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An Analysis on the Characteristic of Multi-response CADIS Method for the Monte Carlo Radiation Shielding Calculation

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1. Introduction

It is well known that the analog Monte Carlo method has a low efficiency to get the appropriate statistical error for the deep penetration problems. To overcome the limitation, lots of variance reduction methods have been developed. As the optimization of the calculation efficiency using the variance reduction techniques, Consistent Adjoint Driven Importance Sampling (CADIS) [1] was developed by Oak Ridge National Laboratory. It uses the deterministic method to calculate adjoint fluxes for the decision of the parameters used in the variance reductions. This is called as hybrid Monte Carlo method. The CADIS method, however, has a limitation to reduce the stochastic errors of all responses. The Forward Weighted CADIS (FW-CADIS) [2] was introduced to solve this problem. To reduce the overall stochastic errors of the responses, the forward flux is used. In the previous study [3], the Multi-Response CADIS (MR-CAIDS) method was derived for minimizing sum of each squared relative error. In this study, the characteristic of the MR-CADIS method was evaluated and compared with the FW-CADIS method.

2. Methods and Results

For analysis of Global variance reduction theory based on CADIS method, this section was consisted as following: 2.1 review of each method, 2.2 comparison of Global variance reduction

2.1 Theories on the CADIS Methods

2.1.1 CADIS Method

In Monte Carlo method, the particle transport equation to calculate response can be expressed as Eq. (1).

$$R = \int \psi(P)\sigma_d(P)dP = \int \psi(P)q^+(P)dP \quad (1)$$

where ψ is particle flux at phase-space (P), $\sigma_d$is some objective function to record the particle in response space, q is a source density function, and + signifies adjoint operator. From the adjoint relationship [4] in Eq. (2), the response equation can be expressed as given in Eq. (3).

$$< \psi^+, q > = < \psi, q^+ > \quad (2)$$

$$R = \int \psi^+(P)q(P)\, dP \quad (3)$$

where <> is an integration operator over all independent variable. The adjoint flux ($\psi^+(P)$), physically, means the expected contribution to response from particle in phase space. An alternative pdf $\hat{q}$ was introduced into Eq. (3) to optimize the particle weight as given Eq. (4). Then, the variance can be calculated with $\hat{q}$ as following Eq. (5).

$$R = \int \left[\frac{\psi^+(P)q(P)}{\hat{q}(P)}\right]\hat{q}(P)dP \quad (4)$$

$$var(R) = \int \left[\frac{{\psi^+}^2(P)q^2(P)}{\hat{q}^2(P)}\right]\hat{q}(P)dP - R^2 \quad (5)$$

The optimized $\hat{q}$ to get minimum variance of response is arranged by importance sampling [5] as given by Eq. (6).

$$\hat{q} = \frac{\psi^+(P)q(P)}{\int \psi^+(P)q(P)dP} = \frac{\psi^+(P)q(P)}{R} \quad (6)$$

In the biasing theory for adjusting PDF, the particle weight must be satisfied as shown Eq. (7)

$$w(P)\hat{q}(P) = w_0(P)q(P) \quad (7)$$

where $w(P)$ and w_0 are biased and initial particle densities, respectively. The initial particle weight should be 1; therefore, the optimized weights were expressed to Eq. (8).

$$w(P) = \frac{\int \psi^+(P)q(P)dP}{\psi^+(P)} = \frac{R}{\psi^+(P)} \quad (8)$$

The equation to decide the weight is called CADIS method. It shows that the particle weights have inverse relationship with to adjoint flux at arbitrary phase space. Thus, the particle in certain phase space was optimally splitting or rouletting according to contribution to response. It is well known that the efficiency of particle transport was considerably increased for a single response.

2.1.2 FW-CADIS Method

In plural response problems, the CADIS method shows irregular statistical error. To achieve uniform accuracy of the responses, Cooper and Larsen [6] suggested a method based on a principle that particle distribution should be uniform throughout the transport system. The total Monte Carlo particle density can be calculated by using following equation:

$$R' = \int \psi(P)\sigma_d{}'(P)dP \quad (9)$$

where $\sigma_d{}'(P)$ is a specific function that transform particle density into Monte Carlo particle density. To set the uniform stochastic error of the responses, Cooper and Larsen uses the following principle. The average particle weight is set to the physical particle density, and then, the Monte Carlo particle densities in the responses approximately become a constant. This can be expressed to Eq. (10).

