# Density Models for Velocity Analysis of Jet Impinged CEDM Missile

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## 1. Introduction

A missile protection is a main safety issue for the design of nuclear power plants. A control element drive mechanism (CEDM) can be a potential missile in the reactor head area during one of the postulated accidents. The CEDM is propelled by the high speed water jet discharged from a broken upper head nozzle. The energy from the CEDM missile is characterized by the velocity and mass of the water jet acting on the missile object. Then, a CEDM missile flies until impacting the missile shield. The jet expansion models to predict the missile velocity have been investigated by Kang et al. [1].

The previous work of Kang et al. showed a continuous increase in missile velocity as the CEDM missile travels. But it is not natural in that two phase flow from the nozzle break exit tends to disperse and the thrust force on the missile decreases along the distance of the travel. The jet flow also interacts with the air surrounding itself. Therefore, the density change has to be included in the estimation of the missile velocity.

In this paper, two density change models of the water jet are introduced for the jet expansion models along with the distance from the nozzle break location. The first one is the direct approximation model. And the second is the pressure approach model. After obtaining the missile velocities from these models, the results will be discussed and the more proper model for the engineering purpose will be recommended.

#### 2. Methods and Results

It is well known that a two phase flow phenomenon occurs when high pressure reactor coolant undergoes partial changes in pressure. And the density of the flow from the jet exit decreases due to the air entrainment during travel by the interaction with the air.

Leu et al. [2] and Rajaratnam et al. [3] have discussed the anatomy of high speed water jets in air. According to their discussion, the water jet can be divided into three distinctive regions as shown in Fig. 1.

As a core region, the water jet velocity in Region-1 is equal to the nozzle exit velocity. In Region-2, the jet transfers momentum to the surrounding air more actively. This phenomenon decreases the jet velocity, and therefore the jet expands. Last part of the jet is Region-3 where jet is in the state of complete disintegration into very small droplets, and therefore their velocity is negligible. Empirical equations are applied to predict the density changes in the water jet because it is hard to numerically describe the density changes due to the air interaction occurring in region-2. In section 2.1, an empirical equation of the density change for a high speed water jet is applied. In section 2.2, an empirical equation of the pressure change for the centerline of a water jet is applied. Then both empirical equations are modified to include the effects of the jet expansion for each model.



Fig. 1. Anatomy of High Speed Water Jet in Air

# 2.1 Direct Approximation Model

The empirical equation was developed by Shavlovsky et al. to predict the density change. The density change as a function of distance above the jet exit plane and nozzle exit diameter is given in the form of

$$\frac{\rho_x}{\rho_e} = \frac{x}{D_e \times \left(2.7 \times \frac{x}{D_e} - 20\right)} + \frac{8 \times D_e}{x}$$
(1)

as presented in Ref. [4]. This equation gives the average density at a certain distance from the nozzle break plane only for a un-expanded uniform jet model. This equation is only valid for a reasonably short distance due to its intrinsic nature.

Then calculated density changes are curve fitted and extrapolated to the maximum distance the CEDM can fly in APR1400 nuclear reactor which is about 5 feet.

The curve fitted equation is modified using the area ratio of the nozzle exit area over the jet expanded area to consider the jet expansion because the density is inversely proportional to the volume increase. And the density equation is transformed into

$$\rho_x = \rho_e \times x^{C_0} \times \left[ \frac{R_e^2}{\left(R_e + x \tan \phi\right)^2} \right]$$
(2)

The Eq.(2) represents that the density tends to change by the power of  $C_0$  along the distance. The constant  $C_0$ can vary according to the experimental conditions. However, it is still useful to approximate the density change in a conservative way since the term has been developed for the uniform jet as mentioned above.

The calculated densities using Eq.(2) are plotted against the distance traveled by the CEDM missile from the nozzle break plane with the jet expansion angles of  $10^{\circ}$  through  $50^{\circ}$  in Fig. 2. The density rapidly decreases in a very short travel distance.



Fig. 2. Density Changes for Direct Approximation Model

The CEDM missile velocities can be estimated from the equations developed for the uniform jet model and the jet expansion model [1]. The uniform jet model can be applied to the expansion jet model in the case where the expanded jet area is less than the possible maximum impingement area on the missile object and the equation developed for this model is expressed as:

$$\left[\left(1 - \frac{V_x}{V_e}\right) - ln\left(1 - \frac{V_x}{V_e}\right)\right] - 1 = \int_0^x \left(\frac{\rho_x A_e}{m}\right) dx \qquad (3)$$

which is given in Ref. [1].

