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Interfacial Friction Factor Model for Two-dimensional Film Flow

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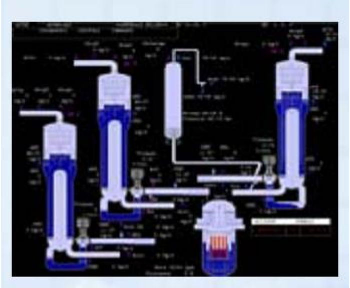
Introduction

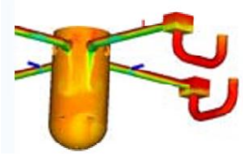
Trend of thermal hydraulic analysis in nuclear engineering

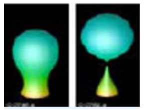
- Multi-scale, multi-dimensional, multi-phase, multi-physics …

  → HIGH PRECISION ANALYSIS!!
- Two-phase flow analysis


System Scale


Component Scale


CFD Scale


DNS Scale

V&V Multi-dimensional modules

- RELAP-5
- MARS
- TRACE
- CATHARE3
- SPACE

Computational Multi-Fluid Dynamics

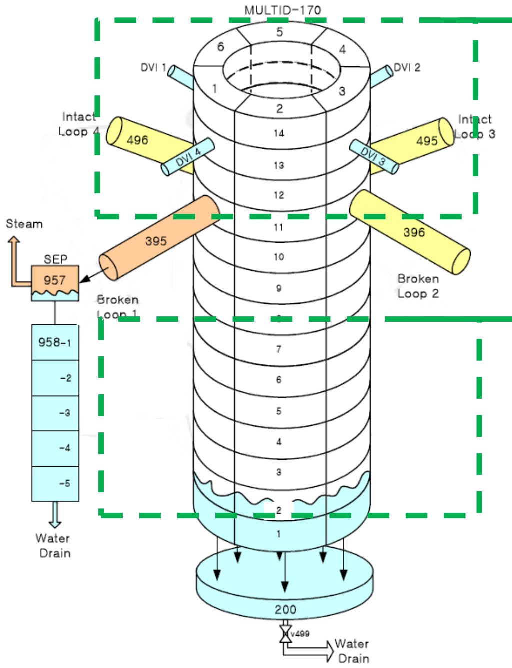
- Neptune-CFD
- STAR-CCM
- CUPID

Introduction

Motivation of this study

- **To produce benchmark data for multidimensional codes validation.**
- **To develop constitutive models based on multidimensional local measurements.**

⇨ Verification & Validation (V&V) of multidimensional codes



Upper downcomer
① MIDAS (KAERI)

Lower downcomer
① ROCOM (Rossendorf)
② DOBO (KAERI)
③ DYNAS (KAERI)

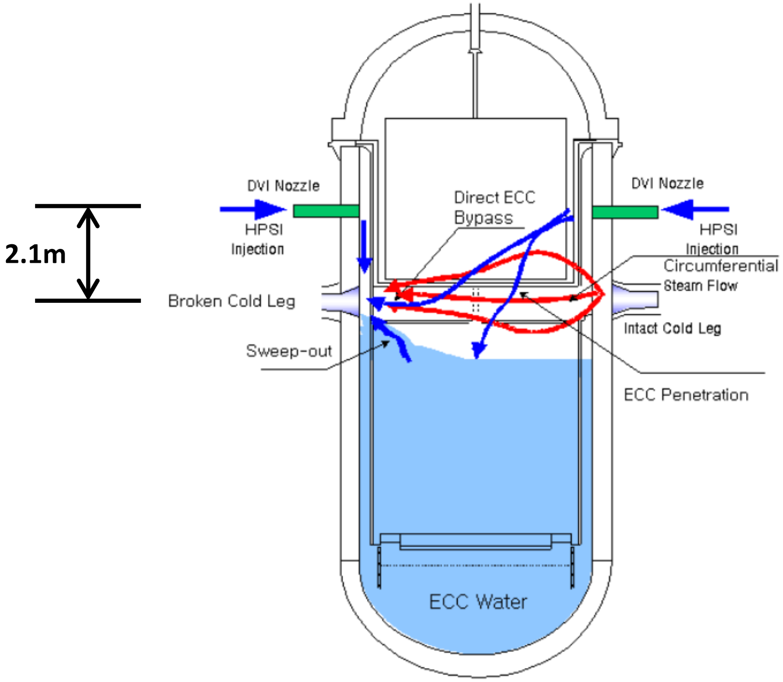
Challenges…

- ✓ Local measurement data in upper downcomer were insufficient.
- ✓ 1D empirical models have been used.

Introduction

❖ Upper downcomer multidimentional two-phase flow

- ECC(Emergency Core Coolant) bypass occurs during reflood phase of cold-leg LBLOCA
- 2.1m elevated DVI(Direct Vessel Injection) line in APR1400
- 2-fluid, 2-D film flow: Vertical liquid flow vs Circumferential vapor flow



Introduction

Objectives

2D Local Experimental Data Acquisition in Downcomer

↓

Liquid Film Experiment with Lateral Air Flow
Liquid film velocity
Liquid film thickness
Air velocity

↓

2D Film Flow Interfacial Friction Factor

↓

Validation of Multidimensional Code

2D Film Flow Experiments

Experimental Features

- 1/10 scaled down facility of unfolded downcomer
- Local liquid film velocity & thickness measurement
- Test conditions

	Real conditions	Scaled-down conditions*
Nozzle diameter (m)	0.22	**0.022**
Water inlet velocity (m/s)	2	**0.63**
Lateral air velocity (m/s)	15 ~ 45	**5 ~ 15**

*Modified linear scaling

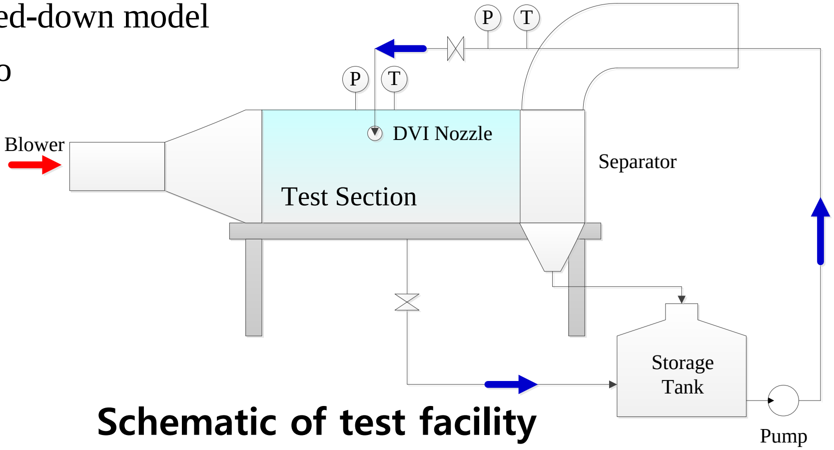
(B.J. Yun et al, "Scaling for the ECC bypass phenomena during the LBLOCA reflood phase," *NED, 2004)*

V_m : velocity of scaled-down model

V_p : velocity of proto

S : scaling ratio

$$j_k^* = \frac{\dot{m}_k}{\rho_k A_{flow}} \left[\frac{\rho_k}{(\rho_f - \rho_g) g D_{gap}} \right]^{1/2} \Rightarrow V_m = \frac{V_p}{\sqrt{S}}$$



Schematic of test facility

Flow Shape Change with Lateral Air Velocity

Hydrodynamic condition

- Inlet air velocity: 5 ~ 15 m/s (real scale 15 ~ 45m/s)

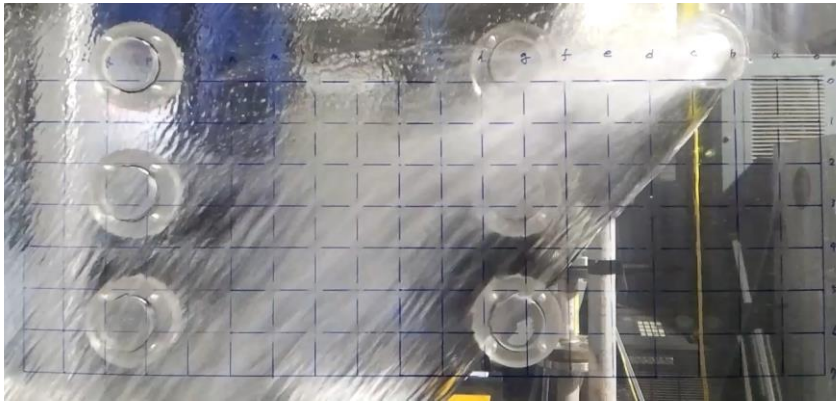
0 m/s

5 m/s

7 m/s

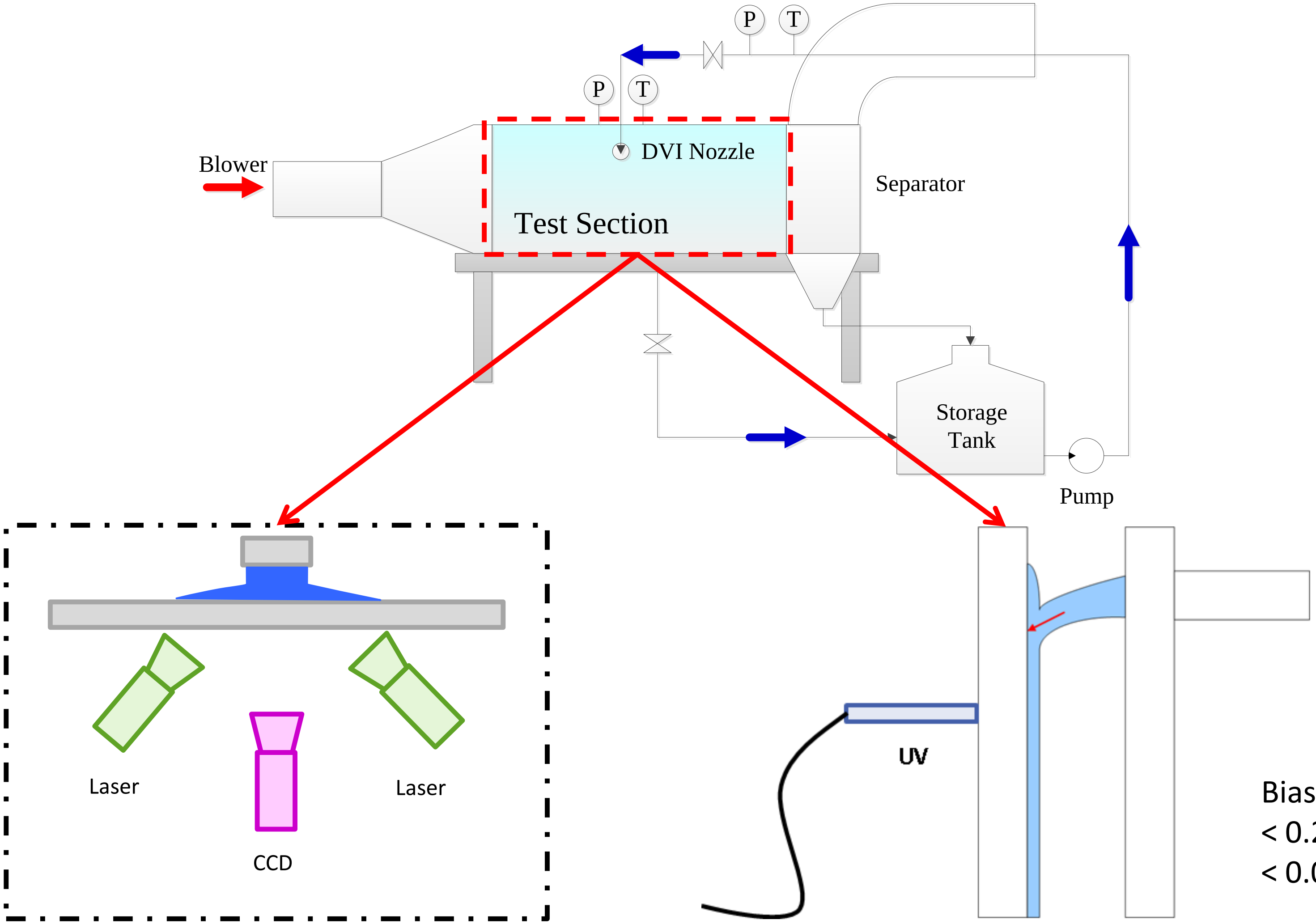
9 m/s

15 m/s



Experimental Measurement Methods

❖ Liquid film velocity and thickness



Uncertainty
5.65%

Bias error
< 0.2 %(Temp.)
< 0.04 mm(Time)

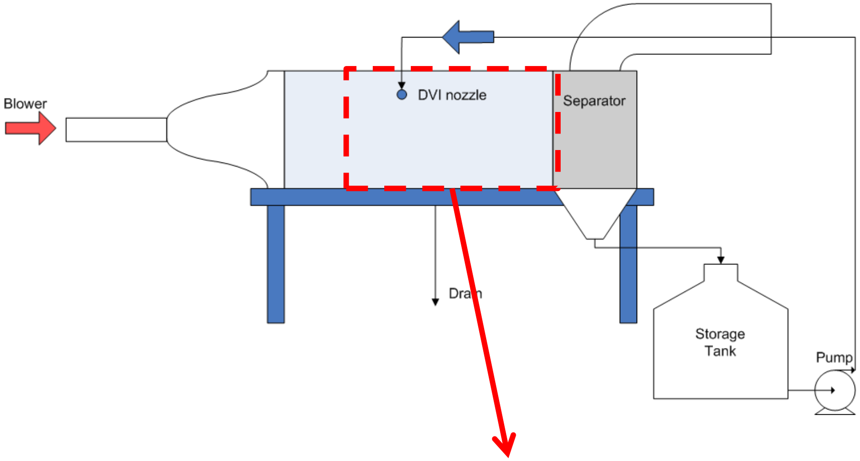
Liquid film velocity measurement with volume-averaged PIV* method

*Particle Image Velocimetry

Liquid film thickness measurement with ultrasonic thickness gauge

Liquid Film Velocity Measurement

❖ Control volume for liquid film velocity measurement



- 560nm long-pass filter
- Pulse: 500 μs
- Sampling: 100 Hz
- 500 pairs (5s)
- Adaptive PIV
  - Min. 32 pixels
  - Max. 64 Pixels

30 x 30 mm

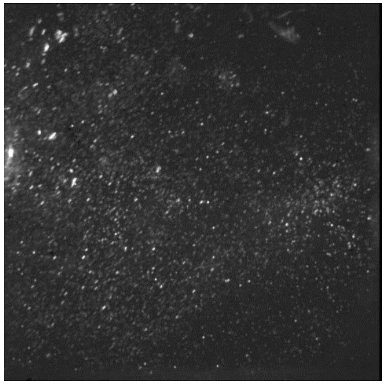
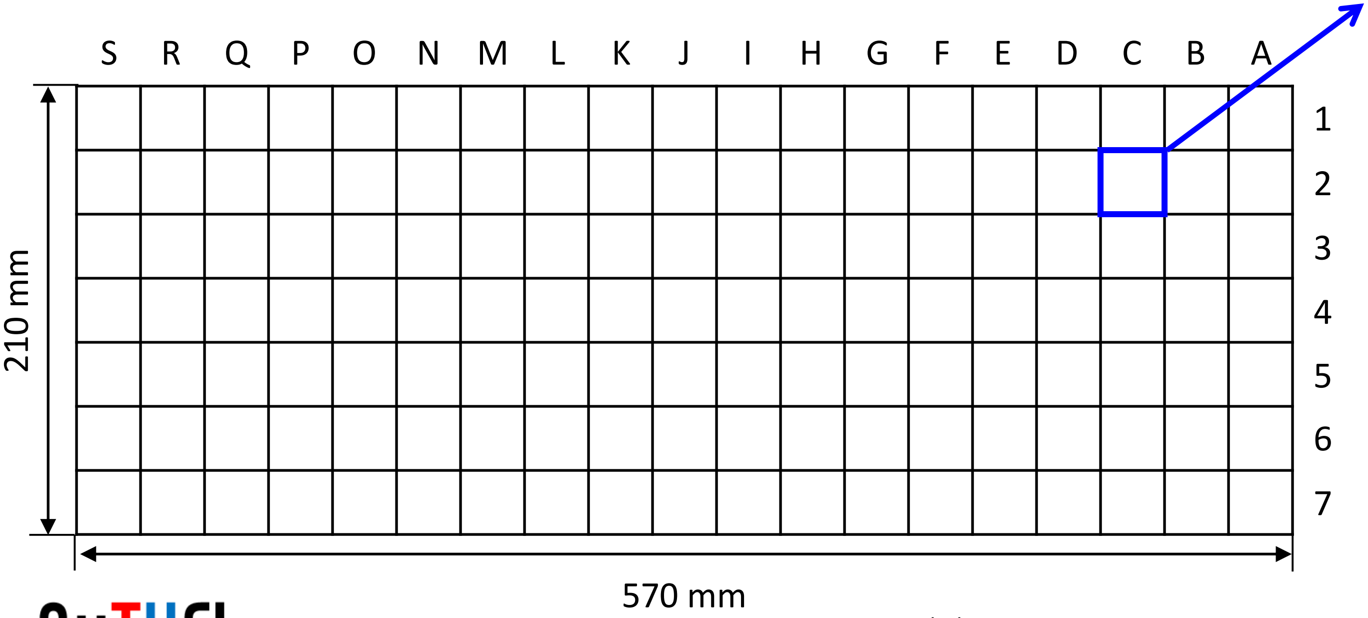
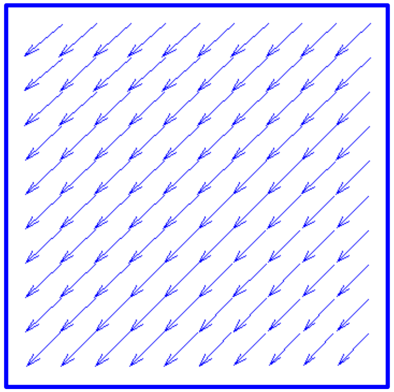


