Development and Verification of Smoothed Particle Hydrodynamics Code for Analysis of Tsunami near NPP

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1. Introduction

After the Fukushima Daiichi NPP accident, the failure due to natural disaster (especially tsunami accompanied earthquake) is being serious concern in nuclear society. In order to safely control large tsunami, an accurate and proper simulation should be equipped. Unfortunately, traditional grid-based numerical code (CFD) encounters its limitation when dealing with free surface or rapidly moving interface. It becomes more complicated when considering the shape and phase of the ground below the seawater. Therefore, some different attempts are required to precisely analyze the behavior of tsunami.

This paper introduces an on-going activities on code development in SNU based on an unconventional mesh-free fluid analysis method called Smoothed Particle Hydrodynamics (SPH) and its verification work with some practice simulations.

2. Modelling Concept of SPH

Smoothed Particle Hydrodynamics (SPH) is one of the fluid simulation method based on Lagrangian formulation. In SPH, the state of system is expressed by a group of particles and particles are points in the domain with each material properties. Fig. 1 shows a simple example of SPH simulation. This mesh-free, Lagrangian fluid analysis code has several advantages compared to traditional mesh-based Eulerian method. It is capable of dealing with the situations that have free surface or deformable boundary as well as large deformation. It also has high applicability in the presence of multi-phases. [1]

In SPH, the properties of each particle is determined by averaging the neighboring particles. Fig. 2 shows smoothing process. In order to get the properties of particle i, the properties of particles inside the smoothing radius is averaged. Various type of kernel functions are used in averaging process to consider the distance. The kernel smoothing of property A basically governed by the following identity [2]

$$< \mathrm{A} >_i = \sum A_j W(r_{ij})\, m_j/\rho_j \qquad (1)$$

where subscript j means neighboring particles, m is a mass of each particles, ρ is a density of each particles, r_{ij} is a distance vector, and W is a kernel function of smoothing process. The first and second derivative of the properties are given below. [2]

$$< \nabla \mathrm{A} >_i = \sum A_j \nabla W(r_{ij})\, m_j/\rho_j \qquad (2)$$

$$< \nabla^2 \mathrm{A} >_i = \sum A_j \nabla^2 W(r_{ij})\, m_j/\rho_j \qquad (3)$$

Since the kernel function is valid just in the compact area, the derivatives of properties are obtained by simply calculating the derivatives of kernel functions.

Not only the properties of particles, but also most of the physical models which have interactions between them go through the smoothing process. Detailed formulas are introduced in following chapter.

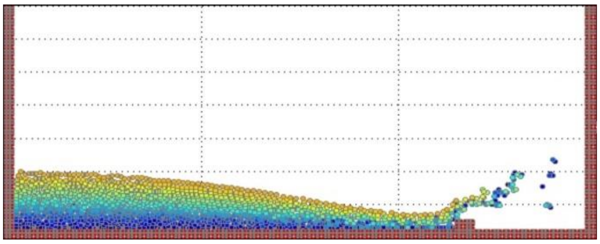

Fig. 1. Example of SPH

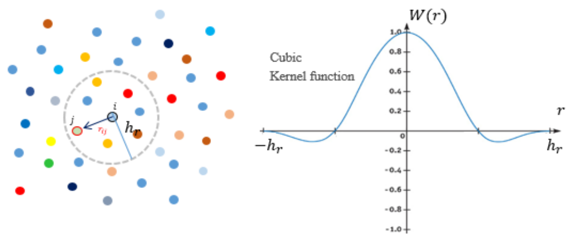

Fig. 2. Smoothing Process of SPH

3. Physical Models and Verification

In order to model SPH, a 3-dimensional in-house code has been newly developed in this study using MATLAB. This code currently incorporates most of the basic models including mass, momentum, and energy equations along with pressure, viscous, surface tension forces. Since the SPH based on Lagrangian formulation, the convection term was not taken into account.