$$m(P) \propto \frac{n(P)}{\overline{w}(P)}\left[\frac{\psi(P)}{n(P)v(P)}\right] = \psi(P)\left[\frac{1}{\overline{w}(P)v(P)}\right]$$

$$= \psi(P)\left[\frac{1}{\psi(P)}\right] \quad (10)$$

where $m(P)$, $n(P)$, $\overline{w}(P)$, $v(P)$ are Monte Carlo particle density, physical particle density, average particle weight and particle velocity, respectively. The total Monte Carlo particle density can be estimated by integration over phase space such as Eq. (11).

$$R' = \int \psi(P)\left[\frac{1}{\psi(P)}\right]dP \quad (11)$$

From the relationship between the Eq. (9) and the Eq. (11), the specific function in Eq. (9) can be defined as Eq. (12).

$$\sigma_d{}'(P) = \frac{1}{\psi(P)} = q'^{+}(P) \quad (12)$$

Thus, the adjoint source function in Eq. (1) can be determined by Eq. (12). For example, the adjoint source function with the FW-CADIS method to tally the fluxes can be estimated using Eq. (13).

$$q^{+}(\vec{r}) = \frac{1}{\int\int \psi^{+}(\vec{r},E,\hat{\Omega})q(\vec{r},E,\hat{\Omega})dEd\hat{\Omega}} \quad (13)$$

Physically, the particle importance of the FW-CADIS approach was adjusted as much as inverse of forward flux (or forward response). Hence, the uncertainties of responses are uniformly decrease by the FW-CADIS method.

2.1.3 MR-CADIS method

The goal of MR-CADIS method was same as FW-CADIS Method. In this method, first, the error of i^{th} response in discrete space is defined as Eq. (14)

$$R_{err,i} \equiv \int_{V_i} \frac{\sqrt{Var(R(\vec{r}))}}{R(\underline{r})} d\vec{r} \quad (14)$$

Then, minimizing sum of squared relative error was suggested to achieve uniform uncertainty. The sum of squared relative error can be expressed by following equation and rearranging with Eq. (5).

$$\Sigma_{i=1}^{N} R_{err,i}^2 = \Sigma_{i=1}^{N} \frac{Var[R_i]}{\mathrm{R_i^2}}$$

$$= \Sigma_{i=1}^{N}\left[\int \frac{q^2(P)\psi^{+2}_{\ i}(P)}{\hat{q}(P)R_i^2}dP\right] - N \quad (14)$$

A proper $\hat{q}$ to obtain minimum value of Eq. (14) can be calculated by importance sampling [5].

$$\hat{\mathrm{q}}(P) = \frac{q(P)\sqrt{\frac{\Sigma_{i=0}^{N}\psi^{+2}(P)}{R_i^2}}}{\int q(p)\sqrt{\frac{\Sigma_{i=0}^{N}\psi^{+2}_{\ i}(P)}{R_i^2}}dP} \quad (15)$$

Finally, the weight function of this method can be obtained by substituting Eq. (15) into (7).

$$\mathrm{w(P)} = \frac{\int q(p)\sqrt{\Sigma_{i=0}^{N}\psi^{+2}_{\ i}(P)/R_i^2}\,dP}{\sqrt{\Sigma_{i=0}^{N}\psi^{+2}(P)/R_i^2}} \quad (16)$$

2.2 Comparison Global Variance Reduction

For the comparison of FW-CADIS and MR- CADIS methods, a simple shielding problem was used as shown Fig.1. The 100 cm x 100 cm x 100 cm cubical room was surrounded by 50 cm concrete wall at each side. The density of concrete is 2.3 g/cm^3. An isotropic point source, which has 1 MeV photon energy, is located at the center of the room. To obtain the responses, 30 cm (width) x 10 cm (depth) x 30 cm (height) mesh tally was used at the right side of the room. The adjoint fluxes and forward flux are calculated by MCNPX 2.7 [7]. Fig.2 is relative error map calculated by using CADIS, FW-CADIS and MR-CADIS methods at 100 CPU time. The CADIS Method gives low accuracies in the edge of tally region. However, both FW-CADIS and MR-CADIS methods give good accuracy for the overall tally regions.