Image Pairs

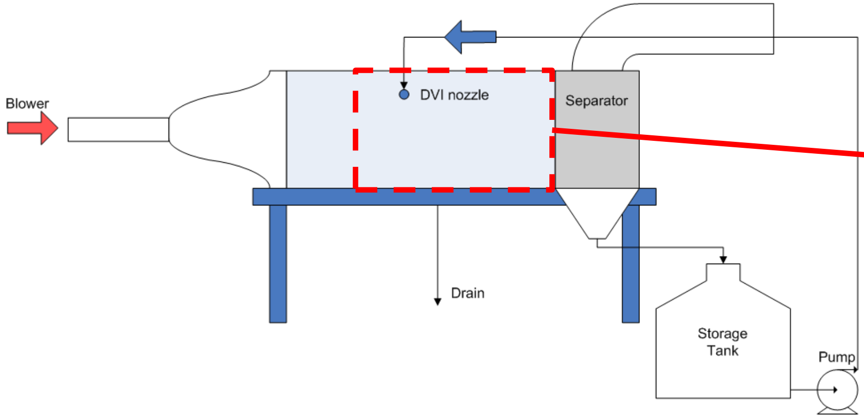


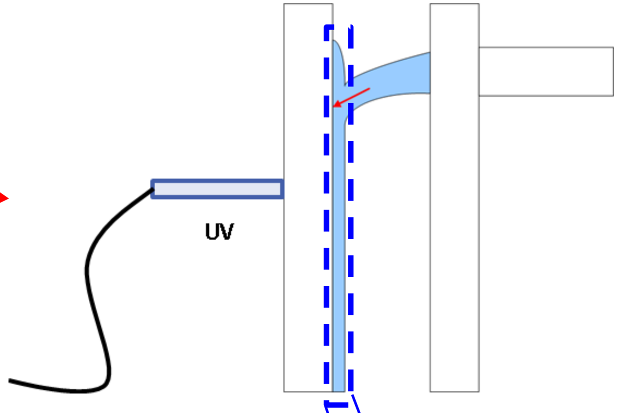


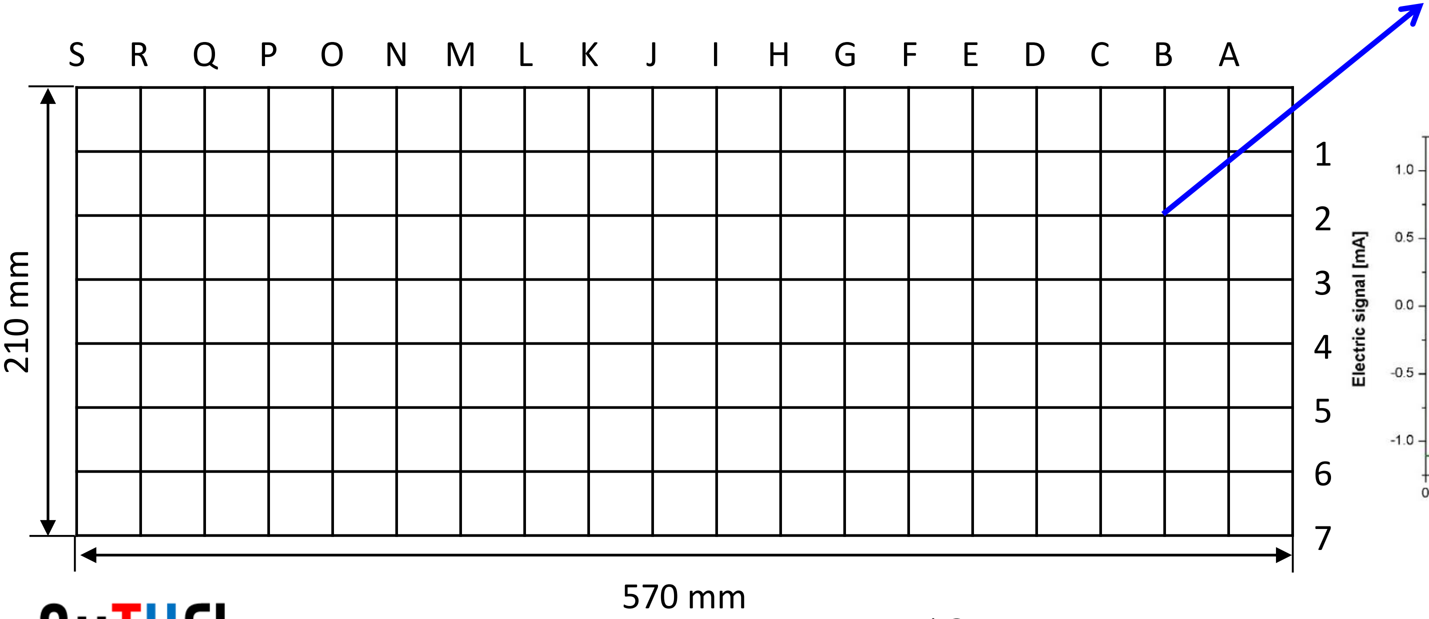
Analysis result

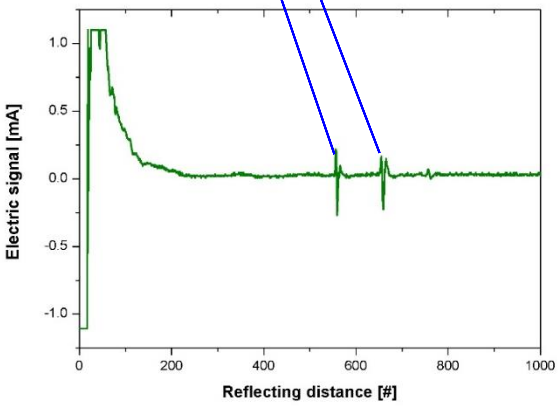
Liquid Film Thickness Measurement

Measurement points of liquid film thickness



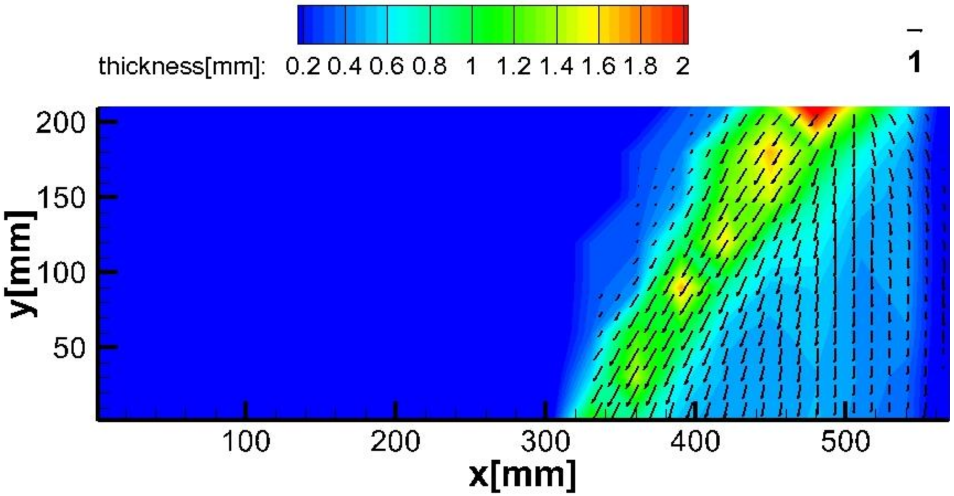


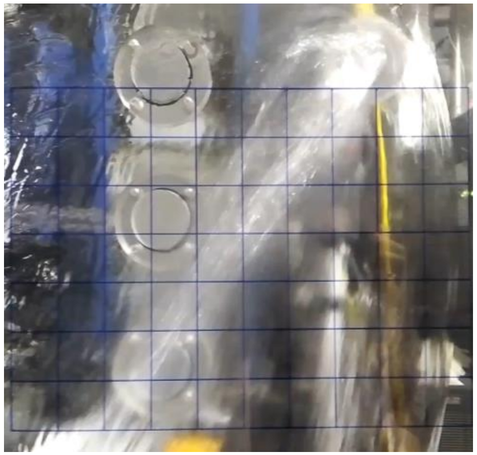




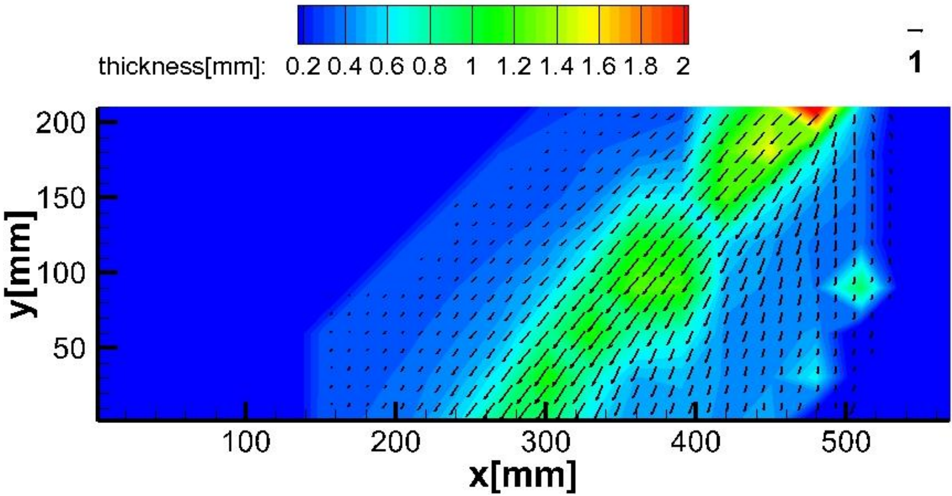
Results of 2D Film Flow Experiments

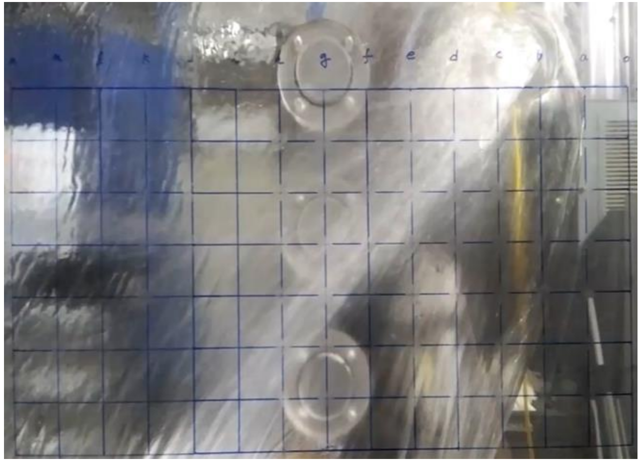
- **Results: 9, 11, 13, 15 m/s lateral air injection (1/2)**





9 m/s

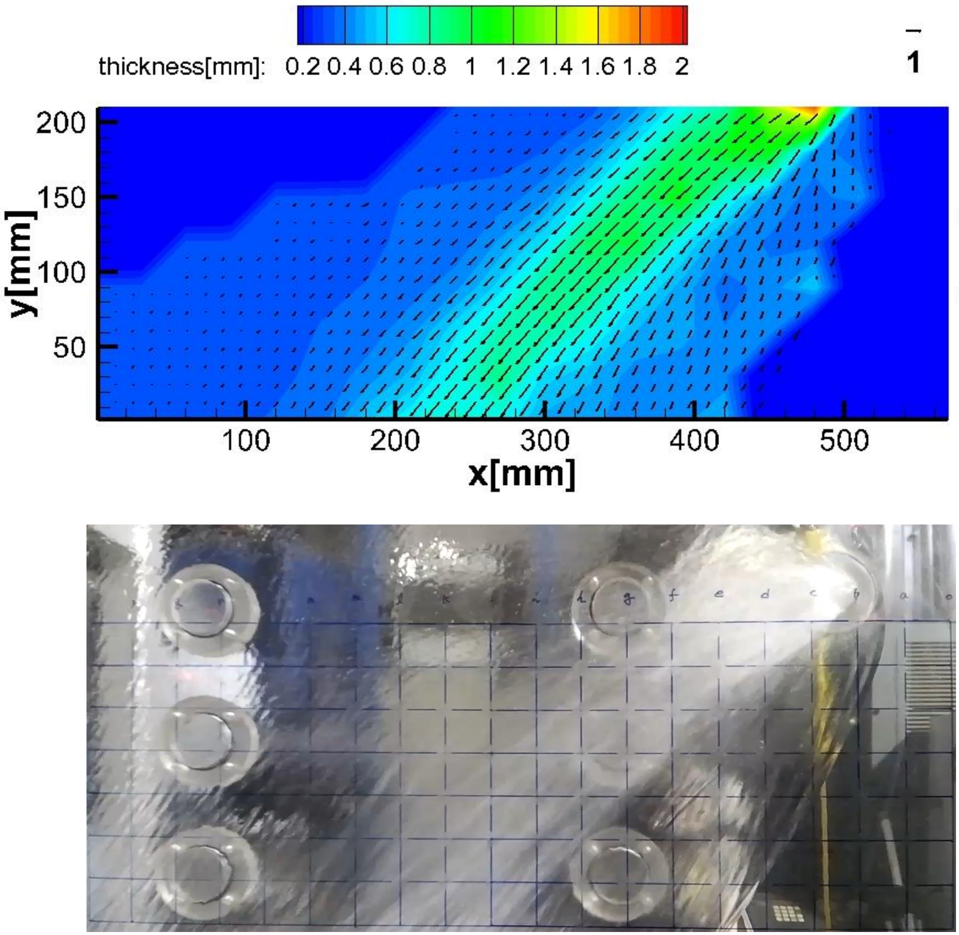




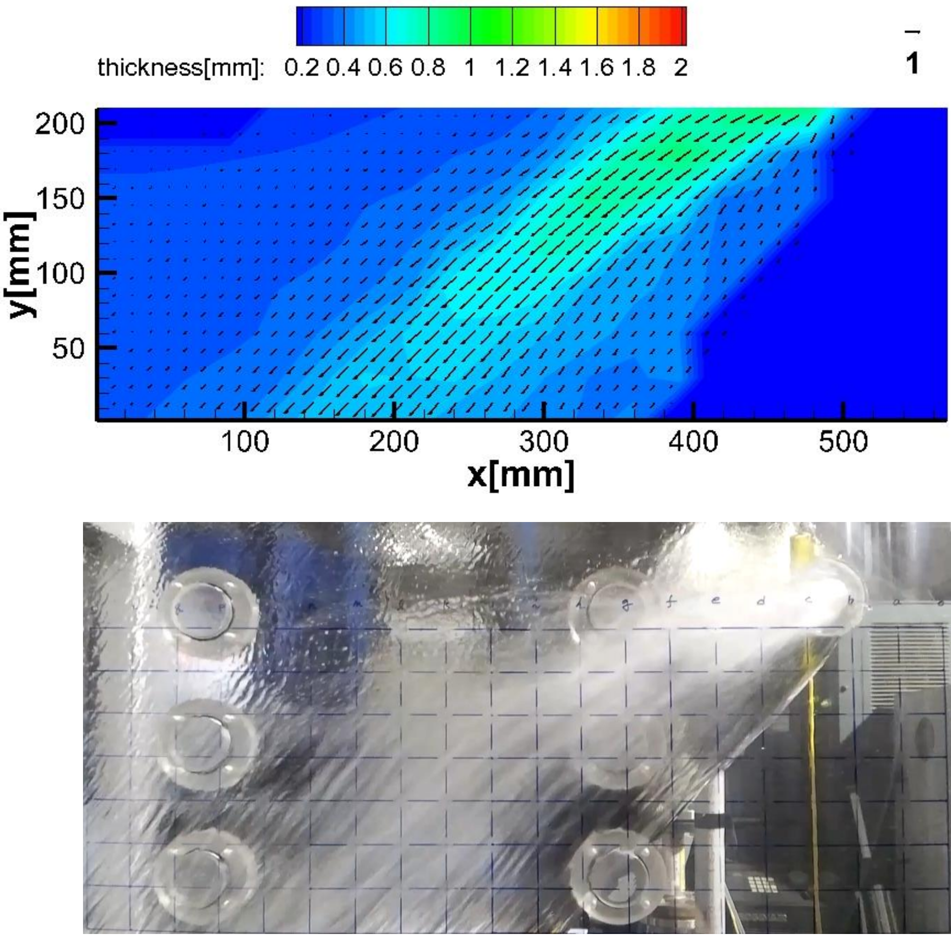
11 m/s

Results of 2D Film Flow Experiments

- **Results: 9, 11, 13, 15 m/s lateral air injection (2/2)**



13 m/s



15 m/s

1. Introduction

2. 2D Film Flow Experiments

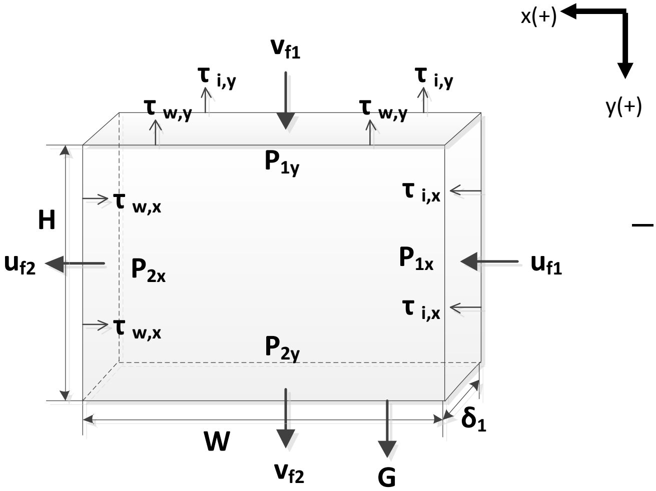
3. MOMENTUM CONSERVATION EQUATIONS

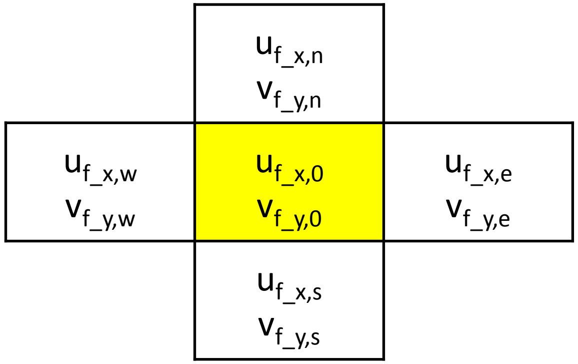
4. Mechanistic Model for Interfacial Friction Factor

5. Conclusions

Momentum Conservation Equations

Force balance of liquid film





	$u_{f\_x,n}$ $v_{f\_y,n}$	
$u_{f\_x,w}$ $v_{f\_y,w}$	$u_{f\_x,0}$ $v_{f\_y,0}$	$u_{f\_x,e}$ $v_{f\_y,e}$
	$u_{f\_x,s}$ $v_{f\_y,s}$	

X-direction

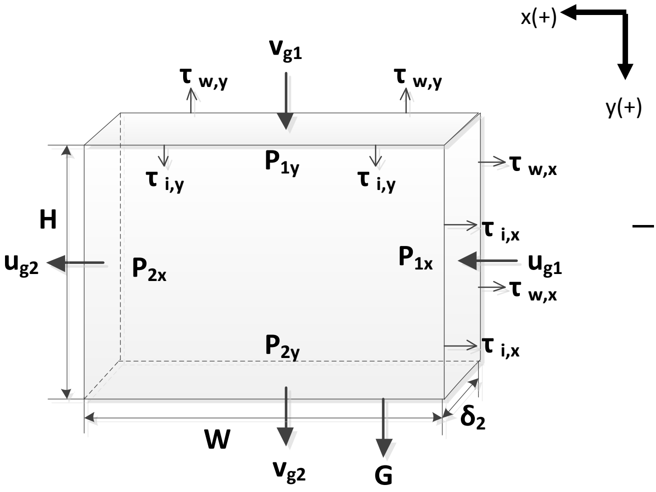
$$-\left(P_2-P_1\right)_x A_1+\int_A \tau_{i,x}dA_1-\int_A \tau_{wf,x}dA_1 = \sum_{in\_x}\left(\rho_f A_1 U_{f\_face}\times U_n\right)-\sum_{out\_x}\left(\rho_f A_1 U_{f\_face}\times U_n\right)$$

Y-direction

$$-\left(P_2-P_1\right)_y A_1-\int_A \tau_{i,y}dA_1-\int_A \tau_{wf,y}dA_1+\rho_f gV = \sum_{in\_y}\left(\rho_f A_1 U_{f\_face}\times U_n\right)-\sum_{out\_y}\left(\rho_f A_1 U_{f\_face}\times U_n\right)$$