3.1 Pressure Force

The pressure force is the force resulting from pressure difference. Since it is the interaction between particles, smoothing scheme is applied. The pressure force in terms of SPH notation is given below. [3]

$$\left(\frac{du}{dt}\right)_i = -\sum_j m_j \left(\frac{p_i}{{\rho_i}^2} + \frac{p_j}{{\rho_j}^2}\right) \frac{\partial W_{ij}}{\partial x} \qquad (4)$$

For given linear pressure gradient like Fig. 3, the pressure force is calculated and compared with the reference value. The width of tetragonal geometry is 9m, and the pressure changes linearly from 0 Pa to -450 Pa. Table I summarizes the result of this example. The pressure force obtained from SPH code has a good agreement with the theoretical value that calculated in Navier-stokes equation.

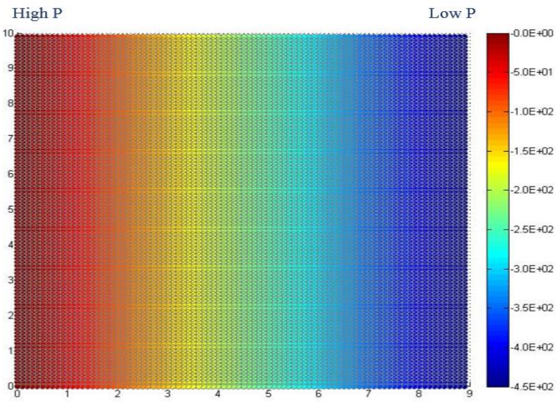

Fig. 3. Linear Pressure Distribution

Table II. Verification of Pressure Force

	SPH	Reference Value
Pressure Force [N/kg]	0.05013	0.05000

In order to model liquid in the SPH method, a nearly incompressible flow approach is generally used. In this approach, local pressure is correlated as a function of changes with even slight density changes, almost incompressible flow can be modeled. The pressure law for incompressible flow is given below [4]

$$p = \frac{{c_s}^2 \rho_0}{\gamma}\left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right] \qquad (5)$$

where c_s is the speed of sound, γ is a sensibility factor that depends on the material, and ρ_0 is the reference density. Fig. 4 shows pressure distribution example in 2D bowl. The fluid is water in room temperature, and there are only gravity and pressure force. Initially, particles move by gravity and pressure force from beginning arrangement. After 3 seconds from calculation started, motion of the particles reduces and it becomes stable. At that time, the slope of pressure-height graph has the value 9836.9 (Pa/m) which is almost correspond with the ρg value of water.
($P = \rho g h$ and $\rho g \cong 9800\ (Pa/m)$ for water.)

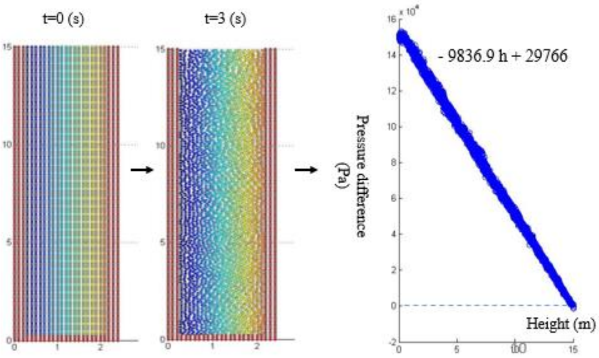

Fig. 4. Pressure Distribution in 2D Bowl

3.2 Viscous Force

The fluid suffers internal friction near boundary, and it decreases the kinetic energy of fluid. This frictional force is viscous force. The SPH notation for viscous force is given below. [4]

$$\frac{du_i}{dt} = \mu \sum_{j \neq i} (\vec{u_j} - \vec{u_i}) \frac{m_j}{\rho_j} \nabla^2 W_{ij} \qquad (6)$$

Following two examples show verification of viscous force. Fig. 5 is for the first example. There are not any other forces except for viscous force. Both sides of boundary are fixed, and initial velocity of fluid is given. Fig. 6 is for the second example. One side of boundary is moving in same velocity with fluid, and all other conditions are identical with the first example. In both cases, the development aspect of flow is very similar with the results of COMSOL simulation in same conditions.

