Two-Dimensional Application of the Functionalized Discontinuity Factors

Woosong Kim and Yonghee Kim*
Dept. of Nuclear and Quantum Eng., KAIST, 373-1 Daehak-ro, Yuseong-gu, Daejeon, Korea, 305-701
**Corresponding author: yongheekim@kaist.ac.kr*

1. Introduction

The nodal equivalence theory is the cornerstone of the modern reactor core analysis when the heterogeneous core is simplified using homogenized fuel assemblies. It is the idea based on preserving the equivalency between an original heterogeneous assembly and a homogenized assembly in terms of reaction rates and node interface currents. Nowadays, simplified equivalence theory (SET) [1] is one of the most widely used technique due to its calculational efficiency. However, the limits of single assembly homogenization of SET become noticeable when the node interface current is not close to zero and the neighborhood effect is rather strong [2]. To overcome this limitation, there were several approaches such as global-local iteration method, color-set fuel assembly calculation, and boundary perturbations theory [3]. In a recent study by W. Kim and Y. Kim, [4] discontinuity factors (DFs) were functionalized according to the node surface current-to-flux ratio (CFR) and it was shown that this functionalized discontinuity factors (FDF) can improve significantly the accuracy of the homogeneous solutions in 1-D slab geometries. In this study, FDF for 2-dimensional geometry is developed and tested for Light Water Reactor (LWR) benchmark problems.

2. Functionalized Discontinuity Factors

Because the conventional assembly discontinuity factors (ADFs) are based on the net-zero current boundary condition, they may be quite different from the reference discontinuity factors (RDFs) which are obtained by using the exact boundary conditions. However, if one can generate the DFs as a function of boundary condition of a fuel assembly in the lattice calculation and the DF can be updated by using the actual current information during the iterative core calculation, it is expected that more accurate DFs will be determined and the resulting nodal equivalence for the homogenized fuel assemblies will be improved, leading to a more accurate core analysis. *This conjecture is based on the general perception that the interface neutron current from a core calculation should be 'closer to the reference' than the 'zero net-current' conditions used for determination of the conventional ADFs*. If the perception is true, update of DFs by using the 'more accurate' interface current resulting from the whole core calculation will provide correspondingly 'improved DFs' to be used in the next iterative core calculation and the accuracy of the resulting core calculation could be improved again and the DF update follows. In this way, the non-linear DF update can be continued until convergence. In a sense, the associated algorithm is based on a 'virtuous cycle' between the DFs and interface currents. As the feasibility of the FDFs in one-dimensional problem was shown, two-dimensional problem test problems are considered in the work.

2.1. Functionalized discontinuity factor of fuel assembly

In this study, we tried to functionalize DFs of a surface by using the current-to-flux ratio (CFR) on the same surface only because the surface DFs are mainly dependent on the surface boundary condition for the same surface. Of course, DF may also depend on the boundary conditions on the other surfaces of the fuel assembly. However, in this work, the FDF is assumed to depend only on the CFR of the same surface for practical and efficient applications of the FDF concept. Fig. 1 shows the schematic illustration of the FDF in a 2-D fuel assembly geometry.

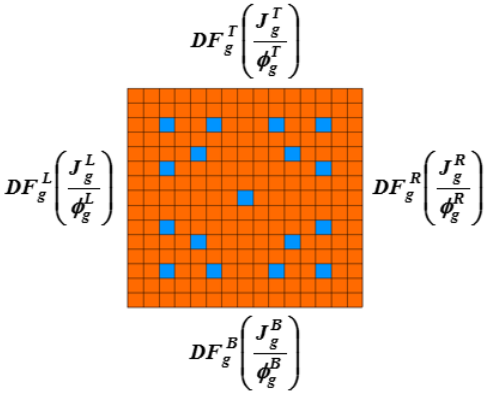

Fig. 1. The FDF of the test problem single assembly

The FDFs are expressed as a function of node interface CFR. For the functionalization of FDFs, following quadratic function is used in this study:

$$DF_g^L = ADF_g\left[a_1 + a_2\left(\frac{J_g^L}{\phi_g^L}\right) + a_3\left(\frac{J_g^L}{\phi_g^L}\right)^2\right], \quad (1)$$

where DF_g^L is the g-th group DFs on the left surface of a homogenized assembly, ADF_g is the conventional g-th group assembly discontinuity factor, a_n are

coefficients, ϕ_g^L is the flux on the left surface of heterogeneous assembly, and J_g^L is the current on the left surface of heterogeneous assembly. The DFs on the other surfaces are also functionalized in the same way.