Momentum Conservation Equations

Force balance of air

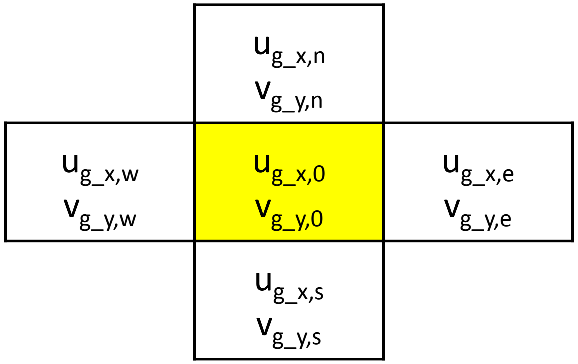


X-direction

$$-\left(P_2 - P_1\right)_x A_2 - \int_A \tau_{i,x} dA_2 - \int_A \tau_{wg,x} dA_2 = \sum_{in\_x}\left(\rho_g A_2 U_{g\_face} \times U_n\right) - \sum_{out\_x}\left(\rho_g A_2 U_{g\_face} \times U_n\right)$$

Y-direction

$$-\left(P_2 - P_1\right)_y A_2 + \int_A \tau_{i,y} dA_2 - \int_A \tau_{wg,y} dA_2 + \rho_g g V = \sum_{in\_y}\left(\rho_g A_2 U_{g\_face} \times U_n\right) - \sum_{out\_y}\left(\rho_g A_2 U_{g\_face} \times U_n\right)$$



	$u_{g\_x,n}$ $v_{g\_y,n}$	
$u_{g\_x,w}$ $v_{g\_y,w}$	$u_{g\_x,0}$ $v_{g\_y,0}$	$u_{g\_x,e}$ $v_{g\_y,e}$
	$u_{g\_x,s}$ $v_{g\_y,s}$	

Momentum Conservation Equations

X-direction force balance

Liquid film

$$\boxed{-\left(P_2-P_1\right)_x} A_1+\int_A \tau_{i,x}\,dA_1-\int_A \tau_{wf,x}\,dA_1 = \sum_{in\_x}\left(\rho_f A_1 U_{f\_face}\times U_n\right)-\sum_{out\_x}\left(\rho_f A_1 U_{f\_face}\times U_n\right)$$

Air

$$\boxed{-\left(P_2-P_1\right)_x} A_2-\int_A \tau_{i,x}\,dA_2-\int_A \tau_{wg,x}\,dA_2 = \sum_{in\_x}\left(\rho_g A_2 U_{g\_face}\times U_n\right)-\sum_{out\_x}\left(\rho_g A_2 U_{g\_face}\times U_n\right)$$

Y-direction force balance

Liquid film

$$\boxed{-\left(P_2-P_1\right)_y} A_1-\int_A \tau_{i,y}\,dA_1-\int_A \tau_{wf,y}\,dA_1+\rho_f gV = \sum_{in\_y}\left(\rho_f A_1 U_{f\_face}\times U_n\right)-\sum_{out\_y}\left(\rho_f A_1 U_{f\_face}\times U_n\right)$$

Air

$$\boxed{-\left(P_2-P_1\right)_y} A_2+\int_A \tau_{i,y}\,dA_2-\int_A \tau_{wg,y}\,dA_2+\rho_g gV = \sum_{in\_y}\left(\rho_g A_2 U_{g\_face}\times U_n\right)-\sum_{out\_y}\left(\rho_g A_2 U_{g\_face}\times U_n\right)$$

Momentum Conservation Equations

$$\frac{\int_A \tau_{i,x} dA_1}{A_1} - \frac{\int_A \tau_{wf,x} dA_1}{A_1} + \frac{\int_A \tau_{i,x} dA_2}{A_2} + \frac{\int_A \tau_{wg,x} dA_2}{A_2} = \sum_{in\_x} \left(\rho_f U_{f\_face} \times U_n\right) - \sum_{out\_x} \left(\rho_f U_{f\_face} \times U_n\right) - \sum_{in\_x} \left(\rho_g U_{g\_face} \times U_n\right) + \sum_{out\_x} \left(\rho_g U_{g\_face} \times U_n\right)$$

$$-\frac{\int_A \tau_{i,y} dA_1}{A_1} - \frac{\int_A \tau_{wf,y} dA_1}{A_1} - \frac{\int_A \tau_{i,y} dA_2}{A_2} + \rho_f gH - \rho_g gH = \sum_{in\_y} \left(\rho_f U_{f\_face} \times U_n\right) - \sum_{out\_y} \left(\rho_f U_{f\_face} \times U_n\right) - \sum_{in\_y} \left(\rho_g U_{g\_face} \times U_n\right) + \sum_{out\_y} \left(\rho_g U_{g\_face} \times U_n\right)$$

$$\int_A \tau_{i,x} dA = \int_A \frac{1}{2} \rho_g f_i \left|\vec{U}_g - \vec{U}_f\right| \left(U_g - U_f\right) dA = \int_A \frac{1}{2} \rho_g f_i \left(u_g(x) - u_f(x)\right) \sqrt{\left(u_g(x) - u_f(x)\right)^2 + v_f^2(y)} dA$$

$$\int_A \tau_{wf,x} dA = \int_A \frac{1}{2} \rho_f f_{wf} u_f(x) \sqrt{u_f^2(x) + v_f^2(y)} dA$$

$$\int_A \tau_{wg,x} dA = \int_A \frac{1}{2} \rho_g f_{wg} u_g^2(x) dA$$

Blasius' equation

$$f_{wg} = 0.79 \,\mathrm{Re}^{-1/4}$$

$$\int_A \tau_{i,y} dA = \int_A \frac{1}{2} \rho_g f_i \left|\vec{U}_g - \vec{U}_f\right| \left(U_g - U_f\right) dA = \int_A \frac{1}{2} \rho_g f_i v_f(x) \sqrt{\left(u_g(x) - u_f(x)\right)^2 + v_f^2(y)} dA$$

$$\int_A \tau_{wf,y} dA = \int_A \frac{1}{2} \rho_f f_{wf} v_f(y) \sqrt{u_f^2(x) + v_f^2(y)} dA$$

Assume that…

① Ignore lateral air flow acceleration

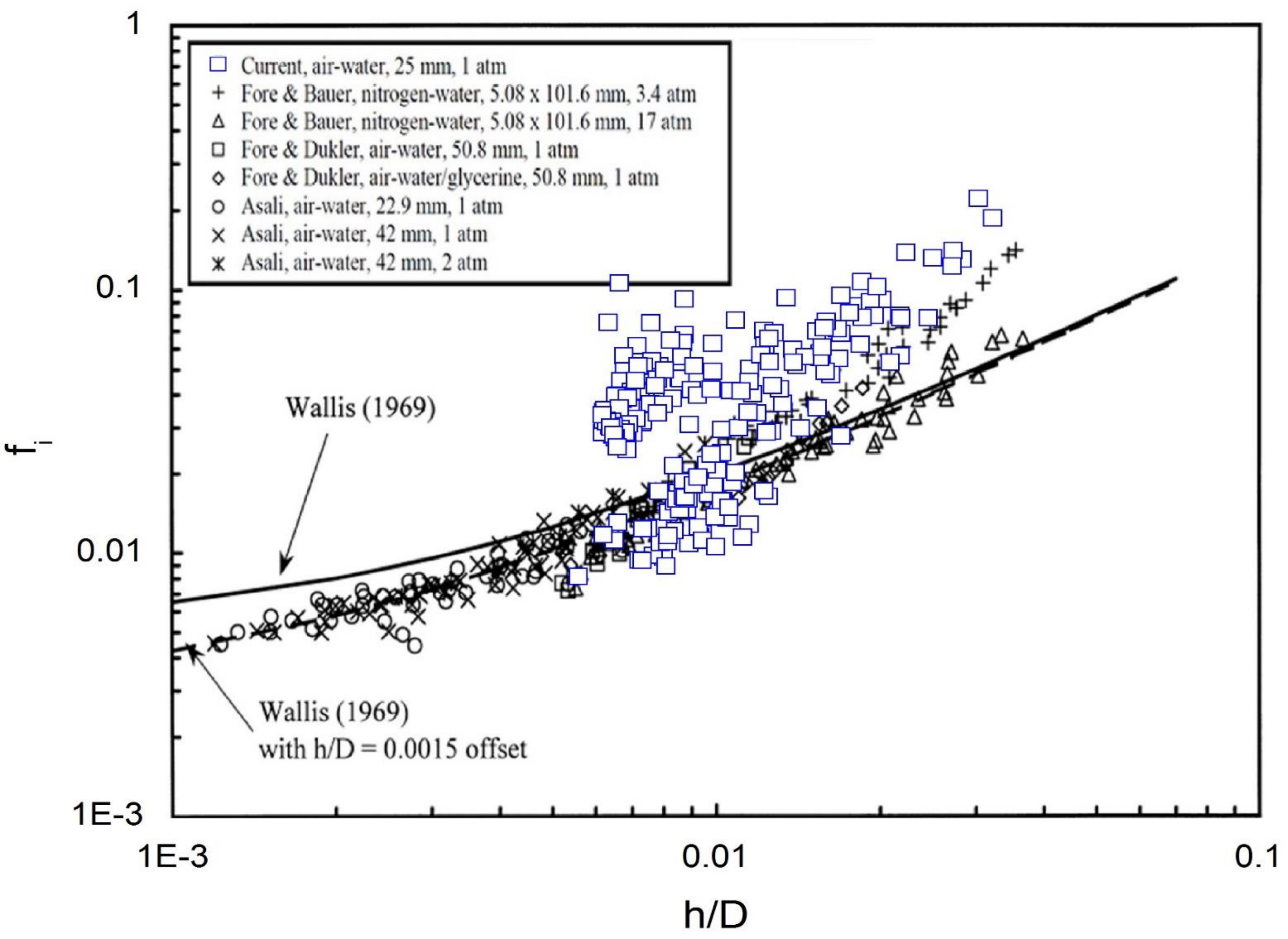
② Linear velocity profile

③ Constant friction coefficient

④ Gas wall friction factor follows Blasius' Eq.

$$\Rightarrow \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \begin{pmatrix} f_i \\ f_w \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Local Interfacial Friction Factors vs. Previous Studies



L.B. Fore, S.G. Beus, R.C. Bauer, “Interfacial friction in gas-liquid annular flow: analogies to full and transition roughness”, International Journal of Multiphase Flow, Vol. 26, p 1755-1769, 2000.

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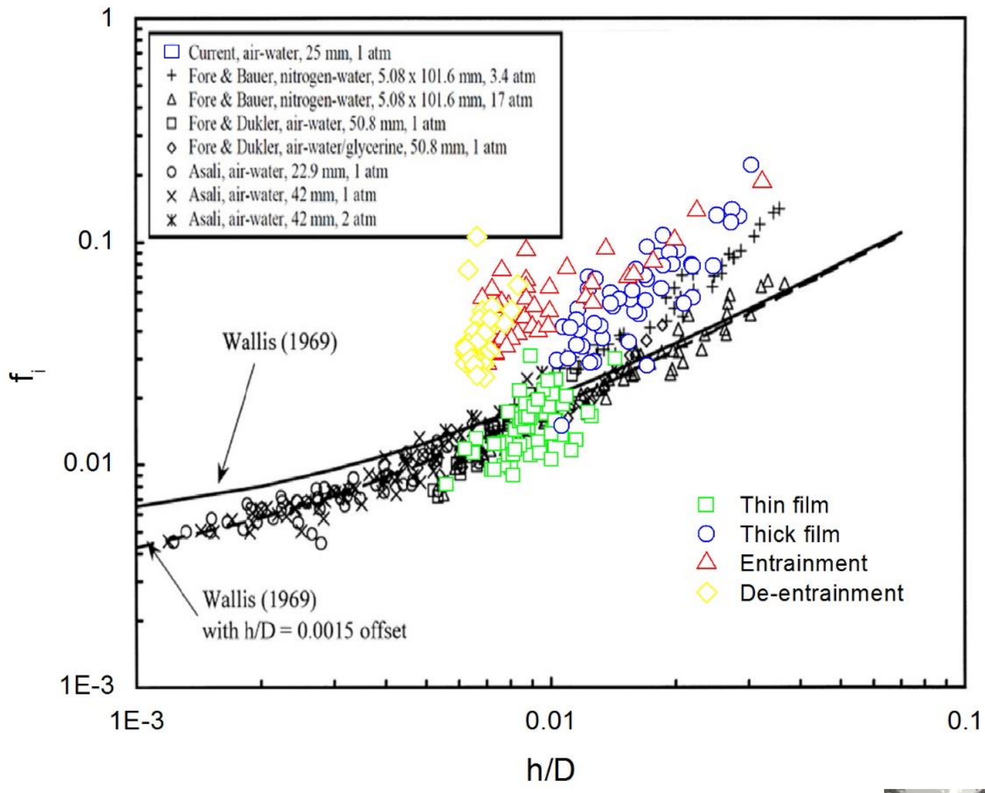
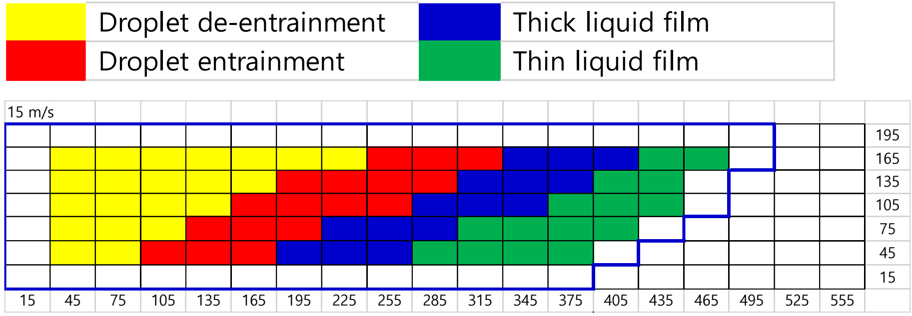
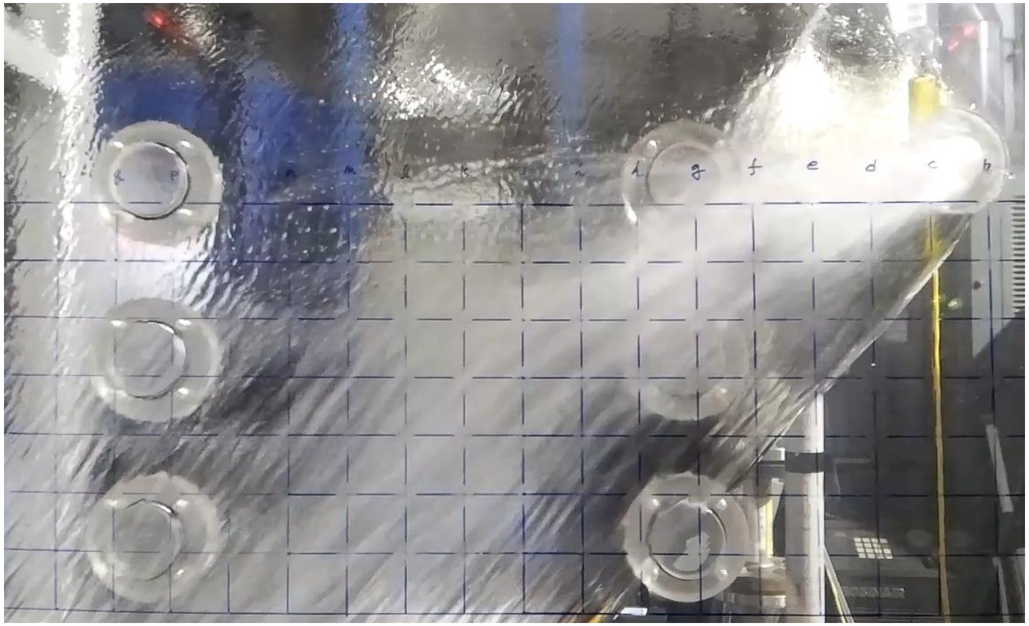
Flow Pattern Definition





🟨	Droplet de-entrainment	🟦	Thick liquid film
🟥	Droplet entrainment	🟩	Thin liquid film

15 m/s																			
																			195
	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩				165
	🟨	🟨	🟨	🟨	🟨	🟥	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩					135
	🟨	🟨	🟨	🟨	🟥	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩	🟩					105
	🟨	🟨	🟨	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩	🟩	🟩						75
	🟨	🟨	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩	🟩	🟩							45
																			15
15	45	75	105	135	165	195	225	255	285	315	345	375	405	435	465	495	525	555	