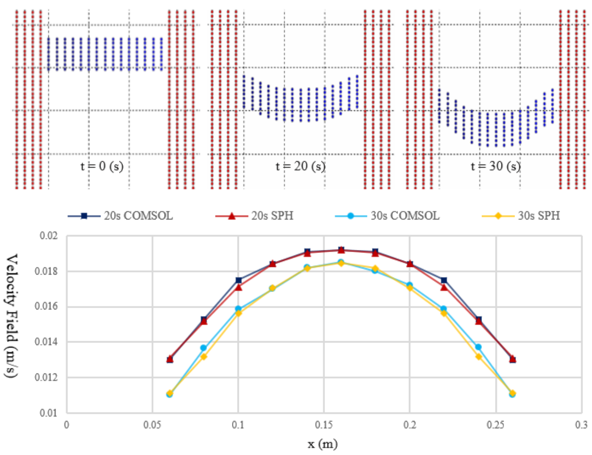

Fig. 5. Viscous Force in Fixed Boundary (Example1)

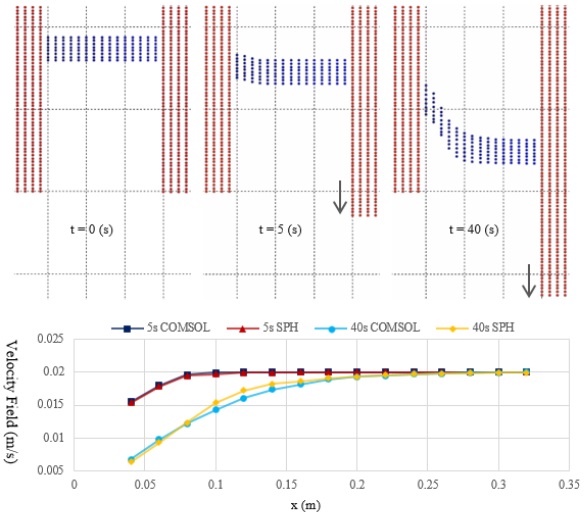

Fig. 6. Viscous Force in moving Boundary (Example2)

3.3 Surface Tension Force

The surface tension is a cohesive force of the surface which allows it to resist external force. The direction of surface tension force heads inward the surface. In SPH, color field c_s is defined to express surface tension. As Fig. 7, the color value is 1 for particles, and 0 for external area. The color field in SPH notation is given below. [4]

$$(c_s)_i = \sum_j m_j W(r_{ij})/\rho_j \qquad (7)$$

The direction of the surface tension is determined by the gradient of color field ($\vec{n}$), and the magnitude of gradient distinguish between surface particles and inner particles. The magnitude of surface tension depends on the curvature of surface. It can be obtained by the laplacian of color filed. Therefore, the surface tension force in SPH notation can be given below. [4]

$$\left(\frac{du}{dt}\right)_i = -\frac{\sigma}{\rho_i}\nabla^2 C_s \frac{\vec{n}}{|n|} \qquad (8)$$

The Fig. 8 shows the simple example of surface tension force. As time passes, square-shaped particles move and form circle at the end.

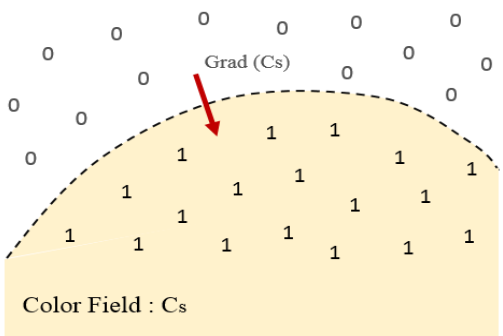

Fig. 7. Color Field

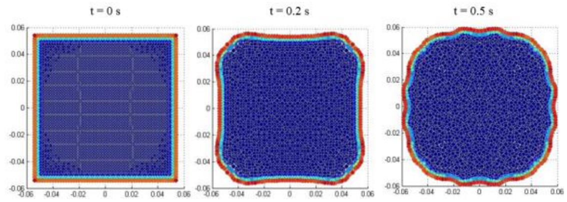

Fig. 8. Change in Shape by Surface Tension

3.4 Gravity Force

Since, the gravity force is just a constant in −z direction, it can be easily implemented.

