\psi_0^+G_0\psi_0>} \quad (13)$$

From the definition of the Σ_r^{*v}, the perturbed parameters in Eq. (13) can be expressed for the $\Sigma_r^{\#u}$ as follows:

$$\delta H^{*v} = \frac{1}{n}\sum_{i=1}^{n} f_t^i(\xi,\mathrm{cov}) - \int dE'\int d\hat{\Omega}\frac{1}{n}\sum_{i=1}^{n} f_s^i(\xi,\mathrm{cov})(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega}) \quad (14)$$
$$= \frac{1}{n}(\delta H^{\#1} + \delta H^{\#2} + \ldots + \delta H^{\#n})$$

$$\delta G^{*v} = \chi(E)\int dE'\nu\frac{1}{n}\sum_{i=1}^{n} f_f^i(\xi,\mathrm{cov})\int d\hat{\Omega}'\psi_0(\vec{r},E',\hat{\Omega}') \quad (15)$$
$$= \frac{1}{n}\chi(E)\int dE'\nu\{f_f^1(\xi,\mathrm{cov}) + f_f^2(\xi,\mathrm{cov}) + \ldots + f_f^n(\xi,\mathrm{cov})\}\int d\hat{\Omega}'\psi_0(\vec{r},E',\hat{\Omega}')$$

Using Eqs. (14) and (15), Eq. (13) can be expressed as the following equation:

$$\delta k^{*v} \simeq \frac{k_0^2<\psi_0^+(\frac{1}{k_0}E[\delta\mathrm{G}^{\#u}] - E[\delta H^{\#u}])\psi_0>}{<\psi_0^+G_0\psi_0>} \quad (16)$$

Also, the average value of $\delta k^{\#u}$ can be written as the follows:

$$E[\delta k^{\#u}] \simeq \frac{1}{n}\frac{k_0^2<\psi_0^+(\frac{1}{k_0}[\delta\mathrm{G}^{\#1}+\delta\mathrm{G}^{\#2}+\ldots+\delta\mathrm{G}^{\#u}] - [\delta H^{\#1}+\delta H^{\#2}+\ldots+\delta H^{\#u}])\psi_0>}{<\psi_0^+G_0\psi_0>} \quad (17)$$
$$\simeq \frac{k_0^2<\psi_0^+(\frac{1}{k_0}E[\delta\mathrm{G}^{\#u}] - E[\delta H^{\#u}])\psi_0>}{<\psi_0^+G_0\psi_0>}$$

Based on Eqs. (16) and (17), the transport result with the proposed method has a relationship with those of conventional sampling based method as follows:

$$\delta k^{*v} = E[\delta k^{\#u}] \quad (18)$$

Eq. (18) means that the average of the perturbed multiplication factors with each sampled cross section is equal to the multiplication factor calculated with the proposed method.

2.3 Result and Analysis

For the verification, GODIVA benchmark problem [5] was estimated with the previous sampling based method and the proposed method. The calculations were performed with McCARD code [2] with ENDF-VII.1 cross section library. The S/U analysis was pursued with the condition given in Fig. 2 and Table I.

Table I: Calculation Condition for the Uncertainty Estimation

Method	Number of Sampled Cross Section Sets	Number of Transport Calculations

Previous Method	2500 #	2500 #
Proposed Method	250 #	250 #

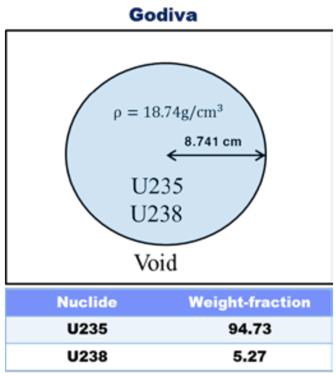
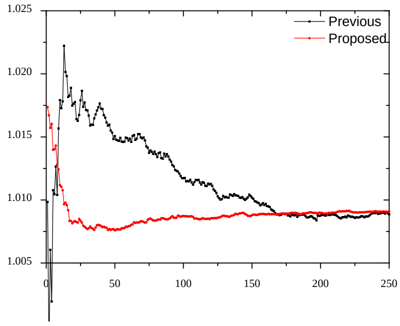


Fig. 2. Overview of the GODIVA Benchmark Problem

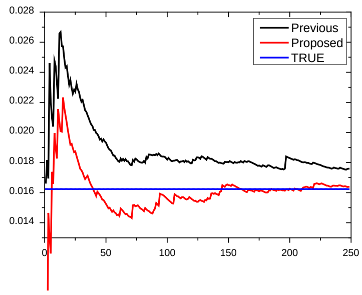
Table II shows the results which are k_{eff} calculated by the previous sampling based method with 10 cross section sets and the proposed method with single cross section set. It shows that both results give a good agreement within the Monte Carlo stochastic error. Fig. 3 is the results of k_{eff} and $\sigma(k_{eff})$ calculated with the Table I condition. It is assumed that the true value is the result calculated by the 2,500 cross section sets. The result with the proposed method was rapidly approach the true value compared with that of the previous method. Analysis shows that the proposed method gives a good accuracy as well as increase the estimation efficiency.

Table II: A Sample of k_{eff} Calculation Results with the Previous Method (*A*) and Proposed Method (*B*)

Method	Number	k_{eff}	Average	Calculation Time
A	1	1.02377	1.01597	928
	2	0.99973		
	3	1.04332		
	4	1.01983		
	5	0.99057		
	6	1.02083		
	7	1.03944		
	8	1.01458		
	9	0.9975		
	10	1.01013		
B	1	1.01603	1.01603	93



(a) k_{eff}



(b) Standard Deviation of k_{eff}

Fig. 3. Results of Multiplication Factor and Uncertainty with Conventional and Proposed Methods

3. Conclusions

In this study, to increase the estimation efficiency of the sampling based S/U method, an advanced sampling and estimation method was proposed. The main feature of the proposed method is that the cross section averaged from each single sampled cross section is used. For the use of the proposed method, the validation was performed using the perturbation theory. Finally, the benchmark problem was evaluated for the verification. The results show that the proposed method gives a good accuracy as well as increasing the estimation efficiency. It is expected that the proposed method can contribute to increase the efficiency of the sampling based sensitivity and uncertainty analysis.

Acknowledgement

This work was supported in part by NHDD Project coordinated by Korea Atomic Energy Research Institute (NRF-2012M2A8A2025679), Energy Efficiency & Resources of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by Korea government Ministry of Knowledge Economy (20121620100070), and Innovative Technology Center for Radiation Safety (iTRS).

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