Test of Poisson Process for Earthquakes in and around Korea

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1. Introduction

Since Cornell's work [1] on the probabilistic seismic hazard analysis (hereafter, PSHA), majority of PSHA computer codes are assuming that the earthquake occurrence is Poissonian. To the author's knowledge, it is uncertain who first opened the issue of the Poisson process for the earthquake occurrence. The early discussion dates back to the 1950s, i.e., Aki [2]. In 1964, Knopoff [3] analyzed whether the earthquakes in southern California follow the Poisson process or not, and concluded 'No'. In 1974, however, he revisited this issue for the southern Californian earthquakes, with aftershocks removed, and concluded 'Yes' this time [4]. Despite of the long-time dispute, no simple conclusion has been reached yet. Rather, it seems to be agreed that, when earthquake clusters are removed, small to intermediate earthquakes follow the Poisson process, while larger earthquakes do not.

The systematic PSHA in Korea, led by the nuclear industry, were carried out for more than 25 year with the assumption of the Poisson process. However, the assumption of the Poisson process has never been tested. Therefore, the test is of significance.

2. Methods and Results

To test the Poissonian occurrence of earthquakes, we used the Chi-square test with the Pearson's test statistic. The earthquake data were selected from the catalog of Korea Meteorological Administration (KMA) [5]. The KMA catalog provides the instrumental earthquake data occurred since 1978. However, since the data of 1978 and 1979 are considered to be highly incomplete, we use the data since 1980 inclusive.

2.1 Pearson's Test Statistic

While Knopoff [3] used ten days as a unit time interval, we used one-year to avoid too many null intervals. For the earthquake catalog of T-year observation, the annual earthquake frequencies, n are counted. The observed statistic, O_n is the number of years in which n earthquakes occurred. The expectation, E_n is given by $T \times P_n$. Here, P_n is the Poisson probability that the event of interest occurs n times;

$$P_n = \Pr(N = n) = \frac{\lambda^{-n} e^{-\lambda}}{n!} \qquad (1)$$

where λ is the mean annual frequency and its estimate is the total number of earthquake, n_e divided by the observation period, T. Then, the Pearson's test statistic, PTS is defined by

$$PTS = \sum_{n=0}^{\infty} \frac{(O_n - E_n)^2}{E_n} \qquad (2)$$

The each term with $O_n \leq 5$ was merged with the next one(s) to have O_n larger than 5. As a result, the summation in Eq. (2) is made over a finite number of terms. If the summation is to be made over m terms, then PTS follows a Chi-square distribution with the degree-of-freedom (n_{df}) of $(m-2)$.

The null hypothesis, H_0 is that O_n follows the Poisson process. For the significance level of α, the null hypothesis cannot be rejected if $H_0 < X^2_{1-\alpha}(m-2)$, and is rejected otherwise. Here, $X^2_{1-\alpha}(m-2)$ is the Chi-square variable corresponding to the $(1-\alpha)$ percentile.

2.2 Results

We applied the Chi-square test to KMA catalog for the 34 years of observation period from 1980 through 2013. The data was composed of 1,091 earthquakes whose magnitude ranged from 1.7 to 5.3. The significance level of 5% (α=0.05) was used.

It is well known that earthquake catalogs are incomplete for smaller magnitude because a portion of small earthquakes are not reported mainly due to the detection limits of seismic networks. The smaller is the magnitude, the more missing are earthquakes from the report. To avoid the distortion due to the incompleteness of catalogs, we introduced the cut-off magnitude, m_{cut}. The earthquakes of magnitude less than m_{cut} are excluded from the catalog. The value of m_{cut} started from the minimum magnitude of the whole catalog, 1.7 in the KMA catalog, then it increased with the increment of 0.1 magnitude unit.

The test results are given in Table 1. For m_{cut} smaller than 2.9, the test said the null hypothesis was rejected, which is denoted by 'R' in Table 1. For m_{cut} of 2.9 or larger, the test said the null hypothesis could not be rejected, which is denoted by 'A'. This fact can be interpreted, if it is true that the earthquakes follow the Poisson process, as an implication of a magnitude of completeness, m_c at m~3. This value is in agreement with Noh *et al*. [6] based on the visual interpretation of annual occurrence rate. Table 1 shows the test results up to m_{cut}=3.0 only. To summarize the omitted results, the null hypothesis could not be rejected up to m_{cut}=4.5. However, the results for m_{cut}=3.6 or larger are of little significance because the degree-of-freedom is too small, i.e., less than 3. This is due to the scarcity of larger earthquakes. From m_{cut}=4.6, the degree-of-freedom is 0 (zero) so that the test could not be carried out.

Table 1. Results of the Poisson process test

m_{cut}	n_e	λ	n_{df}	PTS	X^2	H_0*
1.7	1091	32.1	4	4920	9.488	R
1.8	1090	32.1	4	10660	9.488	R
1.9	1090	32.1	4	10660	9.488	R
2.0	1087	32.0	4	11330	9.488	R
2.1	1045	30.7	4	5429	9.488	R
2.2	972	28.6	4	33270	9.488	R
2.3	879	25.9	3	102.0	7.815	R
2.4	801	23.6	4	242.1	9.488	R
2.5	710	20.9	3	31.09	7.815	R
2.6	604	17.8	3	12.18	7.815	R
2.7	526	15.5	4	12.97	9.488	R
2.8	456	13.4	4	10.58	9.488	R
2.9	376	11.1	4	8.358	9.488	A
3.0	310	9.12	3	4.925	7.815	A

*: A for Accepted (not rejected), R for Rejected

The graphical comparison is shown in Fig. 1. Two graphs compare the observed number of years and the predicted (i.e., Poissonian) number of years. They are the numbers of years before merging those terms in Eq. (2) with $O_n \leq 5$. The left graph shows one of the cases of 'not rejected'. At the first glance, the observed number of years significantly deviates from the Poisson distribution. The right graph shows one of the cases of 'rejected'. The observed number of years is closer to the Poisson distribution than the left graph is. However, discrepancies are still observed. A large discrepancies exit at n=3~5.

3. Conclusions

We tested whether the Korean earthquakes follow the Poisson process or not. The Chi-square test with the significance level of 5% was applied. The test turned out that the Poisson process could not be rejected for the earthquakes of magnitude 2.9 or larger. However, it was still observed in the graphical comparison that some portion of the observed distribution significantly deviated from the Poisson distribution. We think this is due to the small earthquake data. The earthquakes of magnitude 2.9 or larger occurred only 376 times during 34 years. Therefore, the judgment on the Poisson process derived in the present study is not conclusive.

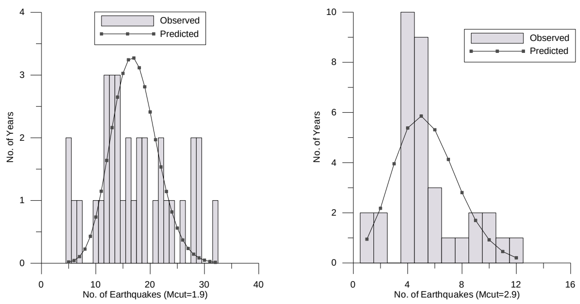

Fig. 1. Comparison of the observed number of years and the predicted (i.e., Poissonian) number of years. Left: example for rejected case, m_{cut}=1.9. Right: example for accepted case, m_{cut}=2.9.

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