$$\left(\frac{du}{dt}\right)_i = -9.8\,\hat{z} \qquad (9)$$

3.5 Heat Conduction Equation

The heat is conducted from high temperature to low temperature, and heat transfer velocity depends on material's heat capacity and thermal conductivity. The heat conduction equation in SPH notation is given below. [4]

$$\left(\frac{dH}{dt}\right)_i = \sum_j \frac{4m_j}{\rho_i\rho_j}\left(\frac{k_i k_j}{k_i+k_j}\right) T_{ij}\frac{\vec{r_{ij}}}{|\vec{r_{ij}}|^2}\frac{\partial W_{ij}}{\partial x} \qquad (10)$$

The following example shows 2D transient conduction situation. The temperature of left side is fixed for 368(K). The right side is copper which has 283(K) of initial temperature. The temperature distribution of each time step is nearly correspond with the results of COMSOL simulation in same conditions.

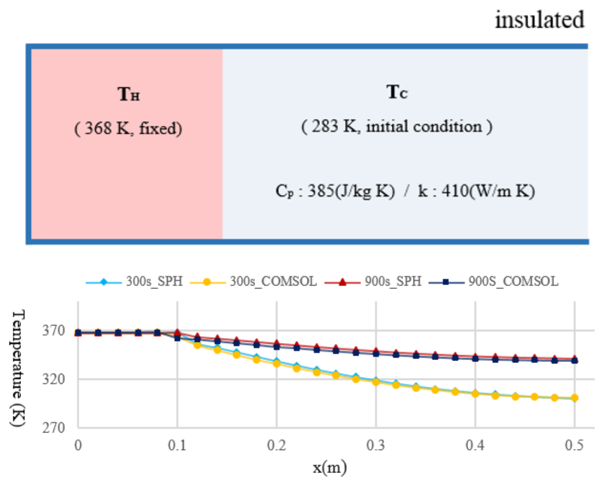

Fig. 9. 2D Conduction Example

4. Practice Simulations

4.1 Dam Break Example

As shown in Fig. 10(a), the fluid particles are arranged in a regular grid initially. As time passes, particles are driven by forces and subside at the end. In this example, the gravity, viscous, and pressure force are applied. The boundary of the system has fixed position, and there are not any other boundary conditions.

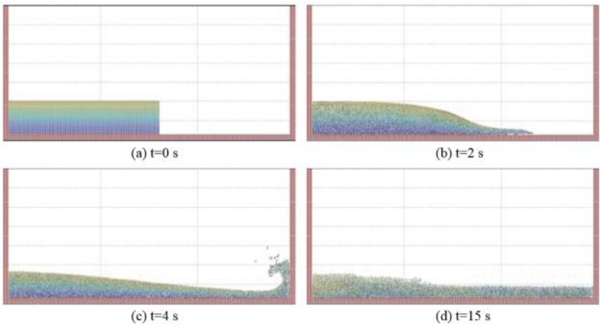

Fig. 10. Dam Break Example

4.2 Tsunami Generation Example

Fig. 11 shows tsunami formation in 2D bowl. The wave is formed by moving boundary, and the intensity of wave can be controlled by velocity of moving boundary. Fig. 11 emphasizes an importance of topography. In case of (a), water particles does not exceed the barrier, but water can overpasses the barrier in case of (b). It means that the shape of ground under the seawater is important factor for determining the height of barrier.