In order to determine the coefficients a_n, the single lattice problem in Fig. 1 is solved by using several CFR boundary conditions on the corresponding surfaces while the reflective boundary condition is applied on the other three surfaces. Because the CFR is a function of albedo in diffusion approximation, a set of albedo boundary condition was used for the set of CFR boundary conditions. The CFR boundary condition can be either positive or negative in actual problems. Therefore, 3 CFR (+, 0, -) boundary conditions are used on each surface, and single lattice problem is solved 3 times with different boundary conditions. Also, the homogeneous surface flux was calculated by simply solving the fixed-source problem of homogeneous assembly with the heterogeneous current as the boundary condition. It is worthwhile to note that, in the case of symmetric fuel assembly, the FDF is identical for all surfaces if a single set of CFR values is used for the FDF determination. As a result, the number of fuel assembly analysis is only increased from one to three. In addition to the conventional zero CFR, 2 CFRs are additionally used for determination of the FDF.

2.2. Functionalized discontinuity factor of reflector

Unlike the symmetric fuel assembly analysis, in the case of heterogeneous baffle-reflector homogenization, a set of current boundary conditions are used instead of albedo boundary condition to determine the homogenized parameters and the coefficients for FDF. As shown in Fig. 2, just like in the conventional baffle-reflector homogenization, a small spectral fuel-reflector model is analyzed. The heterogeneous current at the interface from the spectral geometry is used as reference value. Similarly to the fuel assembly calculation, 3 current (+ε, reference, - ε) boundary conditions are used on the reflector surface to determine the FDF coefficients. Of course, one of the current information is from the fuel-reflector spectral analysis and the other two conditions are determined appropriately. For the reflector nodes, quadratic FDFs are applied, as well.

In addition to the conventional flat baffle-water reflector, there are the concave baffle-water reflectors in the two-dimensional core. They are often approximated to have same homogenized parameters as the flat baffle-water reflector in the conventional reactor core analysis. However, we used separate method to homogenize this concave baffle-water reflector as shown in Fig. 3. First, we decide reference current from the conventional small spectral fuel-reflector model. Then, 3 current (+ε, reference, - ε) boundary conditions are used on the two baffle side reflector surfaces and reflective boundary condition was applied to the other two surfaces. Because the concave baffle-water reflector is diagonally symmetric, the same FDF will be used for the two reflector surfaces.

