Application of CMFD on Continuous Energy Monte Carlo Simulation for Eigenvalue Problems

Hyunsuk Lee^a, and Deokjung Lee^{a*}

^aUlsan National Institute of Science and Technology, UNIST-gil 50, Eonyang-eup, Ulju-gun, Ulsan, 689-798, Korea ^{*}Corresponding author: deokjung@unist.ac.kr

1. Introduction

The Coarse Mesh Finite Difference Method (CMFD) has been widely used to accelerate the convergence of deterministic methods [1-3]. Lee applied the CMFD method to the multi-group Monte Carlo eigenvalue simulation [4-5]. He showed that the CMFD acceleration technique is very effective for fission source convergence. As a next step, the CMFD technique was applied to the continuous energy MC eigenvalue simulation. The effect of CMFD on the continuous energy MC was studied with a 1D homogeneous problem.

2. CMFD Formulation

2.1 Algorithm of CMFD Accelerated MC

The algorithm of CMFD accelerated MC is shown in Fig. 1. The CMFD calculation was performed by periods. If the CMFD period is 10, the CMFD will be performed after every 10 MC cycles. After the CMFD calculation, the fission source distribution will be adjusted by using the fission source distribution of CMFD calculation.



Fig. 1. Algorithm of CMFD accelerated MC.

2.2 Tally Parameter for CMFD

The one-group Finite Difference Method (FDM) was used to accelerate the MC. The parameters for the CMFD calculation were tallied during the MC cycle: one-group cross-section, flux, current. The one-group cross section of reaction α in mesh *m* was tallied as below:

$$\overline{\Sigma}_{\alpha}^{m} = \frac{R_{\alpha}^{m}}{\overline{\phi}^{m} V_{m}} \ . \tag{1}$$

2.3 Correction Factor

In the FDM, the current can be calculated with the diffusion coefficient as shown in Eq. (2-4).

$$J_{s}^{FDM} = -\tilde{D}^{s} \left(\phi^{m_{R}(s)} - \phi^{m_{L}(s)} \right) , \qquad (2)$$

$$\tilde{D}^{s} = \frac{2D_{m_{L}(S)}D_{m_{R}(S)}}{h_{m_{R}(S)}D_{m_{L}(S)} + h_{m_{L}(S)}D_{m_{R}(S)}}, \quad (3)$$

$$D_m = \frac{1}{3\overline{\Sigma}_{tr}^m} , \qquad (4)$$

where $\overline{\Sigma}_{tr}^{m}$ is the one-group total cross section in mesh *m*, and h_{m} is the mesh size.

The FDM current can be adjusted to the MC current by using the correction factor \hat{D}^s as shown in Eq. (5).

$$J_{s}^{MC} = J_{s}^{FDM} - \hat{D}^{s} \left(\phi^{m_{R}(s)} + \phi^{m_{L}(s)} \right) .$$
 (5)

The correction factor can be calculated by using the MC current, MC flux, and FDM current as follows:

$$\hat{D}^{s} = -\frac{J_{s}^{MC} + \tilde{D}^{s} \left(\phi^{m_{R}(s)} - \phi^{m_{L}(s)}\right)}{\phi^{m_{R}(s)} + \phi^{m_{L}(s)}} .$$
(6)

2.4 CMFD Feedback to MC

After the CMFD calculation, the fission source distribution of MC was adjusted to the fission source from CMFD. The fission source can be adjusted by neutron weight as shown in Eq. (7-8).

$$w_m = w_t \frac{p_m^{CMFD}}{N_m} , \qquad (7)$$

$$p_m^{CMFD} = \frac{\psi_m^{CMFD}}{\sum_{m=1}^{N_m} \psi_m^{CMFD}} , \qquad (8)$$

where w_m is the adjusted neutron weight, w_t is the total weight of all meshes, N_m is the number of neutrons in mesh m, p_m^{CMFD} is the CMFD fission source fraction of mesh m, and ψ_m^{CMFD} is the CMFD total fission source in mesh m.

3. Results

3.1 1D Slab Test Problem

The homogeneous 1D slab problem was designed to test the CMFD accelerated MC. The 2.1wt% enriched UO_2 fuel, zirconium cladding, and water were mixed. The material composition of the homogeneous material is in Table I.