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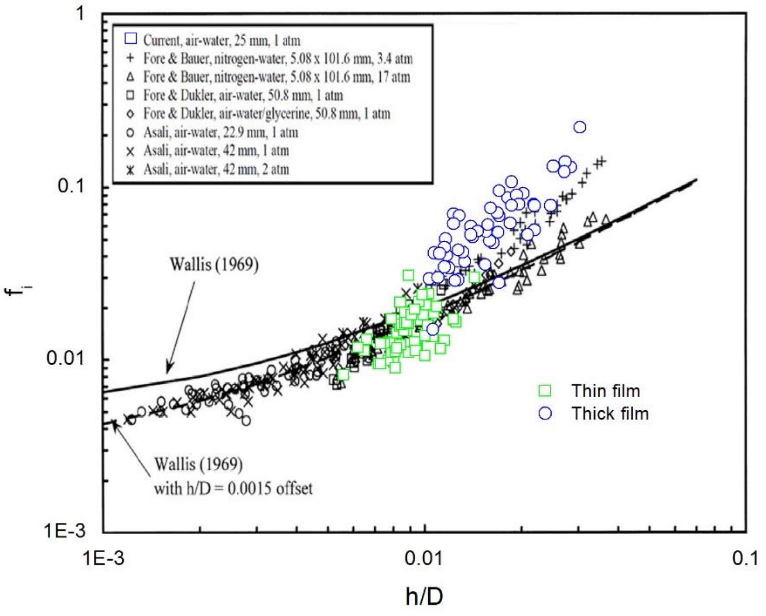
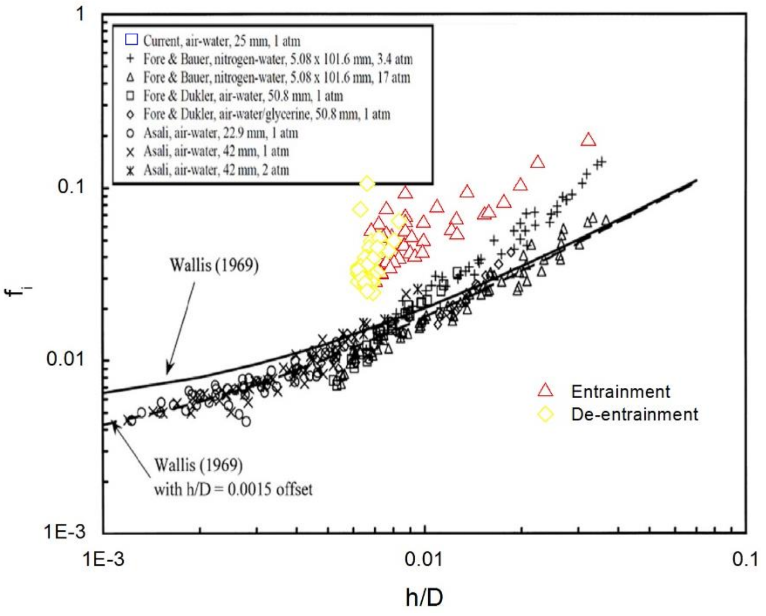
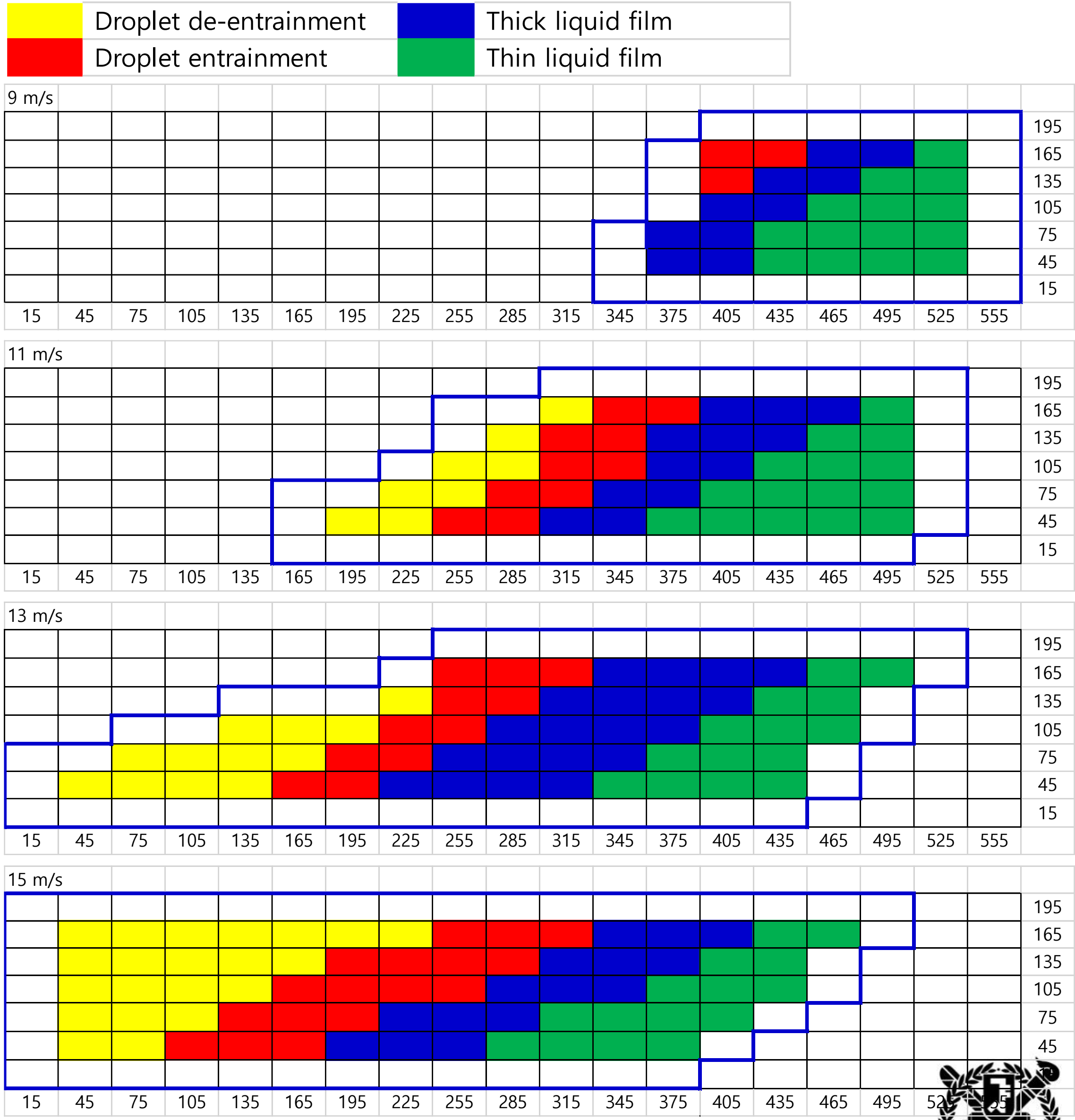
Flow Pattern Definition







🟨	Droplet de-entrainment	🟦	Thick liquid film
🟥	Droplet entrainment	🟩	Thin liquid film

9 m/s

15	45	75	105	135	165	195	225	255	285	315	345	375	405	435	465	495	525	555	
																			195
													🟥	🟥	🟦	🟦	🟩		165
													🟥	🟦	🟦	🟩	🟩		135
													🟦	🟦	🟩	🟩	🟩		105
												🟦	🟦	🟩	🟩	🟩	🟩		75
												🟦	🟦	🟩	🟩	🟩	🟩		45
																			15

11 m/s

15	45	75	105	135	165	195	225	255	285	315	345	375	405	435	465	495	525	555	
																			195
										🟨	🟥	🟥	🟦	🟦	🟦	🟩			165
									🟨	🟥	🟥	🟦	🟦	🟦	🟩	🟩			135
								🟨	🟨	🟥	🟥	🟦	🟦	🟩	🟩	🟩			105
							🟨	🟨	🟥	🟥	🟦	🟦	🟩	🟩	🟩	🟩			75
						🟨	🟨	🟥	🟥	🟦	🟦	🟩	🟩	🟩	🟩	🟩			45
																			15

13 m/s

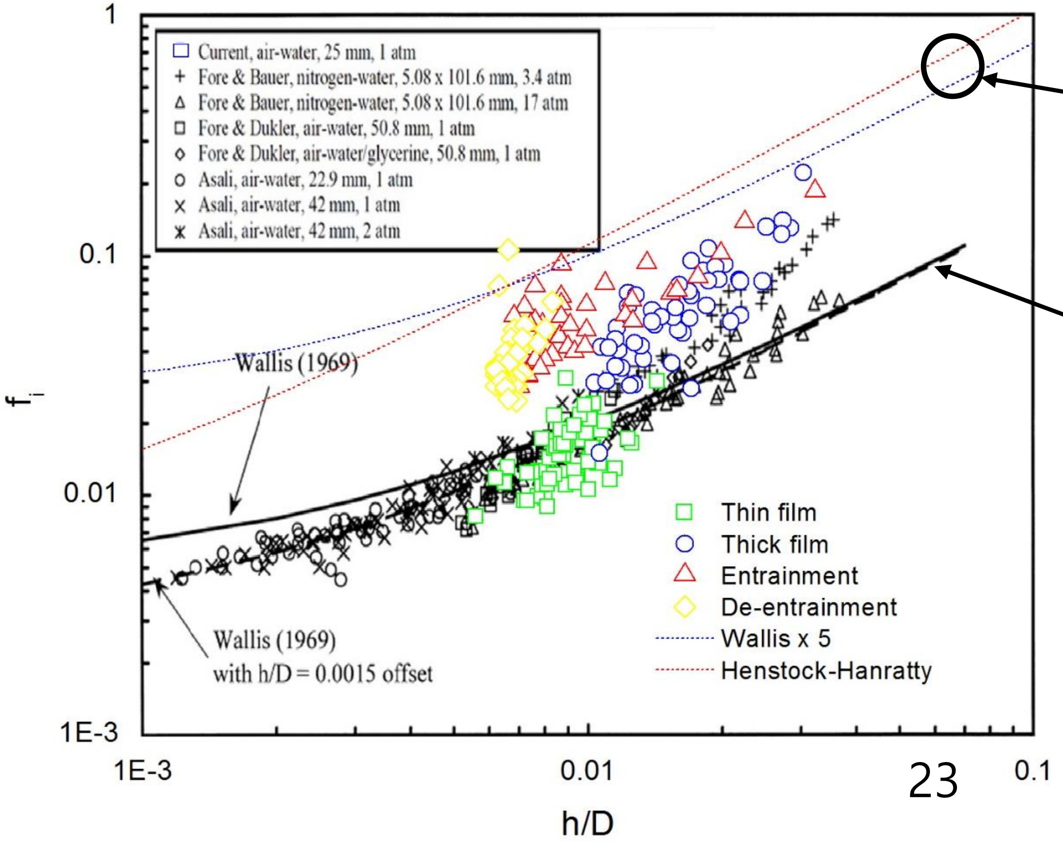
15	45	75	105	135	165	195	225	255	285	315	345	375	405	435	465	495	525	555	
																			195
								🟥	🟥	🟥	🟦	🟦	🟦	🟦	🟩	🟩			165
							🟨	🟥	🟥	🟦	🟦	🟦	🟦	🟩	🟩				135
				🟨	🟨	🟨	🟥	🟥	🟦	🟦	🟦	🟦	🟩	🟩	🟩				105
		🟨	🟨	🟨	🟨	🟥	🟥	🟦	🟦	🟦	🟦	🟩	🟩	🟩					75
	🟨	🟨	🟨	🟨	🟥	🟥	🟦	🟦	🟦	🟦	🟩	🟩	🟩	🟩					45
																			15

15 m/s

15	45	75	105	135	165	195	225	255	285	315	345	375	405	435	465	495	52	55	
																			195
	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩				165
	🟨	🟨	🟨	🟨	🟨	🟥	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩					135
	🟨	🟨	🟨	🟨	🟥	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩	🟩					105
	🟨	🟨	🟨	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩	🟩	🟩						75
	🟨	🟨	🟥	🟥	🟥	🟦	🟦	🟦	🟩	🟩	🟩	🟩							45

Interfacial Friction Factor in MARS-3D

Film flow	Stable	$f_i = 0.0025(1 + 75\alpha_l)$	Wallis
	Unstable	$f_i = f_s \left\{ 1 + 1400F \left[1 - \exp\left(-\frac{1}{G} \frac{(1 + 1400F)^{3/2}}{13.2F} \right) \right] \right\}$	Use the larger one between Henstock-Hanratty and $5 \times$ Wallis.

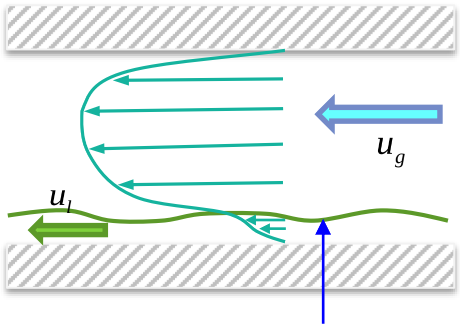


Unstable
: Vertical film flow
Higher relative velocity

Stable

4. MECHANISTIC MODEL FOR INTERFACIAL FRICTION FACTOR

Mechanistic Model for Interfacial Friction Factor



Turbulent boundary layer

$$u^+ = \frac{1}{\kappa}\ln y^+ + B - \Delta B\left(k_s^+\right)$$

$$u^+ = \frac{u_g - u_i}{u^*}$$

B: smooth wall single-phase flow constant, 5.5
κ: Von Karman constant, 0.4
ΔB: effects of the rough wall; increases with roughness, k_s^+

$$\Delta B = \frac{1}{\kappa}\ln\left(1 + Ck_s^+\right)$$

"Thickness of wavy thin liquid film ≈ wall roughness"

$$\mathrm{Re}_t = \frac{\rho t u_{avg}}{\mu} = \int_0^{t^+} u^+ dy^+ = \frac{t^+}{\kappa}\left(\ln t^+ + B\kappa - \Delta B\kappa - 1\right)$$

$$f_i = 2\frac{u^{*2}}{u_{avg}^2} = 2\left(\frac{\rho t u^*}{\mu}\right)^2\left(\frac{\mu}{\rho t u_{avg}}\right)^2 = 2\frac{t^{+2}}{\mathrm{Re}_t^{\,2}} \quad \text{where} \quad u^* = \sqrt{\tau_i / \rho_g} \quad \Rightarrow \quad \mathrm{Re}_t = \sqrt{\frac{2}{f_i}}\,t^+$$

R.Kumar and D.P. Edwards, "Interfacial shear modeling in two-phase annular flow", KAPL Atomic Power Laboratory, KAPL-P-000220, 1996.

$$\therefore \quad \sqrt{\frac{2}{f_i}} = \frac{1}{\kappa}\left[\ln\left(\frac{t^+}{1 + Ck_s^+}\right) + B\kappa - 1\right]$$

Mechanistic Model for Interfacial Friction Factor

- **New approach with local experimental results**

$$\therefore\quad \sqrt{\frac{2}{f_i}}=\frac{1}{\kappa}\left[\ln\left(\frac{t^+}{1+Ck_s^+}\right)+B\kappa-1\right]$$

$$t^+=\mathrm{Re}_t\sqrt{\frac{f_i}{2}}$$

$$k_s^+=\frac{k_s u^*}{\nu}=\frac{tu^*}{\nu}\frac{k_s}{t}=\frac{tu_{avg}}{\nu}\frac{u^*}{u_{avg}}\frac{k_s}{t}=\mathrm{Re}_t\sqrt{\frac{f_i}{2}}\frac{k_s}{t}$$

⇩

$$0.4\times\sqrt{\frac{2}{f_i}}=\ln\left(\frac{\mathrm{Re}_t\sqrt{\frac{f_i}{2}}}{1+C\,\mathrm{Re}_t\sqrt{\frac{f_i}{2}}\frac{k_s}{t}}\right)+1.2$$

Assume that…

① Modeling constant(C) = 6.5(unstable) / 4.5(stable)

② For large $\mathrm{Re}_t\sqrt{4f_i}$

③ $k_s/t \ll 1$

⇩

$$f_i=\frac{1}{4}\left[1.879-4.912\mathrm{C}\frac{k_s}{t}\right]^{-2}$$

$$f_i=\frac{1}{4}\left[1.879-22.104\left(\frac{h}{D}\underline{-0.0015}\right)\right]^{-2}\underline{-0.066}$$

Initial point for single gas flow

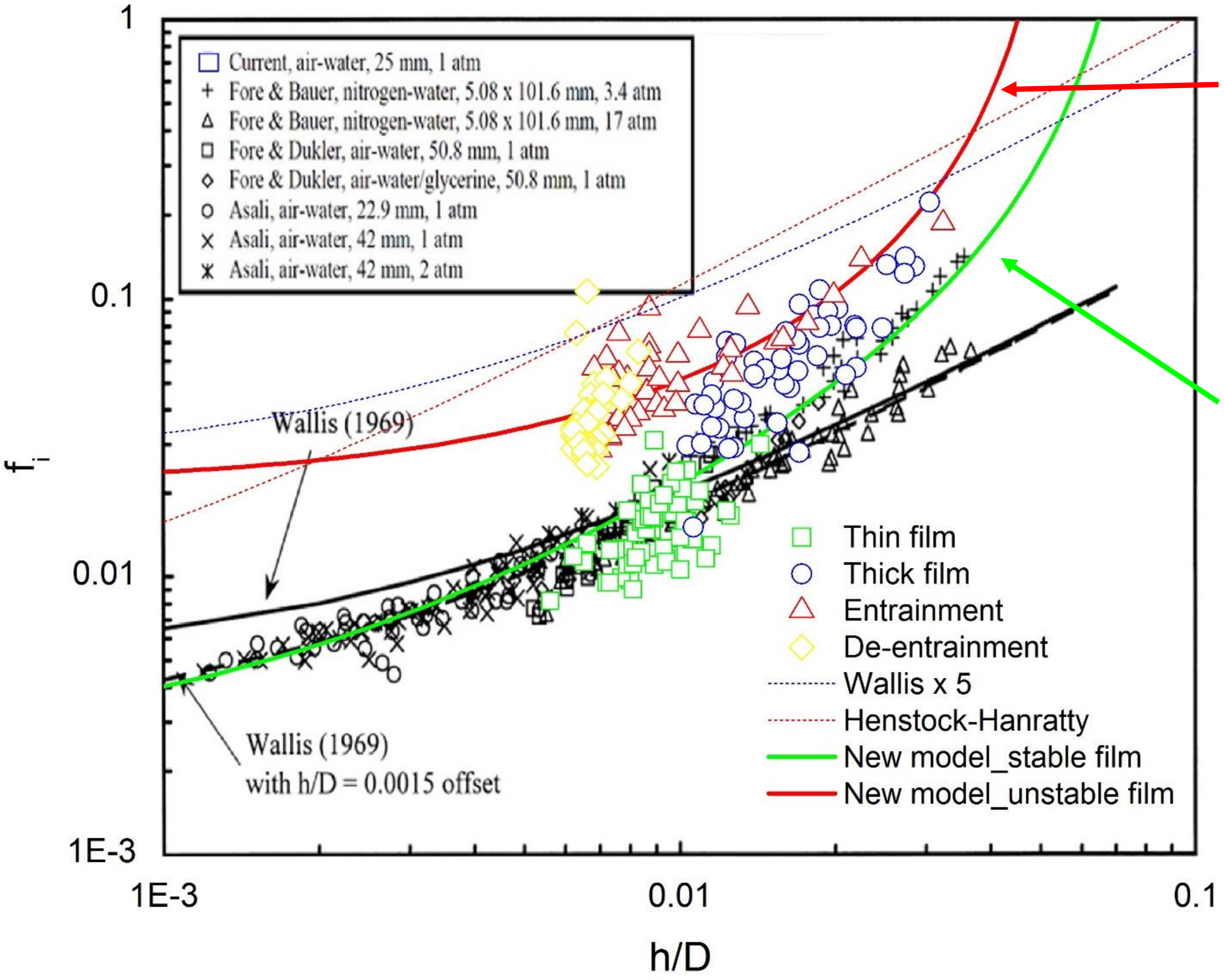
$$\left(f_{sp}=0.005\text{ in stable film}\right)$$

$$f_i=\frac{1}{4}\left[1.879-32.928\left(\frac{h}{D}\underline{-0.0015}\right)\right]^{-2}\underline{-0.046}$$

$$\left(5\times f_{sp}=0.025\text{ in unstable film}\right)$$

$$D=2t,\quad k_s\approx 4h$$

Comparison with Previous Studies



$$f_i = \frac{1}{4}\left[1.879 - 32.928\left(\frac{h}{D} - 0.0015\right)\right]^{-2} - 0.046$$

Unstable film flow
(entrainment, de-entrainment)

$$f_i = \frac{1}{4}\left[1.879 - 22.104\left(\frac{h}{D} - 0.0015\right)\right]^{-2} - 0.066$$

Stable film flow
(thin film, thick film w/o entrainment)

5. CONCLUSIONS

Conclusions

❖ Summary

- Local 2D liquid film velocity & thickness measurement were carried out.
  → Validation data for multidimensional codes were produced.
- Interfacial friction factors were calculated by 2D momentum conservation equations.
- Mechanistic interfacial friction factor model was developed based on law of wall and thin film roughness concept.