Table I is the results of the average and variance of the relative errors in the mesh regions. The average of the relative errors using FW-CADIS method was lower than that of MR-CADIS method. On the contrary, MR-CADIS method gives lowest Variance. Analysis shows that the relative errors with the MR-CADIS method are uniformly reduced than those with the other methods.

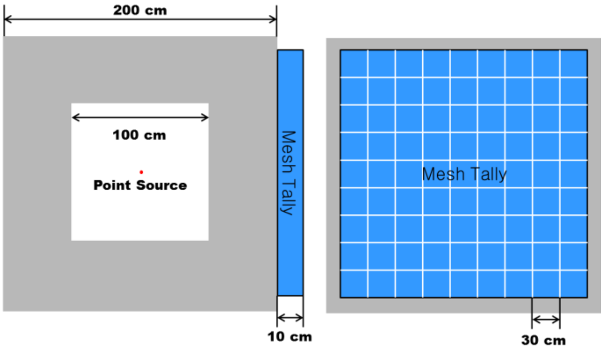
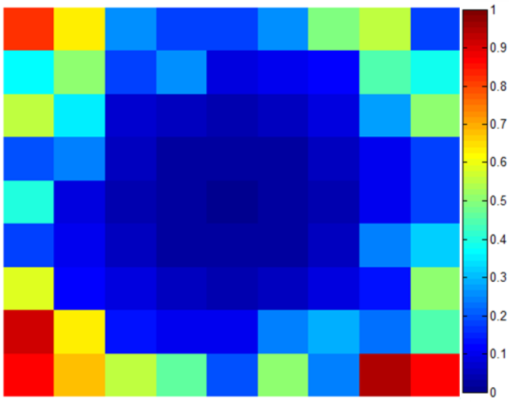
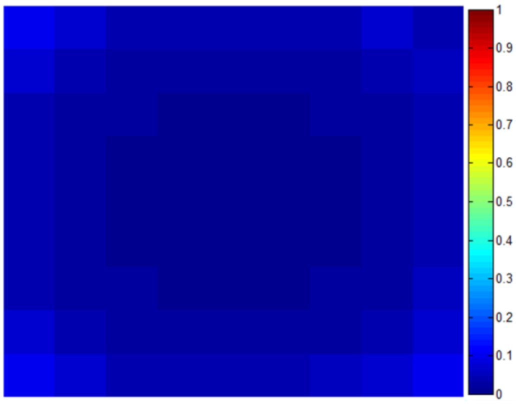


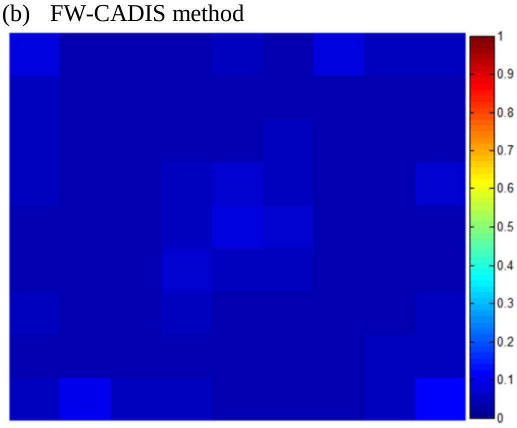
Fig. 1. Schematic drawing of shielding problem



(a) CADIS method



(b) FW-CADIS method



(c) MR-CADIS method

Fig. 2 Relative Error Map of Each Method at 100 CPU Time

Table I: Average and Variance of Relative Error for Each Method

	CADIS	FW-CADIS	MR-CADIS
Average	2.58E-01	**3.21E-02**	4.86E-02
Variance	5.95E-02	4.63E-04	**1.90E-04**

3. Conclusions

In this study, how the CADIS, FW-CADIS, and MR-CADIS methods are applied to optimize and decide the parameters used in the variance reduction techniques was analyzed. In the FW-CADIS Method, it is analyzed that global relative error was decreased by adjusting an adjoint source divided by forward response. The MR-CADIS Method uses a technique that the sum of squared relative error in each tally region was minimized to achieve uniform uncertainty. To compare the simulation efficiency of the methods, a simple shielding problem was evaluated. Using FW-CADIS method, it was evaluated that the average of the relative errors was minimized; however, MR-CADIS method gives a lowest variance of the relative errors. Analysis shows that, MR-CADIS method can efficiently and uniformly reduce the relative error of the plural response problem than FW-CADIS method.

Acknowledgement

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