An Evaluation of the Adjoint Flux Using the Collision Probability Method for the Hybrid Monte Carlo Radiation Shielding Analysis

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1. Introduction

It is noted that the analog Monte Carlo method has low calculation efficiency at deep penetration problems such as radiation shielding analysis. In order to increase the calculation efficiency, variance reduction techniques have been introduced and applied for the shielding calculation. To optimize the variance reduction technique, the hybrid Monte Carlo method was introduced [1]. For the determination of the parameters using the hybrid Monte Carlo method, the adjoint flux should be calculated by the deterministic methods. In this study, the collision probability method [2, 3] is applied to calculate adjoint flux. The solution of integration transport equation in the collision probability method is modified to calculate the adjoint flux approximately even for complex and arbitrary geometries. For the calculation, C++ program is developed. By using the calculated adjoint flux, importance parameters of each cell in shielding material are determined and used for variance reduction of transport calculation. In order to evaluate calculation efficiency with the proposed method, shielding calculations are performed with MCNPX 2.7 [4].

2. Methods and Results

2.1 Introduction of Integration Transport Equation (ITE)

The overview of the transport simulation with the integration transport equation is given in Fig. 1. The form of integration transport equation [5] is expressed as follows:

$$-\frac{d}{dR}\psi(\vec{r}-R\hat{\Omega},\hat{\Omega})+\sigma_t(\vec{r})\psi(\vec{r}-R\hat{\Omega},\hat{\Omega})=q(\vec{r}-R\hat{\Omega},\hat{\Omega}) \quad (1)$$

where R is the distance along the line of neutron travel and σ_t is total cross section. The solution of the *ITE* can be expressed as following equation:

$$\psi(\vec{r},\hat{\Omega})=\int_0^R dR' q(\vec{r}-R'\hat{\Omega},\hat{\Omega})\exp[-\int_0^{R'}\sigma_t(\vec{r}-R''\hat{\Omega})dR'']$$
$$+\psi(\vec{r}-R\hat{\Omega},\hat{\Omega})\exp[-\int_0^{R}\sigma_t(\vec{r}-R''\hat{\Omega})dR''] \quad (2)$$

In case of no internal sources, Eq. (1) can be rewritten to Eq. (3)

$$q(\vec{r}-R'\hat{\Omega},\hat{\Omega})=\int d\hat{\Omega}'\sigma_s(\vec{r},\hat{\Omega}'\to\hat{\Omega})\psi(\vec{r},\hat{\Omega}') \quad (3)$$

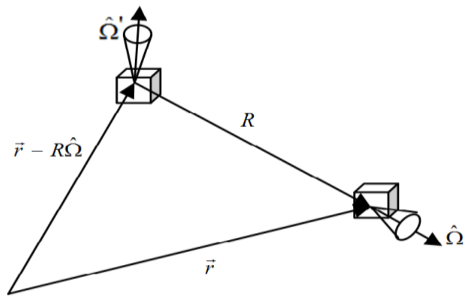

Fig. 1. Description of Transport Calculation from Positions $\vec{r}-R\hat{\Omega}$ to $\vec{r}$ with *ITE*

where σ_s is scattering cross section. In order to calculate adjoint flux of each cell, Eq. (2) is differentiated according to R, and then, it can be expressed as follows:

$$d\psi(\vec{r},\hat{\Omega})=dRq(\vec{r}-R\hat{\Omega},\hat{\Omega})\exp[-\int_0^R\sigma_t(\vec{r}-R'\hat{\Omega})dR'] \quad (4)$$
$$+d\left[\psi(\vec{r}-R\hat{\Omega},\hat{\Omega})\exp[-\int_0^R\sigma_t(\vec{r}-R'\hat{\Omega})dR']\right]$$