Stable film flow (thin film, thick film w/o entrainment)	$f_i = \frac{1}{4}\left[1.879 - 22.104\left(\frac{h}{D} - 0.0015\right)\right]^{-2} - 0.066$
Unstable film flow (entrainment, de-entrainment)	$f_i = \frac{1}{4}\left[1.879 - 32.928\left(\frac{h}{D} - 0.0015\right)\right]^{-2} - 0.046$

❖ Further work

- ✓ Film thickness measurement error in entrainment & de-entrainment region
- ✓ Lateral air velocity acceleration
- ✓ Gas wall friction
- ✓ Transition criterion between stable/unstable film

APPENDIX

1. Experimental methods
2. Mass and Momentum Conservation
3. Uncertainty Analysis
4. Challenges

EXPERIMENTAL METHODS

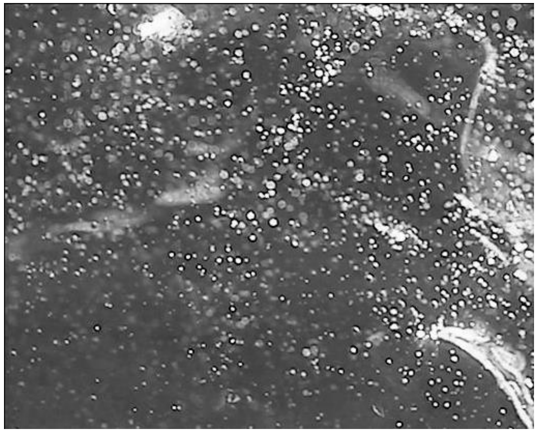
Application of Volume-PIV Method

Volume-averaged PIV measurement method

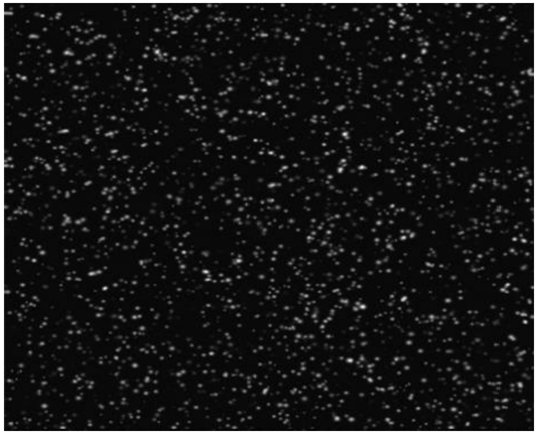
- Continuous wave lasers: 532 nm laser(green) / 10W + 5W
- High speed camera (PHANTOM v211)
- Add fluorescent particles (1~20 μm).

  : luminous particles in range of 570 nm wavelength reflecting 532 nm laser source.
- Use a long-pass filter which cuts off image information under 560 nm wavelength.

Set-up of volume-PIV method



Original image

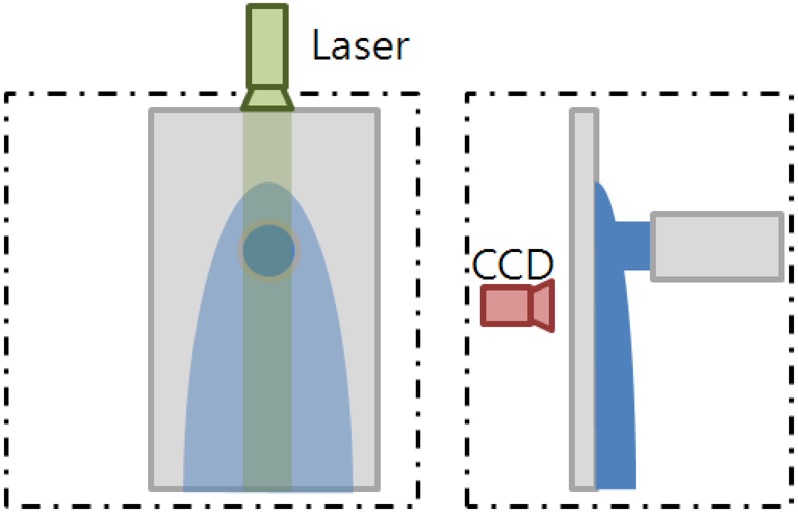


Filtered image

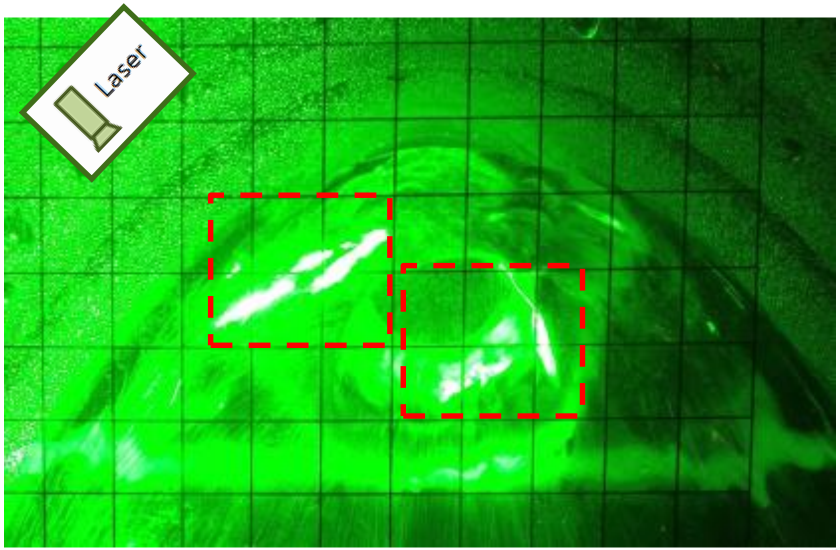
Limitation of Sheet-PIV Method

Why the sheet-PIV is not appropriate method to measure liquid film velocity…

- Geometry
  - Laser sheet cannot help being injected along to the vertical direction.
- Optics
  - Laser is scattered by the oscillating film boundary.
  - Attenuation of laser intensity doesn’t maintain illumination.



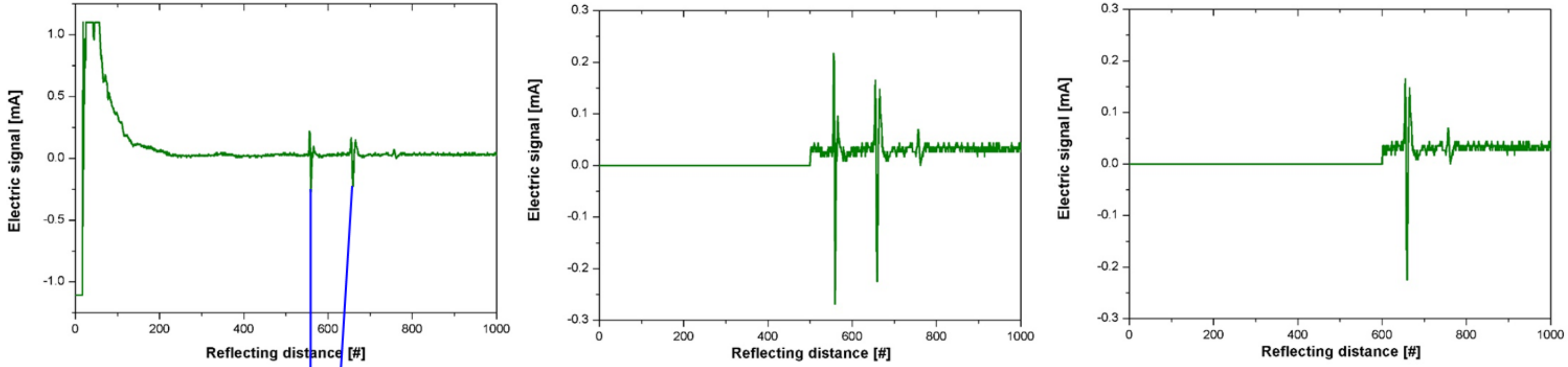
Application of sheet-PIV on liquid film flow



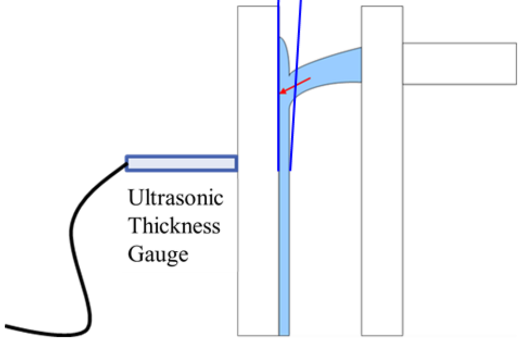
Liquid Film Thickness Measurement

Ultrasonic velocity gauge

- **Signal frequency: 50Mhz / Sample rate: 20Hz (200samples per 10seconds)**



Raw data



① **Remove acryl**
② **Rescaling**
③ **Find first peak point**
④ **Remove tails**
⑤ **Find second peak point**

$$\delta = \frac{v_u}{2}\left(\frac{x_{2p} - x_{1p}}{f_{signal}}\right)$$

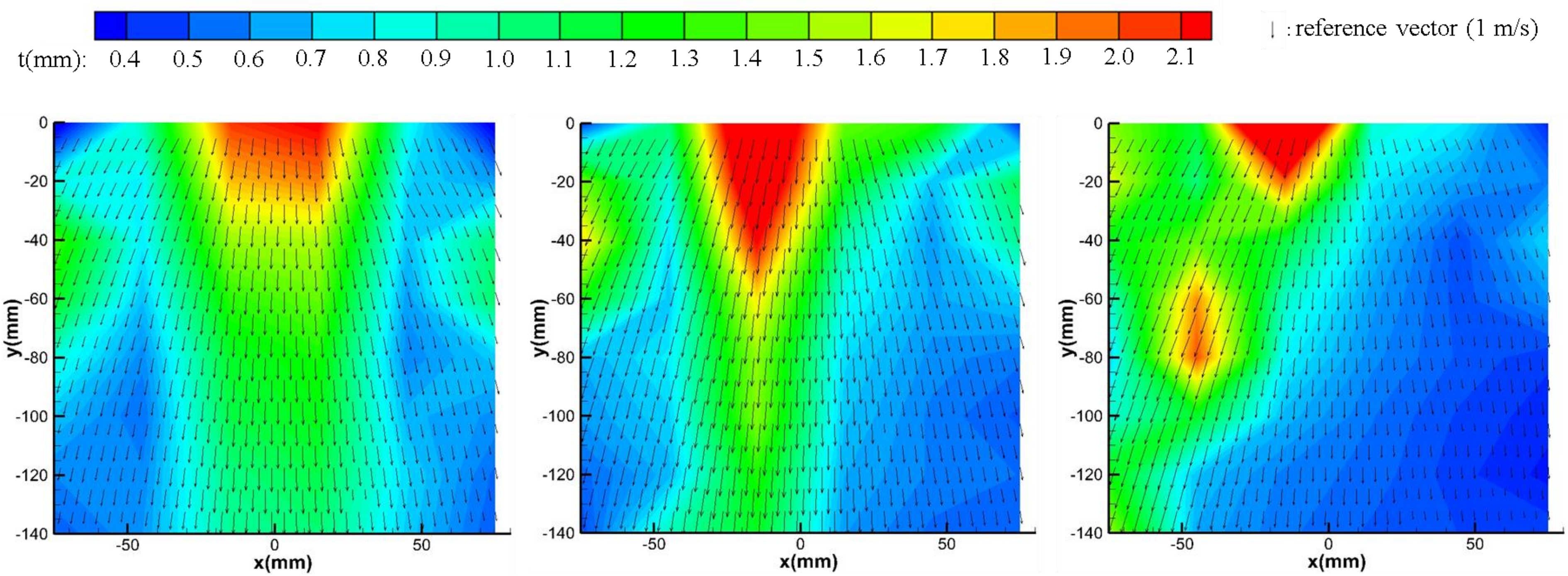
v_u : Ultrasonic velocity of water
$x_{2p} - x_{1p}$: gap distance
f_{signal} : Signal freqeuncy (50MHz)

$$v_u = 1402.38744 + 5.03836171\times T_W - 0.0581172916\times T_W^2 + 0.000334638117\times T_W^3 - 0.00000148259672\times T_W^4 + 0.0000000031658502\times T_W^5$$

MASS CONSERVATION

2D Film Flow Experiments

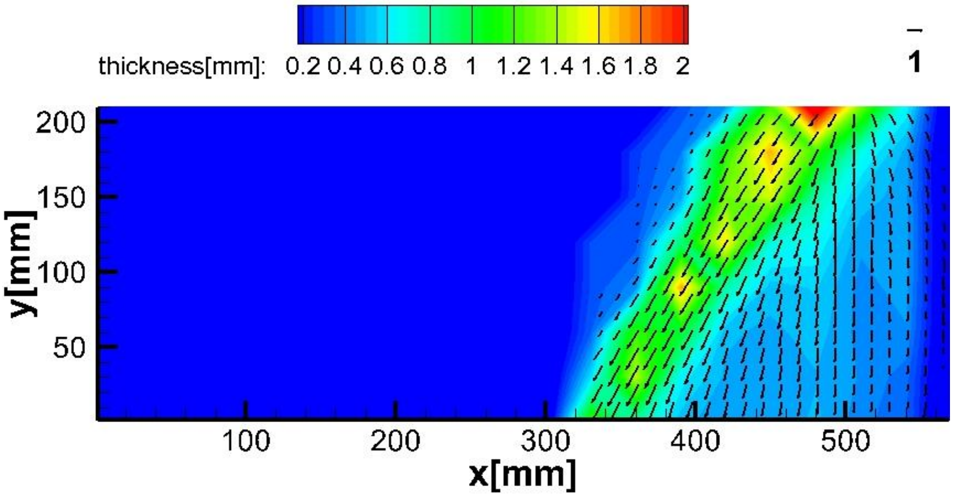
- **Results: 0, 5, 7 m/s lateral air injection**