The equation developed by Kang et al. for the case where expanded jet area is wider than the possible maximum impingement area on the missile object has been modified because their equation is valid only for the case where CEDM missile velocity increases in that the equation only adds the effective jet thrust force beyond a certain travel distance. So, un-effective jet thrust force is subtracted from the jet thrust force computed for the total travel distance instead of adding the effective jet thrust force. This modified equation is described as:

$$\left[ \left(1 - \frac{V_x}{V_e}\right) - ln \left(1 - \frac{V_x}{V_e}\right) \right] - 1 = \int_0^x \left(\frac{\rho_x A_e}{m}\right) dx - \int_{x'}^x \left(\frac{\rho_x A_e}{m}\right) \left(1 - \frac{A_o}{\pi (R_e + x \tan \phi)^2}\right) dx \quad (4)$$

which can be used without considering the tendency of the missile velocity. Then the calculated velocities using Eq.(3) and Eq.(4) are plotted along the distance above the jet exit plane as shown in Fig. 3.

When the travel distance of the potential missile is long, the travel distance of the jet needs to be considered because the jet cannot travel infinitely. However, the jet is assumed as continuous in a conservative way because the potential CEDM missile can fly only a limited short distance. The missile velocities will be increased continuously along the distance if the jet density does not decrease.



Fig. 3. Missile Velocities for Direct Approximation Model

#### 2.2 Pressure Approach Model

The impingement area of the water jet expands along its axial distance from the nozzle break plane. So, density decreases against the augmented area. And the pressure also decreases against the distance. For this model, the density change of the water jet is assumed to be proportional to the product of the nozzle exit area over the jet expanded area and the decreased pressure over the nozzle exit pressure at the centerline of the water jet. At the water jet centerline, where no air entrainment occurs, the density decreases more smoothly.

The assumed equation used for the density change is given as:

$$\rho_{x} = \left[ \left( C_{1} \times \left( \frac{x}{D_{e}} \right) + \frac{P_{e}}{P_{v}} \right) \times \left( \frac{P_{v}}{P_{e}} \right) \right] \times \left[ \frac{R_{e}^{2}}{\left( R_{e} + x \tan \phi \right)^{2}} \right] \times \rho_{e} \quad (5)$$

where the left term of the right-hand side is developed by Anirban et al. [5] explaining the pressure change against the distance above nozzle break plane. And the right term of that compensates for the area increase, and therefore the density decreases. The density change depicted in Fig. 4 by using Eq.(5).



Fig. 4. Density change for Pressure Approach Model

Unlike the missile velocities from the direct approximation model, the velocities have smoothly decreased for this model. The plotted missile velocities using Eq.(3) and Eq.(4) for the pressure approach model are shown in Fig. 5.



Fig. 5. Missile Velocities for Pressure Approach model

# 3. Conclusions

Two density approximation models are introduced to predict the CEDM missile velocity. For each model, the effects of the expanded jet area were included as the area ratio to the exit nozzle area. In direct approximation model, the results have showed rapid decrease in both density and missile velocity. In pressure approach model, the density change is assumed perfectly proportional to the pressure change, and the results showed relatively smooth change in both density and missile velocity comparing to the direct approximation model.

The maximum velocity is the key factor to assure the structural integrity of the missile shield. The maximum missile velocity for the pressure approach model is about 2 times greater than that for the direct approximation model because the air entrainment effects were not considered for this model. Using the

model developed by Kang et al., the maximum missile velocity is about 4 times greater comparing to the pressure approach model since the density is constant as the jet density at the nozzle exit in their model. Pressure approach model has benefits in that this model adopted neither curve fitting nor extrapolation unlike the direct approximation model, and included the effects of density change which are not considered in the model developed by Kang et al. So, this model is recommended for the engineering purpose other than direct approximation model.

Arbitrary jet expansion angles are applied in this paper for the velocity calculation of the potential CEDM missile. Therefore, further investigations and experiments are necessary to predict the jet expansion angles in various conditions.

### REFERENCES

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### Nomenclatures

- $A_o$  = Cross sectional area of the missile object.
- $A_e$  = Cross sectional area of the exit nozzle.
- $C_0$  = Empirical constant
  - (-1.112 obtained from equation 1).
- $C_1$  = Empirical constant (-0.0127 for x is smaller than 50D<sub>e</sub>).
- $D_e$  = Inside diameter of the nozzle exit.
- m = Mass of the missile object.
- $P_e$  = Pressure of the jet at the nozzle exit.
- $P_v$  = Pressure of the coolant inside the vessel.
- $R_e$  = Radius of the nozzle exit.
- $V_e$  = Jet velocity at the exit plane.
- $V_x$  = Jet velocity at distance x.
- x = Distance from the nozzle exit.
- x = Distance from the nozzle exit where the jet expansion area becomes the same as the cross sectional area of the missile objet.
- $\phi$  = Jet expansion angle.
- $\rho_e$  = Jet density at the nozzle exit.
- $\rho_x$  = Jet density at distance x.