Isotope	Density [g/cm ³]
92235	1.88E-01
92238	8.50E+00
93237	2.90E-03
94238	5.99E-04
8016	1.25E+00
40091	6.45E-01
40092	9.97E-01
40094	1.03E+00
40096	1.70E-01
1001	8.27E-02
Total	1.29E+01

Table I: Material composition



Fig. 2. Configuration of 1D slab problem.

3.2 Results

The stand-alone MC simulation and the CMFD accelerated MC simulation were performed. Each simulation used 500 inactive cycles, 5,000 active cycles, and 100,000 histories per cycle. The CMFD calculation was performed with a 10 cycle interval of the MC cycle. Fig. 2 shows the Shannon entropy. In the case of CMFD

accelerated MC, the Shannon entropy converges right after the CMFD calculation which is at cycle 10. As in Table II and Fig. 3, the CMFD accelerated MC can generate the same result as standard MC without bias. The error of flux was less than 0.1%.



Fig. 2. Comparison of Shannon entropy.

Table II: Material composition

	MC	MC w/CMFD
$k_{e\!f\!f}$	1.09801	1.09800
SD [pcm]	1	1



Fig. 3. Comparison of flux distribution.

There are differences between real variance and apparent variance since the inter-cycle correlation. The CMFD accelerated MC adjusts the fission source distribution. Therefore the inter-cycle correlation might be small compared to the standard MC simulation. To see the difference between real and apparent variance, The MC and MC with CMFD were performed 30 times with different seed numbers. Each simulation used 500 inactive cycles, 1,000 active cycles, and 20,000 histories per cycle. Fig. 4 shows the real and apparent errors of flux by MC and CMFD accelerated MC. The apparent error and real error of MC show a large difference. The apparent error and real error of CMFD accelerated MC show smaller differences than the standard MC case. That is because the inter-cycle correlation was reduced by CMFD.



Fig. 4. Real and apparent errors of flux.

4. Conclusions

The CMFD method was applied to the continuous energy MC. The CMFD accelerated MC was tested with a 1D homogeneous problem. The fission source distribution converged very rapidly with CMFD. It was shown that the real variance can be reduced by applying CMFD.

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REFERENCES

[1] J. RHODES, K. SMITH, and D. LEE, "CASMO-5 Development and Applications," *Proc. PHYSOR* 2006, Vancouver, Canada (2006).

[2] H. JOO et al., "Methods and Performance of a Three-Dimensional Whole-Core Transport Code DeCART," *Proc. PHYSOR 2004*, April 25-29, 2004, Chicago, Illinois, USA (2004).

[3] R. J. STAMML'ER et al., "HELIOS Methods," Studsvik Scandpower (1998).

[4] M. J. Lee, et al., "Coarse Mesh Finite Difference Formulation for Accelerated Monte Carlo Eigenvalue Calculation," *Annals of Nuclear Energy*, 65, pp. 101-113 (2014).

[5] M. J. Lee, et al., "CMFD Acceleration of Source Convergence for Three-dimensional Monte Carlo Reactor Calculation," *Proc. ANS 2012 winter meeting*, San Diego, California, USA (2012).

[6] N. CHO, S. YUK, H. YOO, and S. YUN, "Overlapping Local/Global Iteration Framework for Whole-Core Transport Solution," *Nuclear Science and Engineering*, 175, pp. 227-238 (2013).

[7] J. F. BREISMEISTER et al., "MCNP – A General Monte Carlo N-Particle Transport Code, Version 4B," LA-12625-M (1997). [8] H. J. Shim, and C. H. Kim, "Real Variance Estimation Using an Intercycle Fission Source Correlation for Monte Carlo Eigenvalue Calculations," *Nuclear Science and Engineering*, 162, pp. 98-108 (2009).

[9] H. J. Shim, B. S. Han, J. S. Jung, H. J. Park, and C. H. Kim, "McCARD: Monte Carlo code for advanced reactor design and analysis," *Nuclear Engineering and Technology*, 44(2), 161-176, (2012).