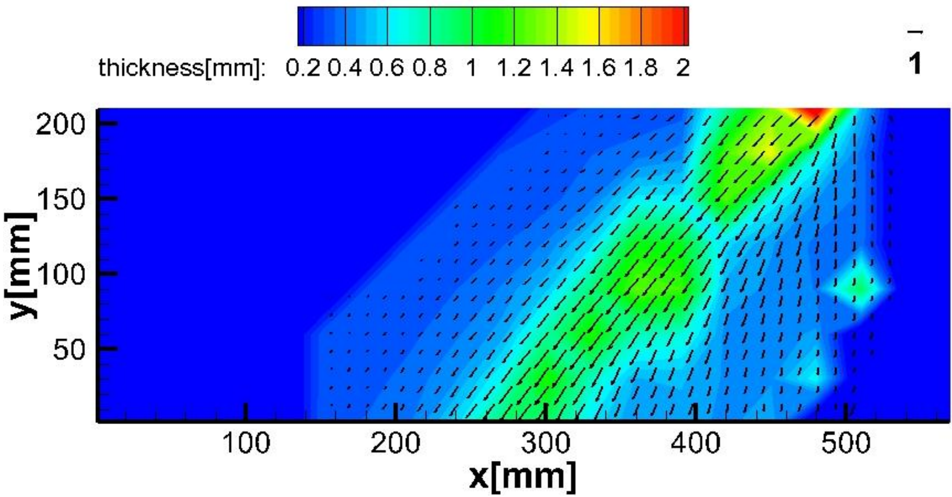
0 m/s **5 m/s** **7 m/s**

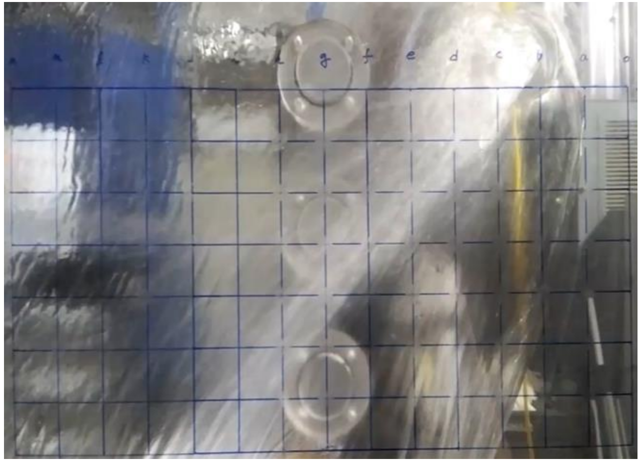
Results of 2D Film Flow Experiments

- **Results: 9, 11, 13, 15 m/s lateral air injection (1/2)**



9 m/s

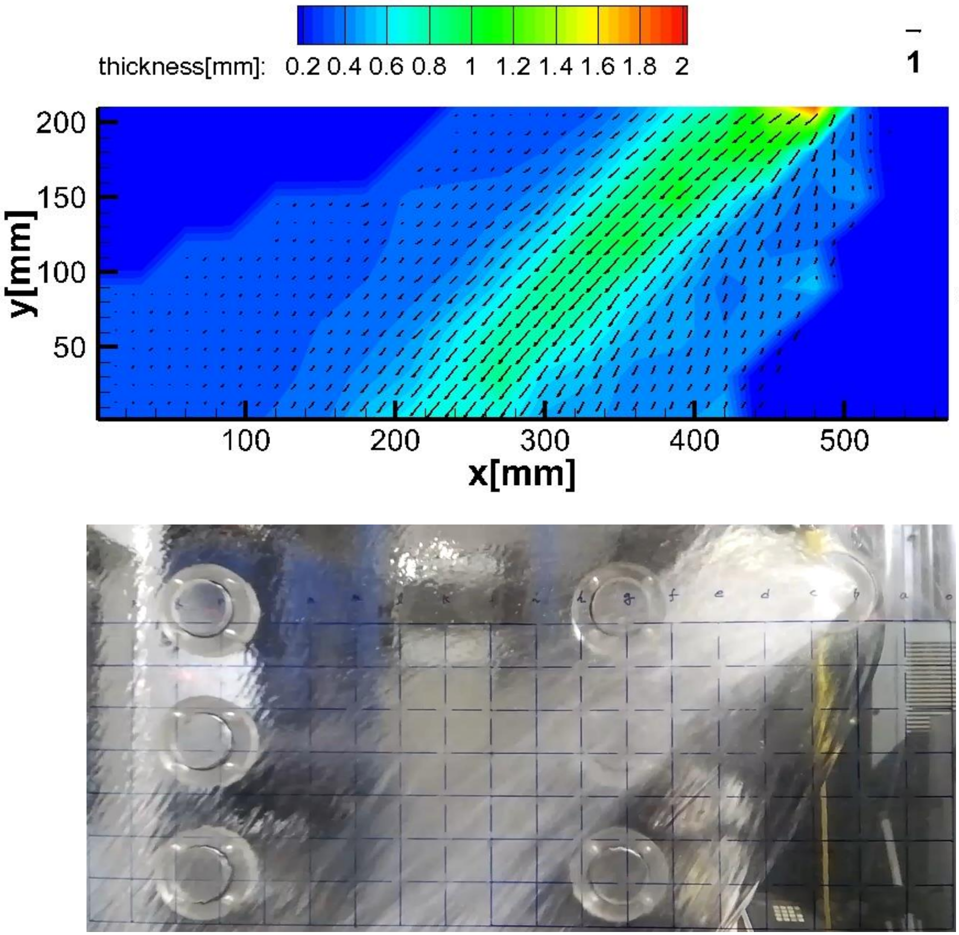




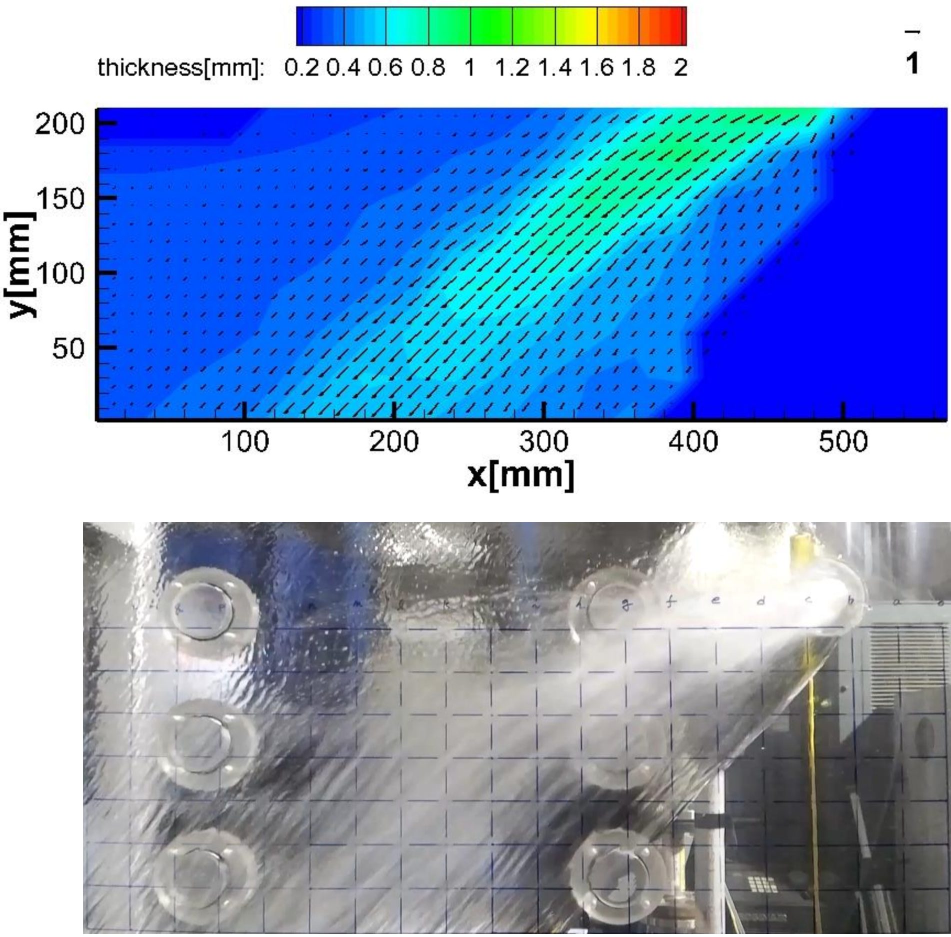
11 m/s

Results of 2D Film Flow Experiments

- **Results: 9, 11, 13, 15 m/s lateral air injection (2/2)**



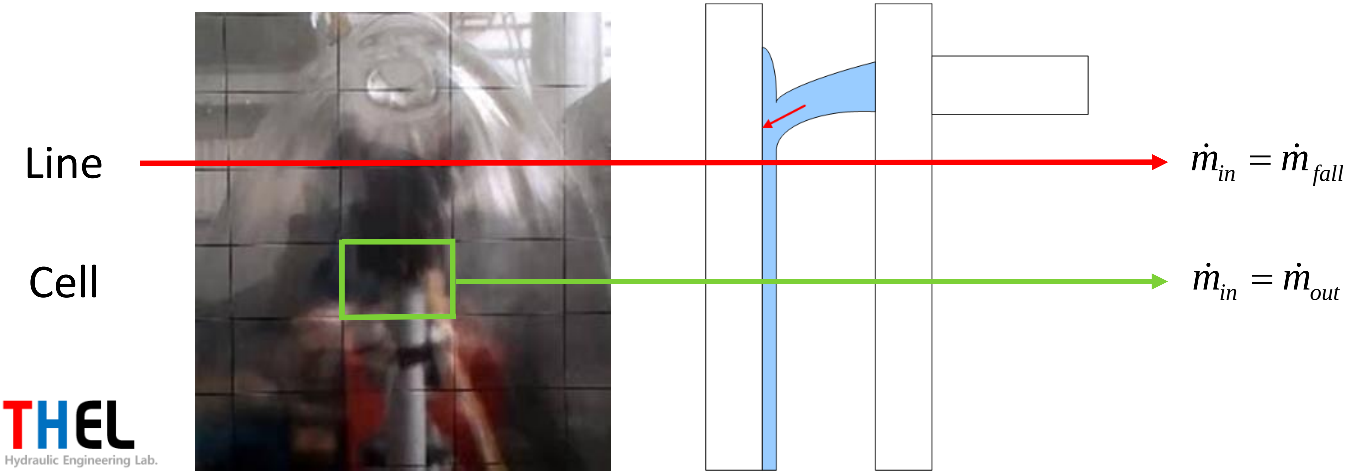
13 m/s



15 m/s

Mass Conservation

Mass conservation[error=(M_in-M_out)/M_in]								
Lateral air velocity		0	5	7	9	11	13	15
Line	M_out	0.00024	0.00025	0.00021	0.00014	0.00011	0.00011	0.00009
	error	0.0017	0.0522	0.1370	0.3985	0.5305	0.5502	0.6212
	%	0.17	5.22	13.70	39.85	53.05	55.02	62.12
					middle			
					0.00018			
					0.2482			
					24.82			
					lower			
					0.00018			
					0.2441			
					24.41			
Cell	Max.	0.122	0.122	0.198	0.195	0.244	0.274	0.216
	Min.	0.002	0.001	0.002	0.003	0.002	0.000	0.002
	Avg.	0.040	0.048	0.049	0.084	0.091	0.070	0.060
	Stdv.	0.032	0.036	0.053	0.049	0.067	0.056	0.050



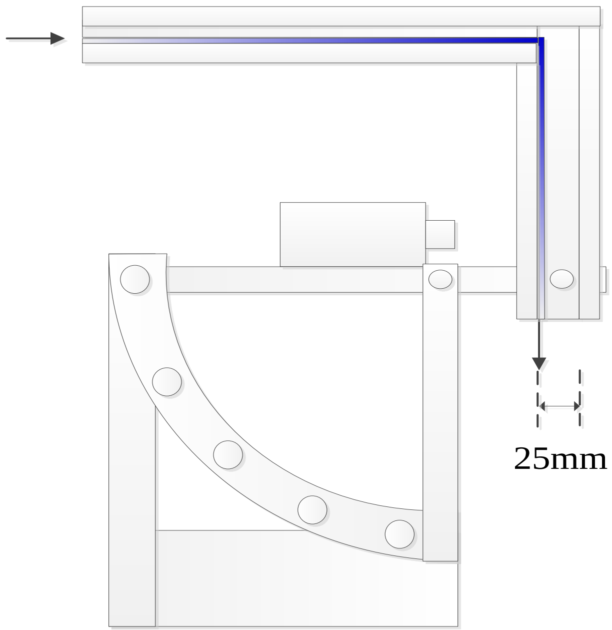
$$\dot{m}_{in} = \dot{m}_{fall}$$

$$\dot{m}_{in} = \dot{m}_{out}$$

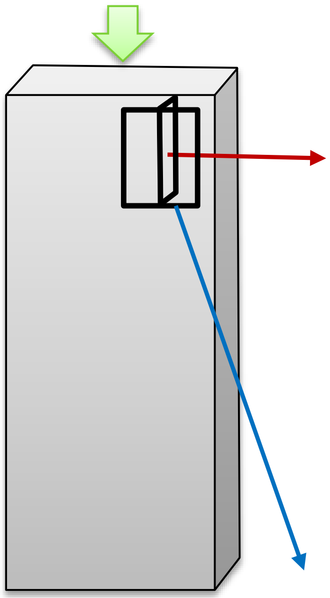
UNCERTAINTY ANALYSIS

Validation Experiment

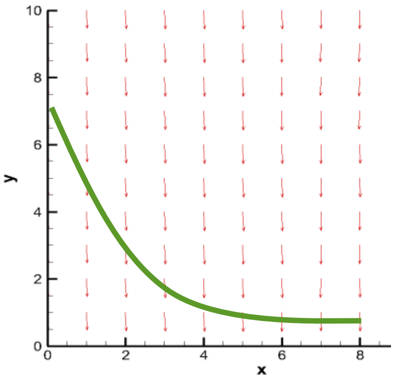
❖ Comparison between sheet-PIV and volume-PIV



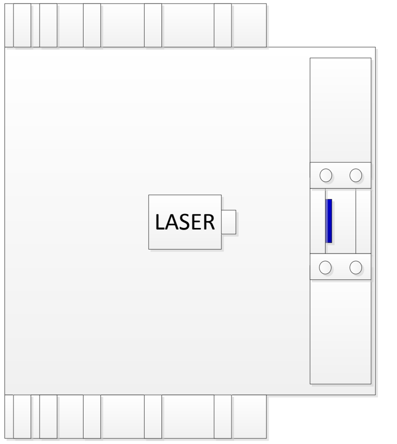
Schematic of validation experimental facility

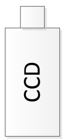


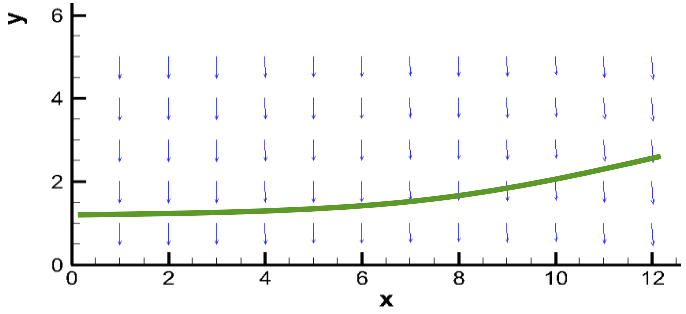
Test section



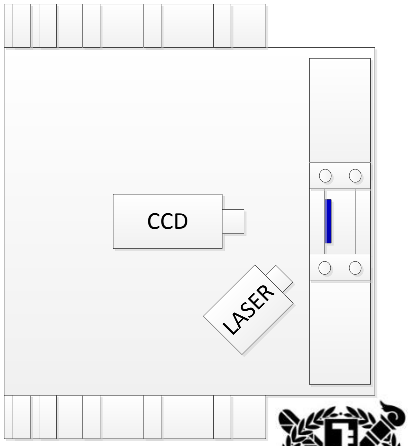
Side view with sheet PIV(depth 3.01mm)







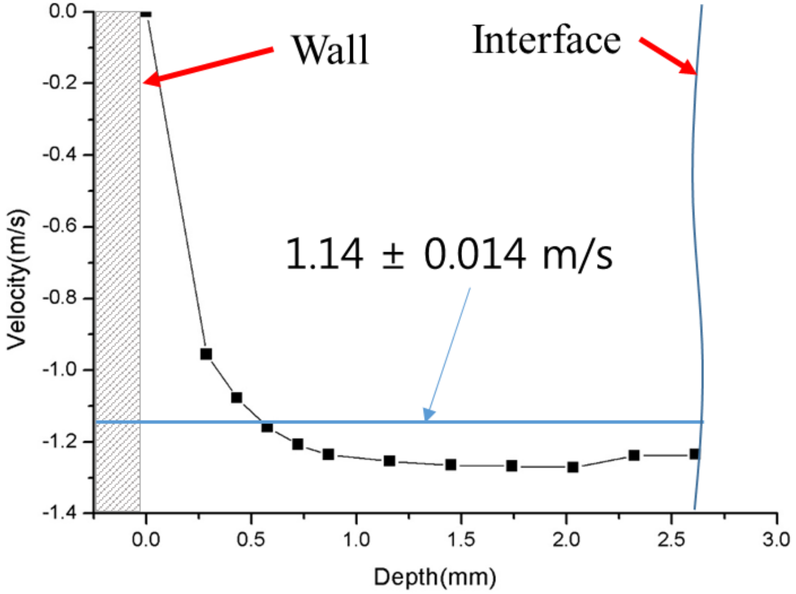
Front view with volume PIV(width 11.85mm)



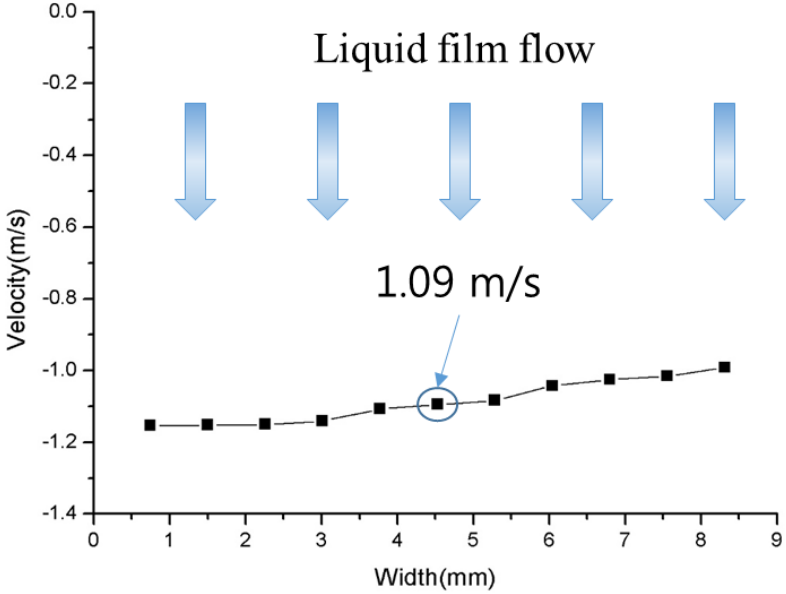
- ✓ Liquid film flow(~ 3mm)
- ✓ Sheet-PIV vs. volume PIV
- ✓ Uncertainty quantification

Validation Experiment

❖ Results



Sheet-PIV along to depth direction

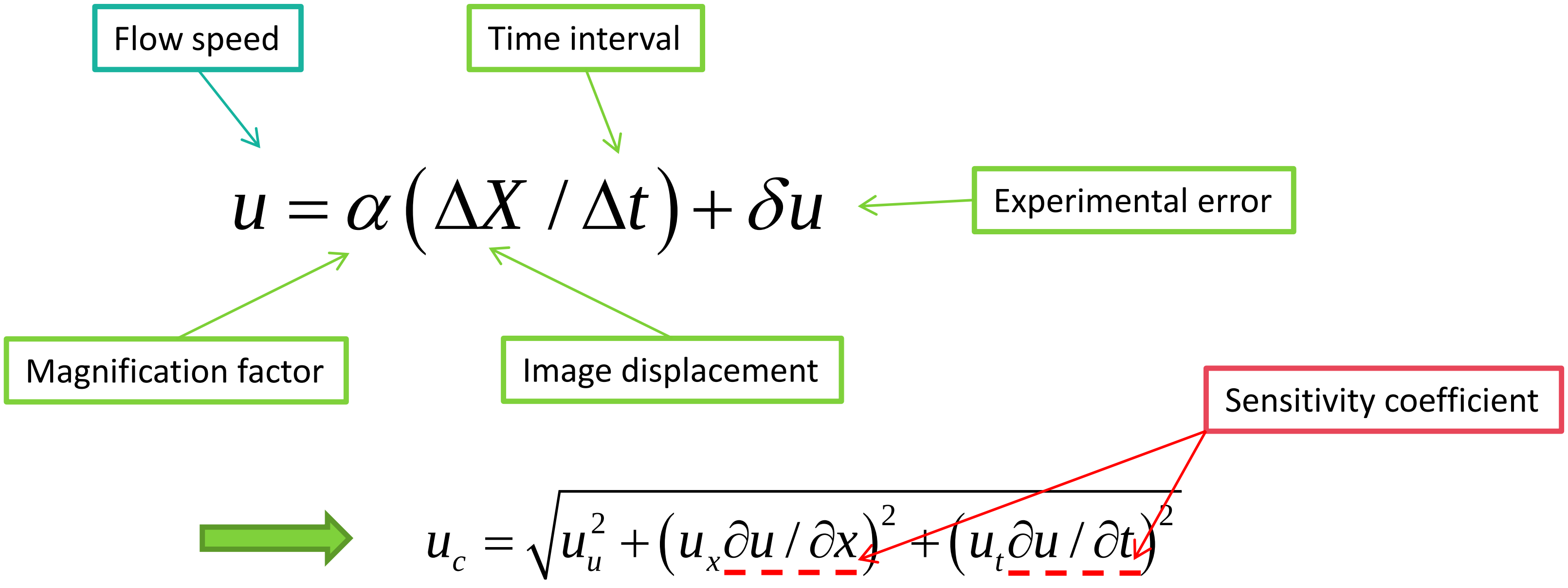


Volume-PIV along to width direction

General Guidelines

Uncertainty Analysis of Particle Image Velocimetry

- Recommended UA procedure was developed by Nishio et al. at 1999.
- Uncertainty of flow speed can be defined by **image displacement**, **time interval** and **magnification factor**.
- Experimental errors should be considered as uncertainty parameters.



Flow speed

Time interval

Experimental error

Magnification factor

Image displacement

$$u = \alpha\left(\Delta X / \Delta t\right) + \delta u$$

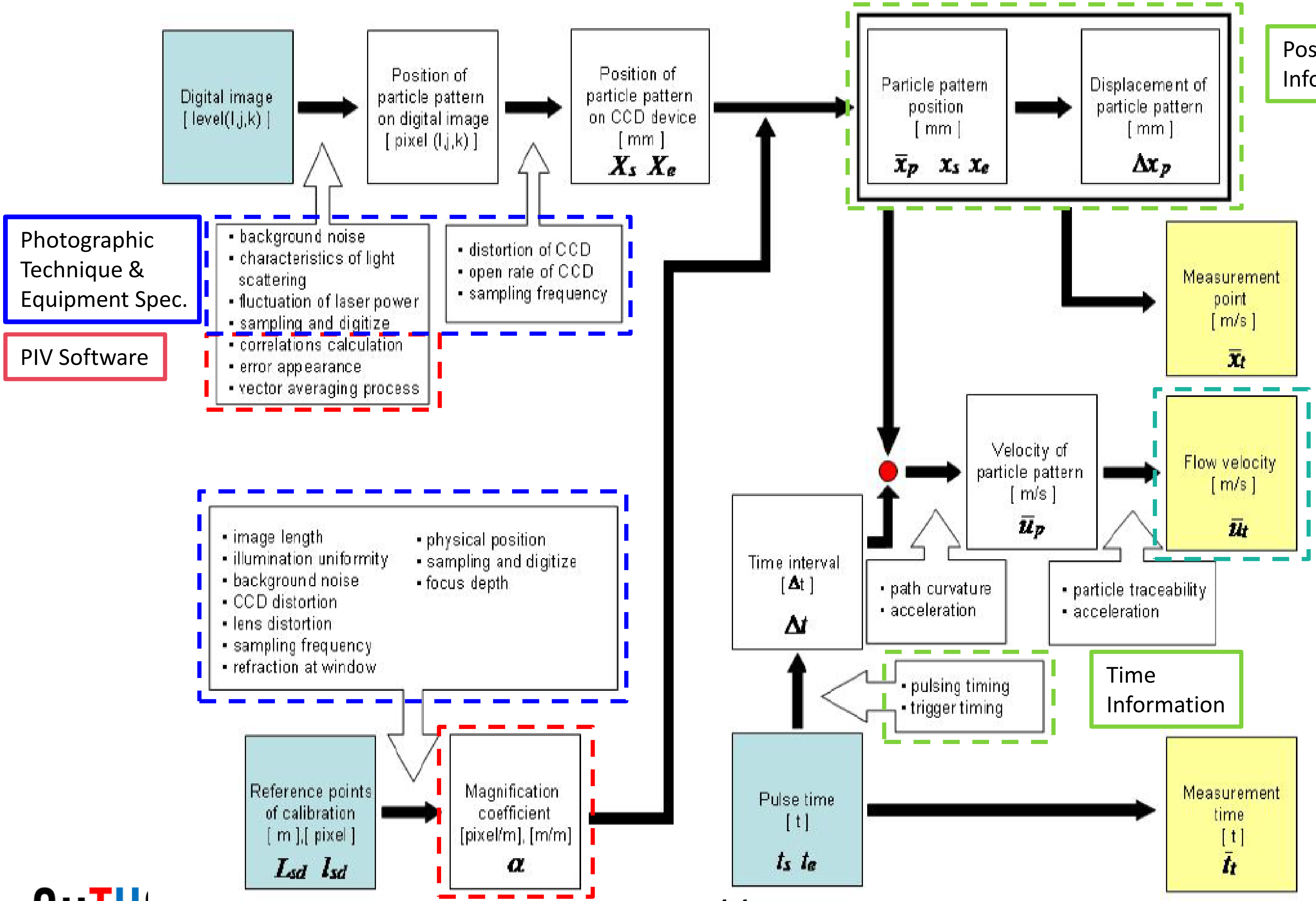
Sensitivity coefficient

$$u_c = \sqrt{u_u^2 + \left(u_x \underline{\partial u / \partial x}\right)^2 + \left(u_t \underline{\partial u / \partial t}\right)^2}$$

u_u, u_x, u_t : uncertainties of u, x, t

ITTC 7.5-01-03-03 (2008)

General Guidelines



Digital image [level(I,j,k)]

Position of particle pattern on digital image [pixel (I,j,k)]

Position of particle pattern on CCD device [mm] X_s X_e

Particle pattern position [mm] $\bar{x}_p$ x_s x_e

Displacement of particle pattern [mm] Δx_p

Position Information

Photographic Technique & Equipment Spec.