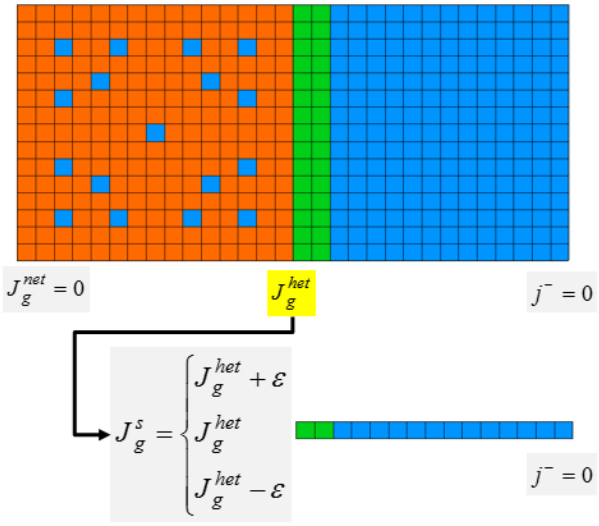

Fig. 2. The heterogeneous flat baffle-water reflector FDF evaluation

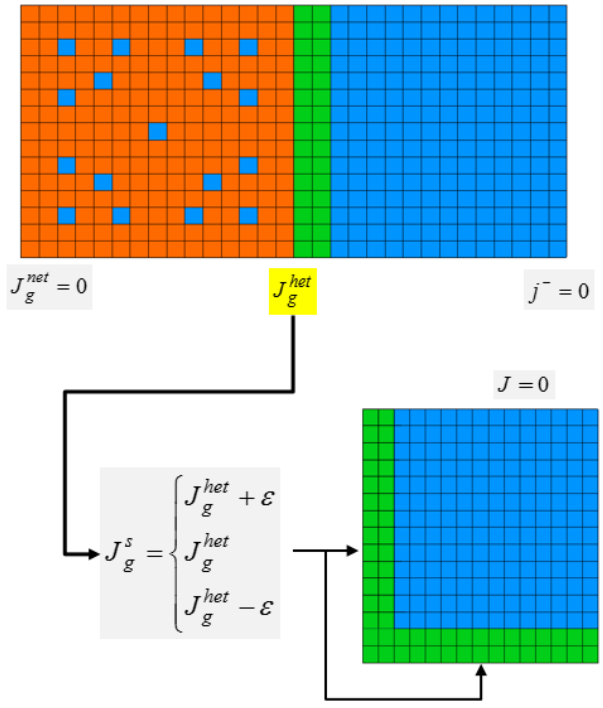

Fig. 3. The heterogeneous concave baffle-water reflector FDF evaluation

2.2. Whole-core calculation with FDFs

In the whole-core analysis based on the well-developed inner-outer nested iterations, the FDFs are updated by using the node interface CFR during iteration (FDF iteration) [4]. The lattice or fuel

assembly calculations with different CFR boundary conditions provide different homogenized group constants for the fuel assembly. However, the other homogenized parameters, such as diffusion coefficients and cross-sections are fixed to the conventional flux-weighted constants (FWCs) determined with zero CFR boundary condition [5].

Fig. 4 shows the iterative solution scheme including an FDF update iteration. In Fig. 4, t is index of "outer iteration" and u is index of "FDF iteration", the update procedure for DFs in Eq. (1). During above iterations, different error criteria are used to check the convergence of outer iteration and FDF iteration.

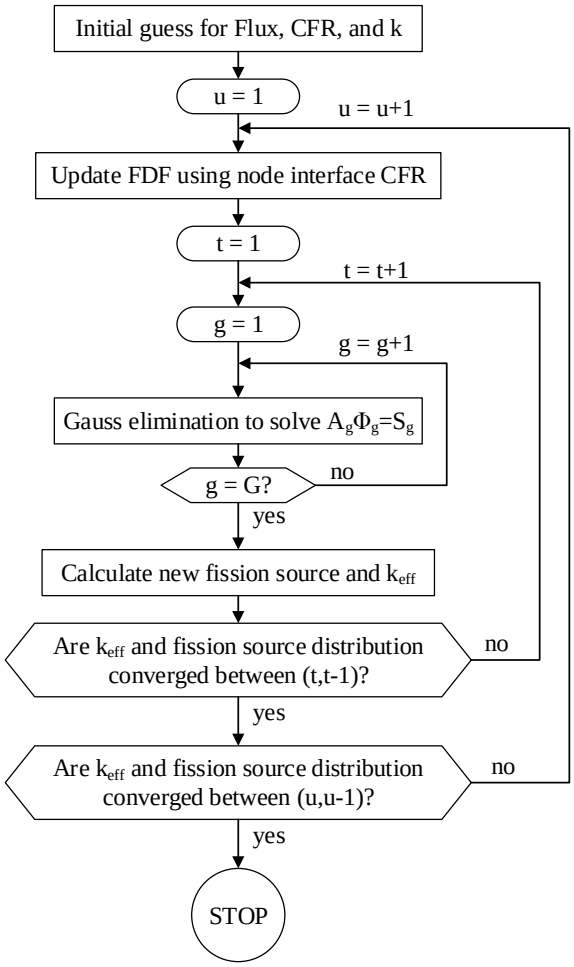

Fig. 4. Flowchart of the FDF iterative solution scheme

3. Results and Discussion

To test the feasibility of the simple 2-D FDF method, two tow-dimensional PWR quarter core benchmark problems were considered. One is EPRI-9 benchmark problem and the other is EPRI-9R benchmark problem, which is almost identical to the EPRI-9 problem except control rods inserted at the center fuel assembly [6]. Fig. 5 shows heterogeneous fuel assembly geometry of benchmark problems. Fuel pins are on the white cells and non-fuel compositions are on the black cells. Non-fuel compositions are either water or control rods, where the black cell at the assembly center is always water. The two-group cross-sections of each material are provided in Table. I.

