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Investigation of Eigenvalue Behavior in the Asymptotic Analysis of PCMI

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1. Introduction

An asymptotic analysis method to evaluate the stress intensification of an LWR fuel rod cladding tube suffered from the expansion of a cracked pellet in a PCMI (Pellet Cladding Mechanical Interaction) [1,2]. As a starting point, a bonded condition was assumed between the wedge and half plane, which were modeled as a pellet fragment and a zirconium alloy cladding tube, respectively. An eigenvalue problem was formed from the model using the geometry and (bonded) contact conditions. As a result, two eigenvalues, associated with the stress singularity at the contact edge, were produced. A finite element analysis technique to calculate the generalized stress intensity factors was also presented in these papers, which would be used as the calibration factors to evaluate the actual stresses when the pellet fragments expand the cladding in the PCMI.

This analysis is further extended in this paper to accommodate a more realistic condition of the PCMI such as a frictional contact between two adjacent pellet fragments and a cladding tube. However, this yields a sophisticated behavior of the eigenvalues depending on the coefficient of friction (incorporating the direction of slipping of each fragment) as well as the angle of the pellet crack. Since the stress field of the cladding is directly determined from the eigenvalues, it is crucial to evaluate and investigate them to analyze the PCMI problem mechanistically, which is pursued in this paper.

2. Eigenvalue problem

2.1 Geometrical model and boundary conditions

Fig. 1 shows a typical section view of an LWR fuel rod including a cracked pellet, and the geometrical model for the present asymptotic analysis.

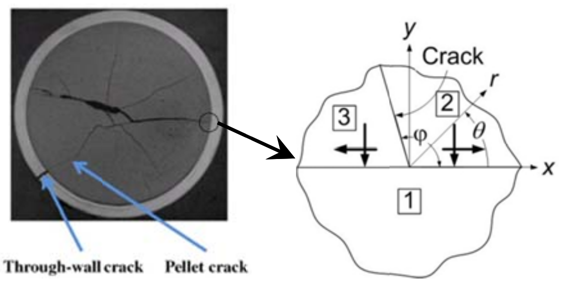

Fig. 1. Typical view of PCMI failure and the geometrical model of the present asymptotic analysis.

Since the contact stress particularly in the vicinity of the edge of a pellet fragment and a cladding tube is concerned in the asymptotic analysis, the cladding tube and pellet fragments are modeled as a half plane (body 1) and two adjacent parts (body 2 and 3) , respectively. The crack angle with respect to the cladding inner surface is defined as φ as shown in Fig. 1. A polar coordinate, (r,θ) is chosen with the origin at the two contact edges of the pellet fragments, and θ increasing in the counterclockwise direction from the contact surface.

As for the boundary conditions, a frictional contact between the fragments and cladding is constituted with a coefficient of friction, f_c. It is set as positive when the right fragment slips away from the origin (to the right). When the pellet expands and pushes the cladding outward, it is regarded that the fragments move apart from each other. Therefore, during the pellet expansion, $+f_c$ is applied to the contact between the right fragment and cladding, while $-f_c$ is applied between the left fragment and cladding. On the other hand, a traction free condition is assumed between each fragment during expansion owing to the crack between them.

The above condition yields the following boundary conditions for the problem definition.

$$\begin{aligned}
&\sigma_{r\theta}^{1}(r,0) = -f_c \cdot \sigma_{\theta\theta}^{1}(r,0),\ \sigma_{r\theta}^{2}(r,0) = -f_c \cdot \sigma_{\theta\theta}^{2}(r,0),\\
&\sigma_{r\theta}^{1}(r,-\pi) = f_c \cdot \sigma_{\theta\theta}^{1}(r,-\pi),\ \sigma_{r\theta}^{3}(r,\pi) = f_c \cdot \sigma_{\theta\theta}^{3}(r,\pi),\\
&\sigma_{\theta\theta}^{2}(r,\varphi) = 0,\ \sigma_{\theta\theta}^{3}(r,\varphi) = 0,\ \sigma_{r\theta}^{2}(r,\varphi) = 0,\\
&\sigma_{r\theta}^{3}(r,\varphi) = 0,\ \sigma_{\theta\theta}^{1}(r,0) = \sigma_{\theta\theta}^{2}(r,0),\\
&\sigma_{\theta\theta}^{1}(r,-\pi) = \sigma_{\theta\theta}^{3}(r,\pi),\\
&u_{\theta}^{1}(r,0) = u_{\theta}^{2}(r,0),\ u_{\theta}^{1}(r,-\pi) = u_{\theta}^{3}(r,\pi).
\end{aligned} \tag{1}$$

where, σ_{ij}, u_i (i, j = r, θ) designate the stress and displacement components, respectively. The superscripts identify the body number.