Expanding the last term of right side in Eq. (4), it can be rewritten as following equation:

$$d\psi(\vec{r},\hat{\Omega})=dRq(\vec{r}-R\hat{\Omega},\hat{\Omega})\exp[-\int_0^R\sigma_t(\vec{r}-R'\hat{\Omega})dR']$$
$$+\exp[-\int_0^R\sigma_t(\vec{r}-R'\hat{\Omega})dR']d\psi(\vec{r}-R\hat{\Omega},\hat{\Omega})$$
$$-\sigma_t(\vec{r}-R\hat{\Omega})\exp[-\int_0^R\sigma_t(\vec{r}-R'\hat{\Omega})dR']\psi(\vec{r}-R\hat{\Omega},\hat{\Omega})dR \quad (5)$$

The differential angular flux can be defined as following equations:

$$d\psi(\vec{r},\hat{\Omega})=\psi_a(\vec{r},\hat{\Omega}) \quad (6)$$

$$d\psi(\vec{r}-R\hat{\Omega},\hat{\Omega})=\psi_a(\vec{r}-R\hat{\Omega},\hat{\Omega}) \quad (7)$$

$\psi_a(\vec{r},\hat{\Omega})$ and $\psi_a(\vec{r}-R\hat{\Omega},\hat{\Omega})$ are adjoint fluxes contributed by particles which lies between $\vec{r}-R'\hat{\Omega}$ and

$\vec r-(R'-dR')\hat\Omega$. Multiplying both side of Eq. (4) by $R^2 d\hat\Omega$,

$$\psi_a(\vec r,\hat\Omega).R^2 d\hat\Omega = \psi_a(\vec r - R\hat\Omega,\hat\Omega)\exp[-\int_0^R \sigma_t(\vec r - R'\hat\Omega)dR']R^2 d\hat\Omega$$

$$+q(\vec r - R\hat\Omega,\hat\Omega)\exp[-\int_0^R \sigma_t(\vec r - R'\hat\Omega)dR'].R^2 d\hat\Omega dR$$

$$-\sigma_t(\vec r - R\hat\Omega)\exp[-\int_0^R \sigma_t(\vec r - R'\hat\Omega)dR']\psi(\vec r - R\hat\Omega,\hat\Omega)R^2 dRd\hat\Omega \quad (8)$$

Substituting $dS = R^2 d\hat\Omega$ and $dV = R^2 d\hat\Omega dR$, then the solution of the integration formulation can be expressed as following equation:

$$\psi_a(\vec r,\hat\Omega).dS = \psi_a(\vec r - R\hat\Omega,\hat\Omega)\exp[-\int_0^R \sigma_t(\vec r - R'\hat\Omega)dR']dS$$

$$+q(\vec r - R\hat\Omega,\hat\Omega)\exp[-\int_0^R \sigma_t(\vec r - R'\hat\Omega)dR']dV$$

$$-\sigma_t(\vec r - R\hat\Omega)\exp[-\int_0^R \sigma_t(\vec r - R'\hat\Omega)dR']\psi(\vec r - R\hat\Omega,\hat\Omega)dV \quad (9)$$

2.2 Computational Method

In order to use the method described in Section 2.1, a simple problem is selected as shown in Fig. 2. Medium of the problem is homogeneous and isotropic scattering.

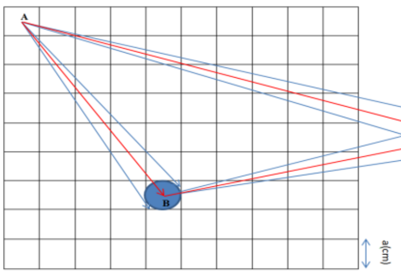

Fig. 2. Geometry of Selected Scattering Problem

For the calculation of adjoint flux from cell A to detector C, Eq. (9) can be rewritten to Eq. (10) by approximating the case which neutron stream flows directly from A to C.

$$\psi_a(\vec r,\hat\Omega) = \psi(\vec r - R\hat\Omega,\hat\Omega)\exp[-\int_0^R \sigma_t(\vec r - R''\hat\Omega)dR''] \quad (10)$$