PIV Software

- background noise
- characteristics of light scattering
- fluctuation of laser power
- sampling and digitize
- correlations calculation
- error appearance
- vector averaging process

- distortion of CCD
- open rate of CCD
- sampling frequency

Measurement point [m/s] $\bar{x}_t$

Velocity of particle pattern [m/s] $\bar{u}_p$

Flow velocity [m/s] $\bar{u}_t$

Time interval [Δt] Δt

- path curvature
- acceleration

- particle traceability
- acceleration

- image length
- illumination uniformity
- background noise
- CCD distortion
- lens distortion
- sampling frequency
- refraction at window
- physical position
- sampling and digitize
- focus depth

- pulsing timing
- trigger timing

Time Information

Reference points of calibration [m],[pixel] L_{sd} l_{sd}

Magnification coefficient [pixel/m], [m/m] α

Pulse time [t] t_s t_e

Measurement time [t] $\bar{t}_t$

ITTC 7.5-01-03-03 (2008)

Summary of Uncertainties

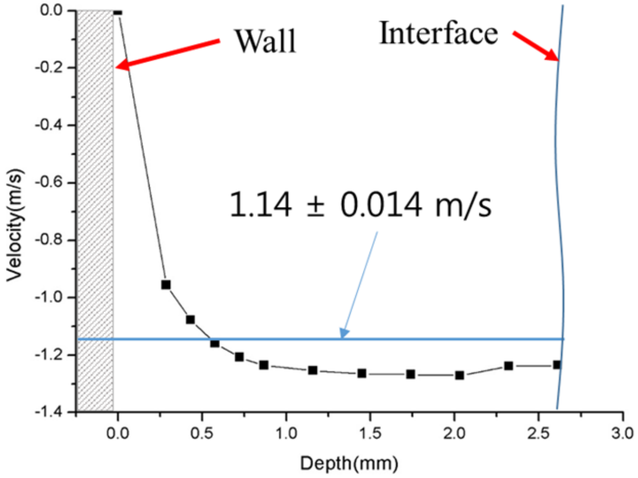
parameter	Category	Error sources	$u(x_i)$	unit	c_i(unit)	unit	$c_iu(x_i)$	u_c
α(mm/pix)	Calibration	Reference image	0.7	pix	1.29E-05	mm/pix^2	9.01E-06	
		Physical distance	0.02	mm	1.60E-03	1/pix	3.21E-05	
		Image distortion by lens	3.12	pix	1.29E-05	mm/pix^2	4.01E-05	
		Image distortion by CCD	0.0056	pix	1.29E-05	mm/pix^2	7.21E-08	
		Board position	0.5	mm	1.51E-05	1/pix	7.55E-06	
		Parallel board	0.035	rad	2.81E-04	mm/pix	9.84E-06	5.37E-05
ΔX(pix)	Acquisition	Particle image distortion	0.000002	mm	1.25E+02	pix/mm	2.49E-04	
		Image distortion by CCD	0.0056	pix	1.29E-05		7.21E-08	
		Normal view angle	0.035	rad	2.81E-04	mm/pix	9.84E-06	
	Reduction	Mis-matching error	0.2	pix	1.00E+00		0.20	
		Sub-pixel analysis	0.03	pix	1.00E+00		0.03	0.20
Δt(s)	Acquisition	Pulse timing accuracy	4.00E-08	s	1.00E+00		4.00E-08	4.00E-08
δu(mm/s)	Experiment	Particle trajectory	0.1	mm/s	1.00E+00		0.1	
		3-D effects	1.10E-03	mm/s	1.00E+00		0.0011	0.10

parameter	Error sources	$u(x_i)$	unit	c_i(unit)	unit	$c_iu(x_i)$	u_c
α	Magnification factor	5.363E-05	mm/pix	158461.1	pix/s	8.4982082	
ΔX	Image displacement	0.20223764	pix	17.644	mm/pix/s	3.5682809	
Δt	Image interval	4.005E-08	s	2795887.6	mm/s^2	0.1119752	
δu	Experiment	0.10000605	mm/s	1		0.100006	9.22

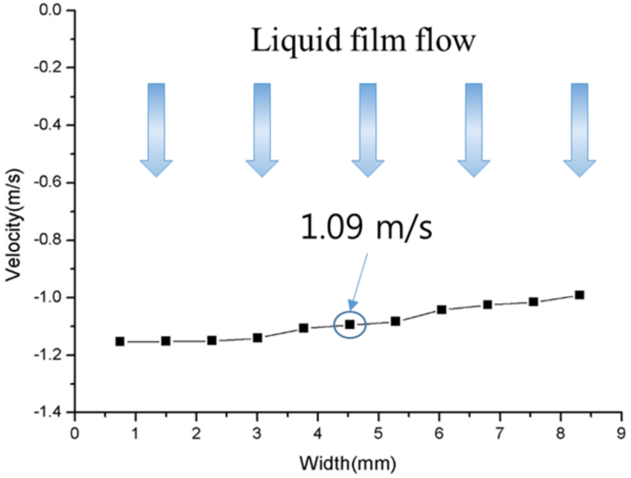
- ✓ **Averaged velocity: 1.14 m/s**
- ✓ **Systematic error: 9.22 x 10^{-3} m/s (0.81%)**

Uncertainty Analysis

Uncertainty quantification



Sheet-PIV along to depth direction



Volume-PIV along to width direction

- Averaged standard deviation

$$B = (\overline{V}_1 + 2\overline{\sigma}_1) - \overline{V}_2 = 0.063\text{m/s}\,(5.53\%)$$

$$2\overline{\sigma}_1 = \frac{\sigma_{RSS}}{n} = 0.014\text{m/s}\,(1.23\%)$$

where $\overline{V}_1$: average velocity by sheet PIV

$\overline{V}_2$: average velocity by volume PIV

$\overline{\sigma}_1$: avraged standard deviation of sheet PIV

- Total error of volume PIV can be defined as 5.65%.

  Averaged velocity = **1.09 ± 0.06 m/s**

CHALLENGES

Challenges

1. Geometry: Cylindrical vs. parallel plates

- 1/10 scale down can bring hydraulic distortion about interaction between two-phase flow.
- Experimental technique(volume-averaged PIV) has limitation to be applied to curved geometry.

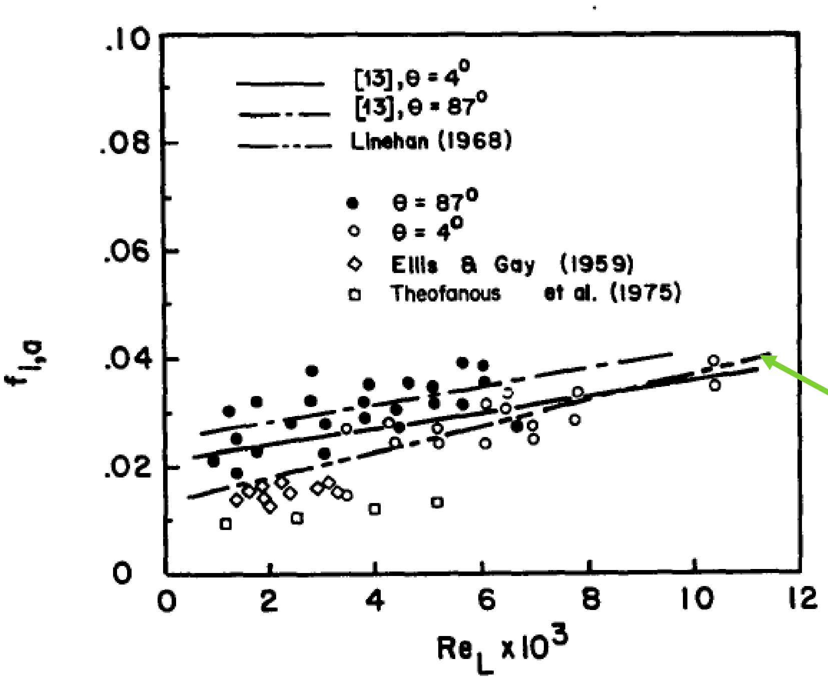
2. Scaling: 1/10 scaled down experimental condition

- 1/5 scale experiments will be carried out.
- Other results of previous 1/5 scaled experiments (MIDAS and DIVA) will be validated.

Challenges

3. Fluid: Steam vs. air

- Steam blocks our view on liquid flow.
- Previous studies on steam & saturated water flow shows similar range of interfacial friction factor.



Steam-water interfacial friction factor

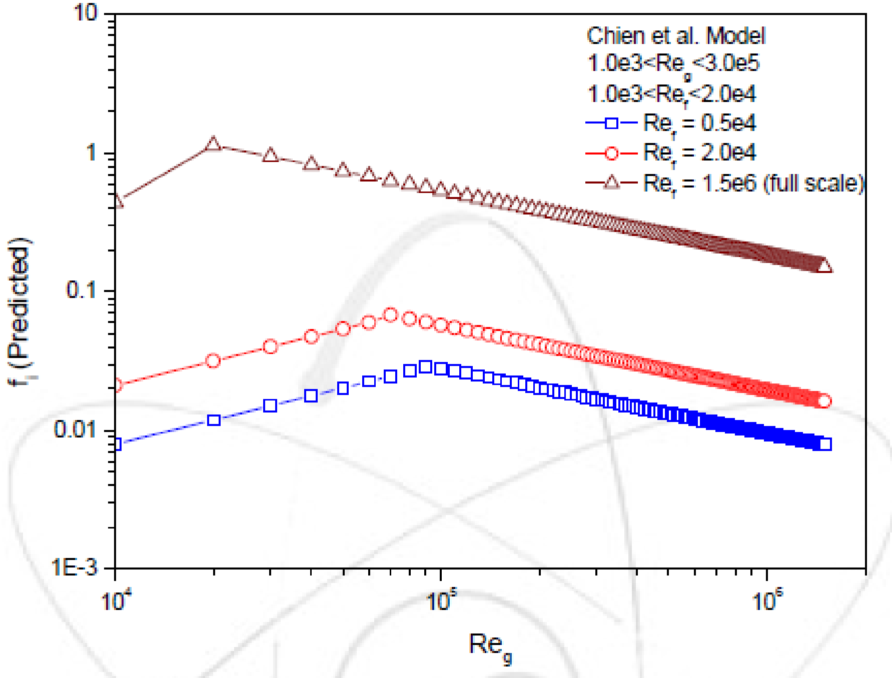
Air-water flow

H. J. KIM, Chang, S. C. LEE, S. G. BANKOFF, 1985. “HEAT TRANSFER AND INTERFACIAL DRAG IN COUNTERCURRENT STEAM-WATER STRATIFIED FLOW”. International Journal of Multiphase Flow, Vol. 11, pp. 593-606.

Challenges

4. Other modeling parameter (1/2)

- Re number, pressure, etc…
- Interfacial friction factor increase with Re number increases.



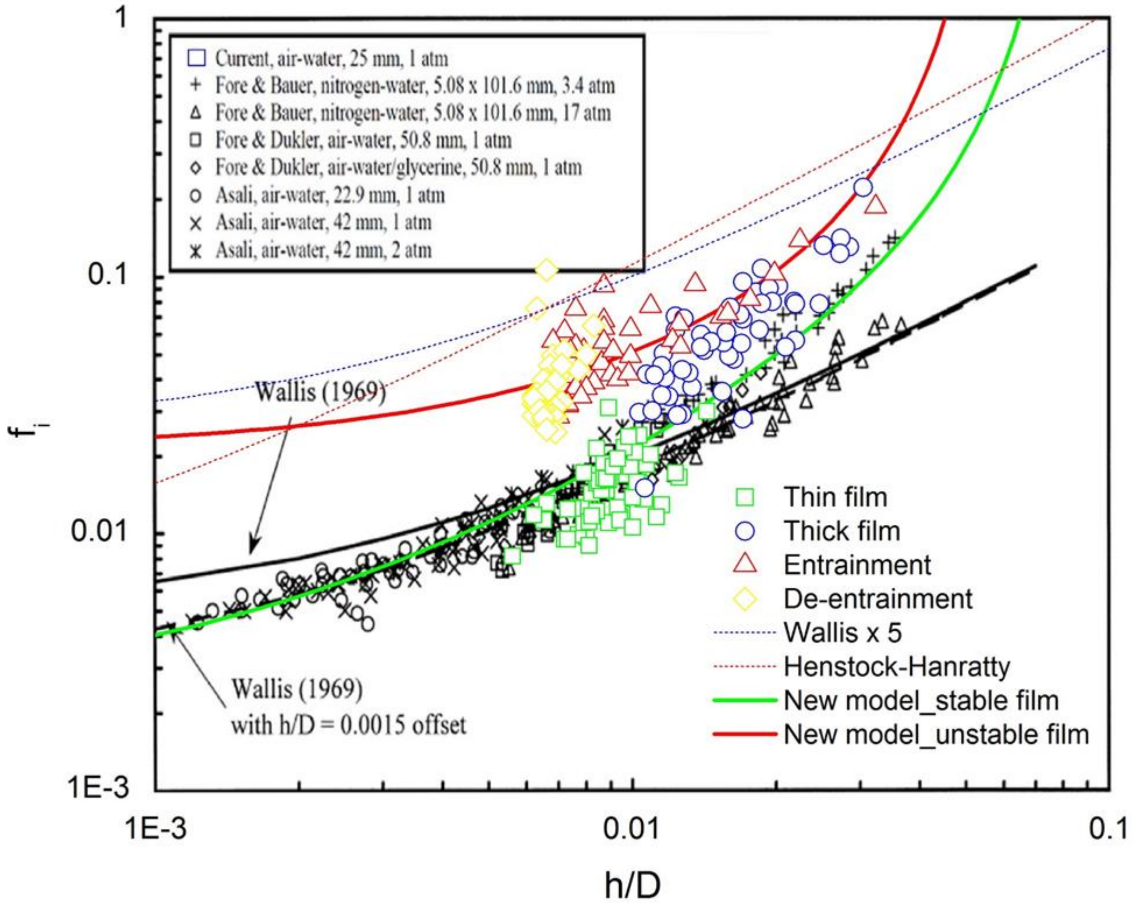
Interfacial friction factor with Re

조형규, 윤병조, 어동진, 권태순, 2005. “LBLOCA 재관수 기간 동안 DVI 방식 원자로에서 발생하는 다차원 안전주입수 우회 현상에 대한 실험 및 이론 연구”. **KAERI/TR-2937/2005**.

Challenges

4. Other modeling parameter (2/2)

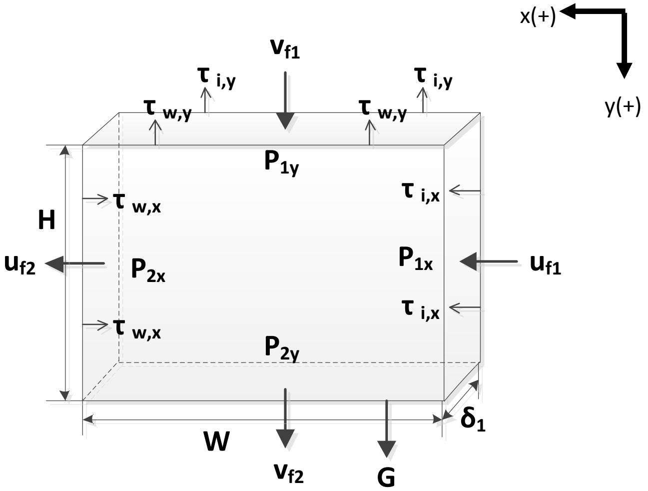
- Re number, pressure, hydraulic diameter etc…
- Results of high pressure experiments has good agreement with Wallis model.

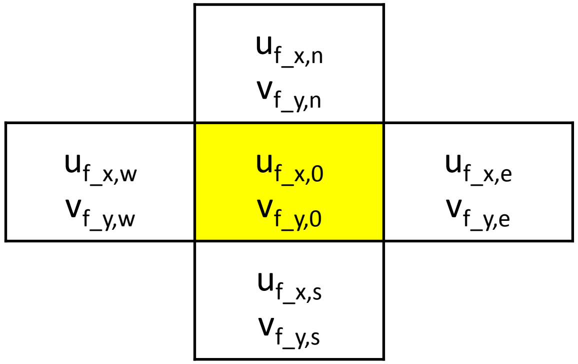


Pressure effect ??