2.2 Formulation

To form a characteristic equation for the asymptotic analysis, the Airy stress potential exploited by Williams [3].is used, which is shown as follows.

$$\begin{aligned}
\Phi = r^{\lambda+1}\{&a\cos(\lambda+1)\theta + b\sin(\lambda+1)\theta\\
&+ c\cos(\lambda-1)\theta + d\sin(\lambda-1)\theta\}
\end{aligned} \tag{2}$$

where, Φ is the Airy stress potential, λ is an eigenvalue that will determine the stress singularity (when $\lambda < 1$) , and a-d are the unknown constants to be determined corresponding to the boundary conditions of the problem.

By applying the well-known formulae of the relationship between the stress and displacement components and the Airy stress potential [4], a simultaneous equation of twelve homogenous equations is obtained from Eq. (1). To obtain non-trivial solutions, a_1-d_3 of the simultaneous equation, the determinant of the coefficient (12×12) should be null, algebraically. This provides a characteristic equation of the eigenvalue, λ. The final form of this is shown below.

$$F(\alpha,\beta,\varphi,f_c;\lambda)=0\ . \tag{3}$$

where α, β are the Dundurs constants that describe the material mismatch of the contacting bodies [5]. For the present material combination, shown in Table 1, it is obtained as $\alpha = 0.432748$, $\beta = 0.0973693$.

Table 1. Material properties of the pellet and cladding tube used for the present analysis

Component	Material	E (MPa)	ν
Pellet	UO2	185288	0.316
Cladding tube	Zry-4	72076	0.34

3. Numerical example and discussion

3.1 Description of asymptotic stress field

From Eq. (3), it is readily known that λ depends on α, β, φ and f_c. To reduce the number of independent variables, the contact angle, φ is fixed as 90°, but the coefficient of friction, f_c is varied as 0-0.9 in this work. It is intended that the variation of f_c will show the influence of shear force on the contact between the pellet and cladding during the pellet expansion. A larger f_c implies more expansion. In previous work where a bonded contact between a wedge and a half plane [1,2] was assumed, there occurred two eigenvalues, say, λ_I and λ_{II} which were associated with the stress singularity, i.e. $0 < \lambda_I < \lambda_{II} < 1$. In this case, λ_I and λ_{II} gave a stronger and a weaker singularity, respectively. Then, the asymptotic stresses were written as follows.

$$\sigma_{ij}(r,\theta)=K_I r^{\lambda_I-1} f_{ij}^I(\theta)+K_{II} r^{\lambda_{II}-1} f_{ij}^{II}(\theta) \tag{4}$$

where, K_I, K_{II} are the generalized stress intensity factors (GSIFs) of mode I and mode II, which calibrate the eigensolutions, $r^{\lambda_k-1} f_{ij}^k(\theta)$, ($k$ = I, II), incorporating the actual loading and dimensional conditions to evaluate the actual stress values.

On the other hand, in the case of a frictional contact between two contacting bodies, there occurred only one $\lambda = \lambda_s$ and $\lambda_I < \lambda_s < \lambda_{II}$ [6]. In this case, the asymptotic stresses are to be written as follows.

$$\sigma_{ij}(r,\theta)=K_s r^{\lambda_s-1} f_{ij}(\theta)\ . \tag{5}$$

3.2 Eigenvalue behavior

Eqs. (4) and (5) are the stress equations in the case of *two* contacting bodies. However, the present configuration of Fig. 1 consists of *three* bodies *mutually* in contact so it is anticipated that the relevant characteristic equation may have a different number of eigenvalues depending on the specific conditions. This kind of a contact problem consisting of three bodies has not been presented previously. Thus, this paper may be the first to provide a solution to the problem of mutually contacted three bodies.