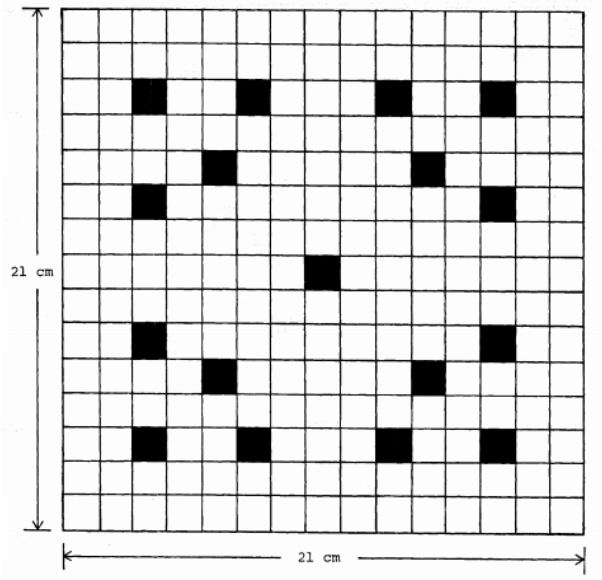

Fig. 5. Heterogeneous PWR assembly geometry

Table I: Heterogeneous, two-group cross-section data for benchmark problems

Material (abbreviation)	Group g	D_g (cm)	Σ_{ag} (cm^{-1})	$\Sigma_{gg'}$ (cm^{-1})	$\nu\Sigma_{fg}$ (cm^{-1})
Fuel-1	1	1.500	0.0130	0.0200	0.0065
(F-1)	2	0.400	0.1800	0.0	0.2400
Fuel-2	1	1.500	0.0100	0.0200	0.0050
(F-2)	2	0.400	0.1500	0.0	0.1800
Water	1	1.700	0.0010	0.0350	0.0
(W)	2	0.350	0.0500	0.0	0.0
Control Rod	1	1.113	0.0800	0.0038	0.0
(CR)	2	0.184	0.9600	0.0	0.0
Baffle	1	1.020	0.0032	0.0000	0.0
(B)	2	0.335	0.1460	0.0	0.0

As shown in Fig. 6, the EPRI-9 and EPRI-9R benchmark problems model the baffle and reflector regions of PWR's. The core configurations for the two problems are similar, but the only difference between them is that the control rods are inserted at the center fuel assembly of EPRI-9R, while no control rods are inserted in EPRI-9 case.

For the analysis of the fuel assembly and core analysis, a 2-D diffusion finite difference method (FDM) code was developed for both whole core and lattice calculations. A fine mesh (0.2 cm) was used in the analysis and the error criteria between FDF iteration steps were $\varepsilon_{keff} < 10^{-7}$ for the multiplication factor and $\varepsilon_{source} < 10^{-6}$ for the source distribution. And during FDF iterations, the FDFs were updated whenever outer iteration satisfies an error criterion which is 10 times

looser than that of FDF iteration. Consequently, the FDF update is triggered only when the fission source is roughly converged.