If a neutron transports from cell A to cell B with a scattering reaction, the scattered neutron are considered as the adjoint fluxes. To calculate such a collision probability, the cross section of the geometrical area should be decided. In this study, to approximate cross section of geometrical area *(dS)* at each cell, the cubic unit cell is replaced to spherical cell with preserving the original unit volume. The number of neutrons passing through from cell A to cell B can be approximated as the following equation:

$$\psi_a(\vec r_B,\hat\Omega')dS_B = \psi_a(\vec r_A,\hat\Omega')\exp[-\int_0^{R_{AB}} \sigma_t(\vec r_B - R\hat\Omega')dR \quad (11)$$

After scattering reaction at cell B, the number of neutrons arriving to detector C can be expressed as following equation:

$$\psi_a(\vec r_C,\hat\Omega)dS_C = \Sigma_s(\vec r,\hat\Omega' \to \hat\Omega)\psi_a(\vec r_B,\hat\Omega')\times$$

$$\times\exp[-\int_0^{R_{BC}} \sigma_t(\vec r_C - R\hat\Omega)dR]dV_B \quad (12)$$

where Σ_S is total scattering cross section on a cell.

2.3 Evaluation of Calculation Efficiency

Using the proposed approximation method with *ITE*, the adjoint flux for a simple shielding problem was calculated. Shielding material is carbon, and the size was set to 10 cm x 10 cm x 10 cm. The cubical mesh is used; and the dimension of the unit mesh is 1x1x1cm. Adjoint fluxes to detector were calculated up to two collisions. The detector is located at a straight line which passes through center of shielding material. The detector is assumed to be a 1 cm x 1 cm x 1 cm cubic.

Using the above condition, the adjoint fluxes for each case of shielding thickness were calculated with the developed C++ program. The result of the adjoint fluxes are given in Table I and Fig. 3. By using the adjoint flux, importance parameter is determined as shown in Table II. The shielding calculations were performed with MCNPX 2.7 code [4] with ENDF-VI cross section library for both analog and biased system. The particle transport results are given in Table III. The results show that the proposed method can enhance the FOM by 16 times compared with analog calculation.

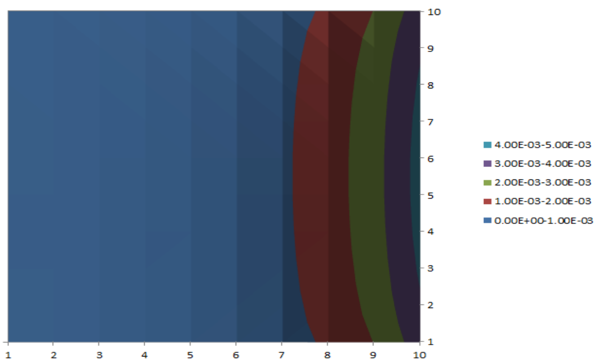

Fig. 3. Map of Calculated Adjoint flux at the Center of Shielding

Table I: Adjoint Flux at Each Shielding Depth

Thickness	1 cm	2 cm	3 cm	4 cm	5 cm
Adjoint Flux	0.003	0.005	0.009	0.015	0.026
Thickness	6 cm	7 cm	8 cm	9 cm	10 cm
Adjoint Flux	0.046	0.082	0.144	0.250	0.420

Table II: Importance Parameter Determined by the Adjoint Flux

Thickness	1 cm	2 cm	3 cm	4 cm	5 cm
IMP	1	1.8	3.1	5.5	9.6
Thickness	6 cm	7 cm	8 cm	9 cm	10 cm
IMP	16.8	29.6	52	90.6	152

Table III: Calculation Results with Analog and Biased System

	NPS	Tally	Error	FOM
Analog MC	4.00E+06	4.31E-06	0.0575	21
Biased MC	1.00E+08	4.21E-06	0.0579	326

3. Conclusions

In this study, a method to calculate the adjoint flux in using the Monte Carlo variance reduction was proposed to improve Monte Carlo calculation efficiency of thick shielding problem. The importance parameter for each cell of shielding material is determined by calculating adjoint flux with the modified collision probability method. In order to calculate adjoint flux with the proposed method, C++ program is developed. The results show that the proposed method can efficiently increase the FOM of transport calculation. It is expected that the proposed method can be utilize for the calculation efficiency in thick shielding calculation.

Acknowledgments

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