Returning back to Eq. (3) for the present contact configuration of Fig. 1, we found that there was only one eigenvalue, λ_s (for $\lambda_s < 1$) when $0 \le f_c \le 0.9$, which is provided in Table 2. Therefore, the asymptotic form of the stress equation will follow Eq. (5). However, two eigenvalues were found when $-0.9 \le f_c \le -0.1$ although they are not included in Table 2.

Table 2. Eigenvalues evaluated for the present contact problem ($\alpha = 0.432748$, $\beta = 0.0973693$, $\varphi = 90°$), Fig. 1.

f_c	0	0.1	0.3	0.5	0.7	0.9
λ_s	0.458	0.479	0.522	0.567	0.615	0.667

If $f_c = 0.5$ is chosen for an example case, the spatial variation of the eigensolution, i.e. $f_{ij}(\theta)$ is as shown in Fig. 2.

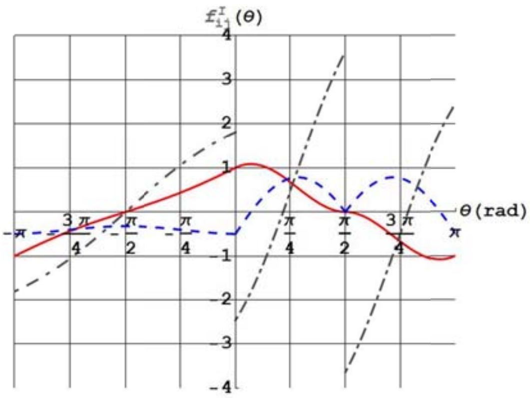

Fig. 2. Spatial variation of $f_{ij}(\theta)$ when $\alpha = 0.432748$, $\beta = 0.0973693$, $\varphi = 90°$ and $f_c = 0.5$; red solid: $f_{\theta\theta}(\theta)$, blue dash: $f_{r\theta}(\theta)$, black dash and dot: $f_{rr}(\theta)$.

To evaluate the actual stress field in the area around the contact surface between the cladding and adjacent pellet fragments, a finite element analysis may be conveniently used to calculate the GSIF, K_s. In the case of the bonded contact of two bodies, mode separation angles were looked for and each GSIF was calculated at each separation angle [1,2].

However, for the present case, the angle of the contact surface (θ = 0°) may be chosen as a proper location to calculate K_s due to the occurrence of a *single* GSIF as well as the present character of a *frictional* contact. The normal stress component at the contact surface, $\sigma_{\theta\theta}$, may be an appropriate parameter for the

GSIF calculation. Thus, K_s can be evaluated from the following equation.

$$K_s = \lim_{r \to 0} \left[\frac{\sigma_{\theta\theta}(r,0) \cdot r^{1-\lambda_s}}{f_{\theta\theta}(0)} \right]. \tag{6}$$

5. Conclusions

In the sequel to the previous work of an asymptotic analysis of a bonded contact between a wedge and a half plane (two bodies in contact) [1.2], a frictional contact problem of three bodies mutually contacted is considered here to simulate a further actual contact configuration of a cracked pellet and a cladding tube in PCMI. As a first step, a corresponding eigenvalue problem is formulated and the behavior of the eigenvalues is investigated in this work. The case of a 90° pellet crack is analyzed as a plausible example. The results are summarized as follows.

1) The number of eigenvalues associated with the stress singularity (i.e., eigenvalues less than unity) varies depending on the coefficient of friction, f_c. There is one eigenvalue when $0 \le f_c \le 0.9$ but two eigenvalues appear when $-0.9 \le f_c \le -0.1$.
2) In the PCMI condition, a positive f_c is thought to be applied to accommodate the pellet expansion. This results in the corresponding asymptotic stress field having the form of $\sigma_{ij}(r,\theta) = K_s r^{\lambda_s - 1} f_{ij}(\theta)$.
3) The spatial variation of $f_{ij}(\theta)$ of the stress equation is provided which will be used for a finite element analysis to calculate the generalized stress intensity factor.

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