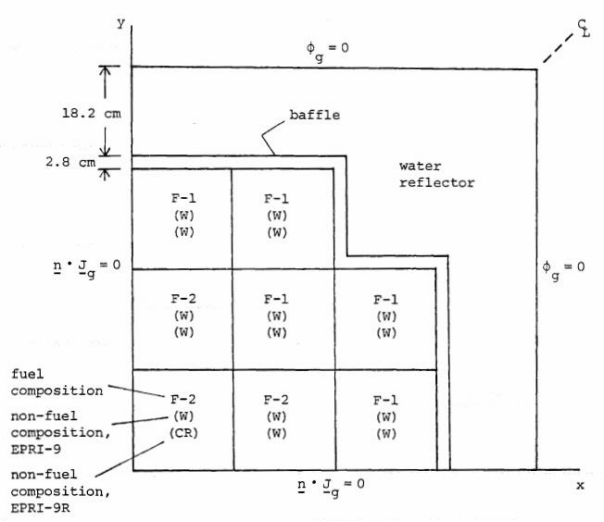

Fig. 6. Flowchart of the FDF iterative solution scheme

3.1. EPRI-9 core benchmark problem

The EPRI-9 benchmark problem was calculated calculation and the results are compared between FDF method and the conventional ADF. Table II compares the effective multiplication factors of the two methods. As a result, the FDF shows 33% improved accuracy compared to the ADF.

Table II: The effective multiplication factor comparison (EPRI-9 benchmark problem)

	k_{eff}	Error (pcm)
Reference	0.927484	-
w/o DF	0.928242	76
ADF	0.927176	-31
FDF	0.927269	-21

And the normalized assembly power distribution is shown in Fig. 7. By using FDF, both maximum relative error and RMS error noticeably decreased.

0.849	0.610	
1.428	4.630	
0.053	-0.859	
-0.075	-0.597	
1.217	1.206	0.610
-1.578	-1.126	4.630
0.229	-0.142	-0.859
0.210	-0.116	-0.597
1.443	1.217	0.849
-1.990	-1.578	1.428
0.397	0.229	0.053
0.336	0.210	-0.075

coredax	MAX err	RMS err
noDF %	4.630	2.673
ADF %	-0.859	0.470
FDF %	-0.597	0.342

Fig. 7. Normalized assembly power distribution (EPRI-9 benchmark problem)

3.2. EPRI-9R core problem

The EPRI-9R benchmark problem was analyzed and the results are compared in Table III between FDF method and the conventional ADF. In this case, the FDF shows about 34% accuracy improvement for the k-eff compared to the ADF.

Table III: The effective multiplication factor comparison (EPRI-9R benchmark problem)

	k_{eff}	Error (pcm)
Reference	0.889747	
w/o DF	0.890463	72
ADF	0.888859	-89
FDF	0.889153	-59

For the EPRI-9R problem, the normalized assembly power distribution is shown in Fig. 8. In the control rod-inserted ERPI-9R problem, FDF shows higher maximum error and RMS error. The maximum error takes place in the rodded assembly where the power is very low. Meanwhile, the error near the baffle-reflector interface decreased noticeably with the FDF.

1.070	0.814	
1.301	4.747	
0.410	-0.553	
0.252	-0.286	
1.138	1.394	0.814
-3.148	-1.755	4.747
0.250	-0.043	-0.553
0.297	0.048	-0.286
0.562	1.138	1.070
-1.609	-3.146	1.302
-0.869	0.251	0.411
-1.458	0.299	0.253

coredax	MAX err	RMS err
noDF %	4.747	3.040
ADF %	-0.869	0.478
FDF %	-1.458	0.570

Fig. 8. Normalized assembly power distribution (EPRI-9R benchmark problem)

4. Conclusions

In this study, a two-dimensional FDF method has been applied to PWR benchmark problems to investigate its feasibility. The DFs are still expressed as a function of node interface current-to-flux ratio of same surface only. Even for the two-dimensional fuel assembly, the evaluation of FDF required only two more lattice calculations when it is symmetric. Also, the DFs at baffle-water reflector interface were functionalized, not only for the flat baffle-water reflector but also for the concave baffle-water reflector. The feasibility of two-dimensional FDF was evaluated by using two benchmark problems. It has been shown that a simple implementation of the 2-D FDF noticeably improves the k-eff prediction accuracy and the reactor power profile can also be more accurately determined. In future, FDF evaluation using transport lattice calculation needs to be studied for practical application of FDF method. In addition, efficient way to determine more accurate FDFs in 2-D problem should be developed, as well.

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