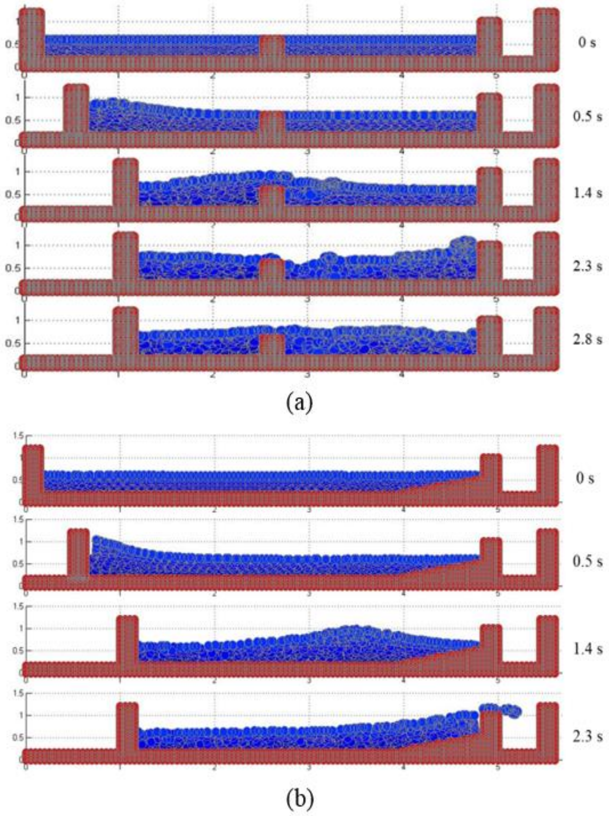

Fig. 11. Tsunami in Different Shape of Ground

4.3 3D-Dam Break Example

The newly developed SPH code in SNU can perform 3-dimensional calculation. Fig. 12 shows simple dam break simulation in 3D.

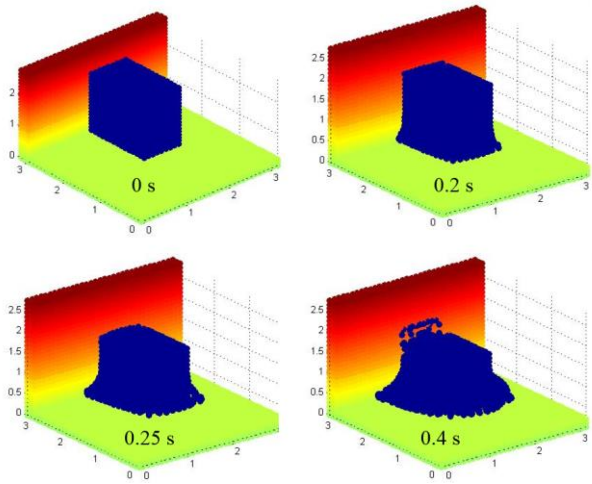

Fig. 12. 3D-Dam Break Example

5. Implementation Environment

5.1 GPU Parallel Computing

Parallel Computing is a form of computation in which calculations are performed simultaneously. Therefore, large problems can be divided into smaller ones, and thus the time is saved. [5] Own SPH code is practicable for parallel computing, and also GPU programming is possible. (NVIDIA GEFORCE GTX 770 is used for GPU programming),

5.2 Graphical User Interface (GUI)

As shown in Fig. 13, a simple GUI is equipped for the current SPH code so that users can set up the various parameters more easily. Some simple post-processing are also available using the GUI.

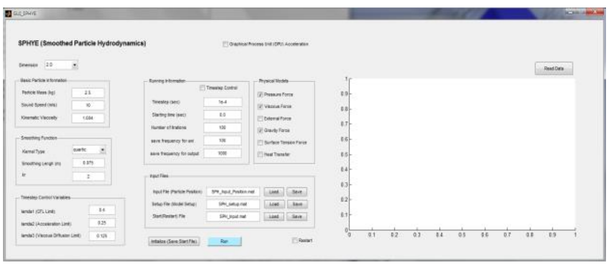

Fig. 13. Graphical User Interface (MATLAB)

6. Summary and Conclusions

This paper summarizes the on-going development and verification activities on Lagrangian mesh-free SPH code in SNU. The newly developed code can cover equation of motions and heat conduction equation so far, and verification of each models is completed. In addition, parallel computation using GPU is now possible, and GUI is also prepared. If users change input geometry or input values, they can simulate for various conditions geometries.

A SPH method has large advantages and potential in modeling of free surface, highly deformable geometry and multi-phase problems that traditional grid-based code has difficulties in analysis. Therefore, by incorporating more complex physical models such as turbulent flow, phase change, two-phase flow, and even solid mechanics, application of the current SPH code is expected to be much more extended including molten fuel behaviors in the sever accident.

ACKNOWLEDGEMENT

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