Momentum Conservation Equations

Force balance of liquid film





	$u_{f\_x,n}$ $v_{f\_y,n}$	
$u_{f\_x,w}$ $v_{f\_y,w}$	$u_{f\_x,0}$ $v_{f\_y,0}$	$u_{f\_x,e}$ $v_{f\_y,e}$
	$u_{f\_x,s}$ $v_{f\_y,s}$	

X-direction

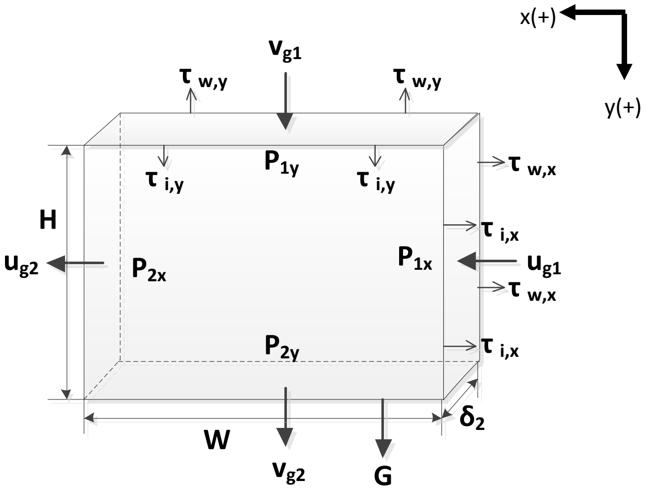
$$
\begin{aligned}
-\left(P_2-P_1\right)_x \delta_1 H+\int_A \tau_{i,x} dA-\int_A \tau_{wf,x} dA
&=-\rho_f\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)\delta_1 H\times\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)+\rho_f\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)\delta_1 H\times\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)\\
&\quad-\rho_f\left(\frac{u_{f\_n}+u_{f\_0}}{2}\right)\delta_1 W\times\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)+\rho_f\left(\frac{u_{f\_0}+u_{f\_s}}{2}\right)\delta_1 W\times\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)
\end{aligned}
$$

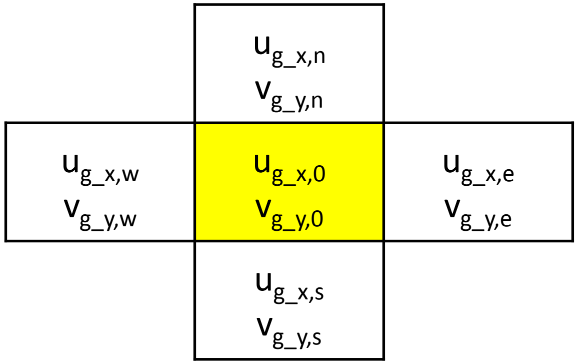
Y-direction

$$
\begin{aligned}
-\left(P_2-P_1\right)_y \delta_1 W-\int_A \tau_{i,y} dA-\int_A \tau_{wf,y} dA+\rho_f g\delta_1 HW
&=-\rho_f\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)\delta_1 W\times\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)+\rho_f\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)\delta_1 W\times\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)\\
&\quad-\rho_f\left(\frac{v_{f\_e}+v_{f\_0}}{2}\right)\delta_1 H\times\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)+\rho_f\left(\frac{v_{f\_0}+v_{f\_w}}{2}\right)\delta_1 H\times\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)
\end{aligned}
$$

Momentum Conservation Equations

Force balance of air





X-direction

$$-(P_2-P_1)_x\,\delta_2 H-\int_A \tau_{i,x}\,dA-\int_A \tau_{wg,x}\,dA$$
$$=-\rho_g\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)\delta_2 H\times\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)+\rho_g\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)\delta_2 H\times\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)$$
$$-\rho_g\left(\frac{u_{g\_n}+u_{g\_0}}{2}\right)\delta_2 W\times\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)+\rho_g\left(\frac{u_{g\_0}+u_{g\_s}}{2}\right)\delta_2 W\times\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)$$

Y-direction

$$-(P_2-P_1)_y\,\delta_2 W+\int_A \tau_{i,y}\,dA-\int_A \tau_{wg,y}\,dA+\rho_g g\delta_2 HW$$
$$=-\rho_g\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)\delta_2 W\times\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)+\rho_g\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)\delta_2 W\times\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)$$
$$-\rho_g\left(\frac{v_{g\_e}+v_{g\_0}}{2}\right)\delta_2 H\times\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)+\rho_g\left(\frac{v_{g\_0}+v_{g\_w}}{2}\right)\delta_2 H\times\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)$$

Momentum Conservation Equations

X-direction force balance

Liquid film

$$\boxed{-\left(P_2-P_1\right)_x}\delta_1 H+\int_A \tau_{i,x}dA-\int_A \tau_{wf,x}dA=-\rho_f\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)\delta_1 H\times\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)+\rho_f\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)\delta_1 H\times\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)$$
$$-\rho_f\left(\frac{u_{f\_n}+u_{f\_0}}{2}\right)\delta_1 W\times\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)+\rho_f\left(\frac{u_{f\_0}+u_{f\_s}}{2}\right)\delta_1 W\times\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)$$

Air

$$\boxed{-\left(P_2-P_1\right)_x}\delta_2 H-\int_A \tau_{i,x}dA-\int_A \tau_{wg,x}dA=-\rho_g\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)\delta_2 H\times\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)+\rho_g\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)\delta_2 H\times\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)$$
$$-\rho_g\left(\frac{u_{g\_n}+u_{g\_0}}{2}\right)\delta_2 W\times\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)+\rho_g\left(\frac{u_{g\_0}+u_{g\_s}}{2}\right)\delta_2 W\times\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)$$

$$\Rightarrow\quad -\left(P_2-P_1\right)_x+\frac{\int_A \tau_{i,x}dA}{\delta_1 H}-\frac{\int_A \tau_{wf,x}dA}{\delta_1 H}=-\rho_f\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)^2+\rho_f\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)^2-\rho_f\left(\frac{u_{f\_n}+u_{f\_0}}{2}\right)\frac{W}{H}\times\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)+\rho_f\left(\frac{u_{f\_0}+u_{f\_s}}{2}\right)\frac{W}{H}\times\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)$$

$$-)\quad -\left(P_2-P_1\right)_x-\frac{\int_A \tau_{i,x}dA}{\delta_2 H}-\frac{\int_A \tau_{wg,x}dA}{\delta_2 H}=-\rho_g\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)^2+\rho_g\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)^2-\rho_g\left(\frac{u_{g\_n}+u_{g\_0}}{2}\right)\frac{W}{H}\times\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)+\rho_g\left(\frac{u_{g\_0}+u_{g\_s}}{2}\right)\frac{W}{H}\times\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)$$

$$\frac{\int_A \tau_{i,x}dA}{\delta_1 H}-\frac{\int_A \tau_{wf,x}dA}{\delta_1 H}+\frac{\int_A \tau_{i,x}dA}{\delta_2 H}+\frac{\int_A \tau_{wg,x}dA}{\delta_2 H}=-\rho_f\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)^2+\rho_f\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)^2-\rho_f\left(\frac{u_{f\_n}+u_{f\_0}}{2}\right)\frac{W}{H}\times\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)+\rho_f\left(\frac{u_{f\_0}+u_{f\_s}}{2}\right)\frac{W}{H}\times\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)$$
$$+\rho_g\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)^2-\rho_g\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)^2+\rho_g\left(\frac{u_{g\_n}+u_{g\_0}}{2}\right)\frac{W}{H}\times\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)-\rho_g\left(\frac{u_{g\_0}+u_{g\_s}}{2}\right)\frac{W}{H}\times\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)$$

Momentum Conservation Equations

Y-direction force balance

Liquid film

$$\boxed{-(P_2-P_1)_y}\delta_1 W-\int_A\tau_{i,y}dA-\int_A\tau_{wf,y}dA+\rho_f g\delta_1 HW=-\rho_f\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)\delta_1 W\times\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)+\rho_f\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)\delta_1 W\times\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)$$

$$-\rho_f\left(\frac{v_{f\_e}+v_{f\_0}}{2}\right)\delta_1 H\times\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)+\rho_f\left(\frac{v_{f\_0}+v_{f\_w}}{2}\right)\delta_1 H\times\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)$$

Air

$$\boxed{-(P_2-P_1)_y}\delta_2 W+\int_A\tau_{i,y}dA-\int_A\tau_{wg,y}dA+\rho_g g\delta_2 HW=-\rho_g\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)\delta_2 W\times\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)+\rho_g\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)\delta_2 W\times\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)$$

$$-\rho_g\left(\frac{v_{g\_e}+v_{g\_0}}{2}\right)\delta_2 H\times\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)+\rho_g\left(\frac{v_{g\_0}+v_{g\_w}}{2}\right)\delta_2 H\times\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)$$

⇨

$$-(P_2-P_1)_y-\frac{\int_A\tau_{i,y}dA}{\delta_1 W}-\frac{\int_A\tau_{wf,y}dA}{\delta_1 W}+\rho_f gH=-\rho_f\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)^2+\rho_f\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)^2-\rho_f\left(\frac{v_{f\_e}+v_{f\_0}}{2}\right)\frac{H}{W}\times\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)+\rho_f\left(\frac{v_{f\_0}+v_{f\_w}}{2}\right)\frac{H}{W}\times\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)$$

-)

$$-(P_2-P_1)_y+\frac{\int_A\tau_{i,y}dA}{\delta_2 W}+\rho_g gH=-\rho_g\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)^2+\rho_g\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)^2-\rho_g\left(\frac{v_{g\_e}+v_{g\_0}}{2}\right)\frac{H}{W}\times\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)+\rho_g\left(\frac{v_{g\_0}+v_{g\_w}}{2}\right)\frac{H}{W}\times\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)$$

$$-\frac{\int_A\tau_{i,y}dA}{\delta_1 W}-\frac{\int_A\tau_{wf,y}dA}{\delta_1 W}-\frac{\int_A\tau_{i,y}dA}{\delta_2 W}+\rho_f gH-\rho_g gH=-\rho_f\left(\frac{v_{f\_n}+v_{f\_0}}{2}\right)^2+\rho_f\left(\frac{v_{f\_0}+v_{f\_s}}{2}\right)^2-\rho_f\left(\frac{v_{f\_e}+v_{f\_0}}{2}\right)\frac{H}{W}\times\left(\frac{u_{f\_e}+u_{f\_0}}{2}\right)+\rho_f\left(\frac{v_{f\_0}+v_{f\_w}}{2}\right)\frac{H}{W}\times\left(\frac{u_{f\_0}+u_{f\_w}}{2}\right)$$

$$+\rho_g\left(\frac{v_{g\_n}+v_{g\_0}}{2}\right)^2-\rho_g\left(\frac{v_{g\_0}+v_{g\_s}}{2}\right)^2+\rho_g\left(\frac{v_{g\_e}+v_{g\_0}}{2}\right)\frac{H}{W}\times\left(\frac{u_{g\_e}+u_{g\_0}}{2}\right)-\rho_g\left(\frac{v_{g\_0}+v_{g\_w}}{2}\right)\frac{H}{W}\times\left(\frac{u_{g\_0}+u_{g\_w}}{2}\right)$$

Momentum Conservation Equations

$$\frac{\int_A \tau_{i,x} dA}{\delta_1 H} - \frac{\int_A \tau_{wf,x} dA}{\delta_1 H} + \frac{\int_A \tau_{i,x} dA}{\delta_2 H} + \frac{\int_A \tau_{wg,x} dA}{\delta_2 H} = -\rho_f \left(\frac{u_{f\_e} + u_{f\_0}}{2}\right)^2 + \rho_f \left(\frac{u_{f\_0} + u_{f\_w}}{2}\right)^2 - \rho_f \left(\frac{u_{f\_n} + u_{f\_0}}{2}\right)\frac{W}{H} \times \left(\frac{v_{f\_n} + v_{f\_0}}{2}\right) + \rho_f \left(\frac{u_{f\_0} + u_{f\_s}}{2}\right)\frac{W}{H} \times \left(\frac{v_{f\_0} + v_{f\_s}}{2}\right)$$
$$+ \rho_g \left(\frac{u_{g\_e} + u_{g\_0}}{2}\right)^2 - \rho_g \left(\frac{u_{g\_0} + u_{g\_w}}{2}\right)^2 + \rho_g \left(\frac{u_{g\_n} + u_{g\_0}}{2}\right)\frac{W}{H} \times \left(\frac{v_{g\_n} + v_{g\_0}}{2}\right) - \rho_g \left(\frac{u_{g\_0} + u_{g\_s}}{2}\right)\frac{W}{H} \times \left(\frac{v_{g\_0} + v_{g\_s}}{2}\right)$$

$$-\frac{\int_A \tau_{i,y} dA}{\delta_1 W} - \frac{\int_A \tau_{wf,y} dA}{\delta_1 W} - \frac{\int_A \tau_{i,y} dA}{\delta_2 W} + \rho_f gH - \rho_g gH = -\rho_f \left(\frac{v_{f\_n} + v_{f\_0}}{2}\right)^2 + \rho_f \left(\frac{v_{f\_0} + v_{f\_s}}{2}\right)^2 - \rho_f \left(\frac{v_{f\_e} + v_{f\_0}}{2}\right)\frac{H}{W} \times \left(\frac{u_{f\_e} + u_{f\_0}}{2}\right) + \rho_f \left(\frac{v_{f\_0} + v_{f\_w}}{2}\right)\frac{H}{W} \times \left(\frac{u_{f\_0} + u_{f\_w}}{2}\right)$$
$$+ \rho_g \left(\frac{v_{g\_n} + v_{g\_0}}{2}\right)^2 - \rho_g \left(\frac{v_{g\_0} + v_{g\_s}}{2}\right)^2 + \rho_g \left(\frac{v_{g\_e} + v_{g\_0}}{2}\right)\frac{H}{W} \times \left(\frac{u_{g\_e} + u_{g\_0}}{2}\right) - \rho_g \left(\frac{v_{g\_0} + v_{g\_w}}{2}\right)\frac{H}{W} \times \left(\frac{u_{g\_0} + u_{g\_w}}{2}\right)$$

$$\int_A \tau_{i,x} dA = \int_A \frac{1}{2} \rho_g \boxed{f_i} \left|\vec{U}_g - \vec{U}_f\right| \left(U_g - U_f\right) dA = \int_A \frac{1}{2} \rho_g f_i \left(u_g(x) - u_f(x)\right) \sqrt{\left(u_g(x) - u_f(x)\right)^2 + v_f^2(y)} dA$$

$$\int_A \tau_{wf,x} dA = \int_A \frac{1}{2} \rho_f \boxed{f_{wf}} u_f(x) \sqrt{u_f^2(x) + v_f^2(y)} dA$$

$$\int_A \tau_{wg,x} dA = \int_A \frac{1}{2} \rho_g \boxed{f_{wg}} u_g^2(x) dA$$

Blasius' equation

$$f_{wg} = 0.79\,\mathrm{Re}^{-1/4}$$

Assume that…

① Ignore lateral air flow acceleration

② Linear velocity profile

③ Constant friction coefficient

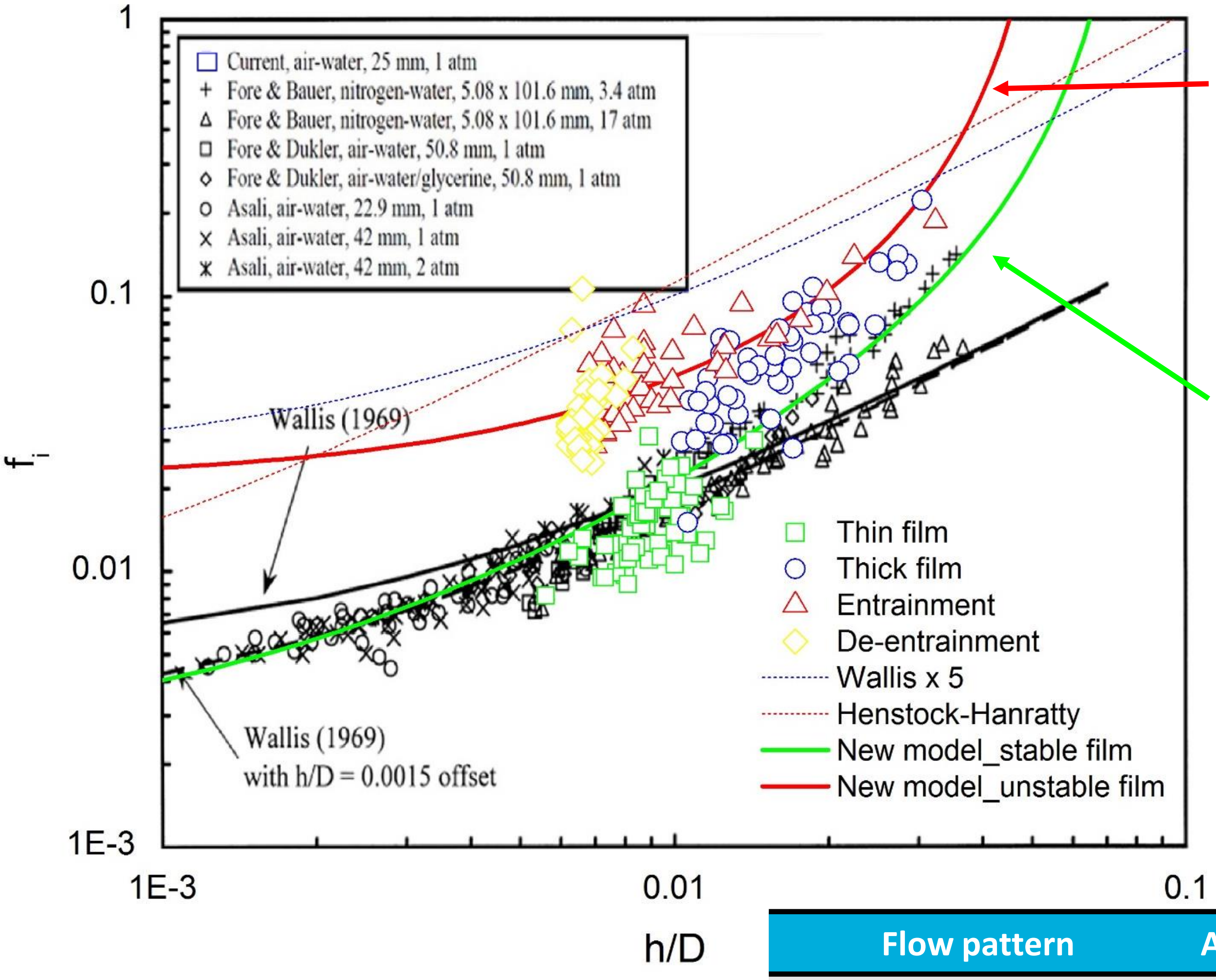
④ Gas wall friction factor follows Blasius' Eq.

$$\int_A \tau_{i,y} dA = \int_A \frac{1}{2} \rho_g f_i \left|\vec{U}_g - \vec{U}_f\right| \left(U_g - U_f\right) dA = \int_A \frac{1}{2} \rho_g \boxed{f_i v_f}(x) \sqrt{\left(u_g(x) - u_f(x)\right)^2 + v_f^2(y)} dA$$

$$\int_A \tau_{wf,y} dA = \int_A \frac{1}{2} \rho_f \boxed{f_{wf}} v_f(y) \sqrt{u_f^2(x) + v_f^2(y)} dA$$

$$\Rightarrow \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \begin{pmatrix} f_i \\ f_w \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

Comparison with Previous Studies



$$f_i = \frac{1}{4}\left[1.879 - 32.928\left(\frac{h}{D} - 0.0015\right)\right]^{-2} - 0.046$$

Unstable film flow
(entrainment, de-entrainment)

$$f_i = \frac{1}{4}\left[1.879 - 22.104\left(\frac{h}{D} - 0.0015\right)\right]^{-2} - 0.066$$

Stable film flow
(thin film, thick film w/o entrainment)

Flow pattern	Avg. error (%)	Std. error (%)
Thin film	-36	32
Thick film	32	23
Entrainment	-2	24
De-entrainment	-15	26