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A Feasibility Study on a New Diagnostic Method for Convergence of Fission Source Distribution Using Kolmogorov-Smirnov Test

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1. Introduction

In the Monte Carlo (MC) criticality calculation, the power iteration method is conventionally used. First, a fission source distribution (FSD) is assumed in the initial cycle. After the neutron transport calculation, a new FSD is sampled and updated to use it for the next cycle. If a FSD is not converged to fundamental mode FSD in inactive cycles, the response biases can be occurred. Therefore, the diagnostics whether FSD is converged are important for MC eigenvalue simulation.

There are three kinds of the diagnostic methodologies based on the cell-wised number density are known: the posterior method [1], the anterior method [2], and the entropy method [3]. In the cell-wised methods, it should determine how to define cells and to divide regions. Also, some inaccuracy can be generated due to the loss of the distribution data in each cell. Because of these inconveniences and inaccuracy, source center of mass [4], center distance sum method [5], and A&S method [6] were proposed. In this study, not only to overcome limitation of cell-wised method, but also to increase the judgment ability of the FSD convergence, a feasibility study on a new diagnostic strategy is performed.

2. Methods and Results

2.1 Proposal on a New Diagnostic Method

The eigenvalue equation for the criticality calculation is given as the follows:

$$ks(r) = Hs(r) \tag{1}$$

where k is multiplication factor, *s(r)* is fission source number density, r is position and H is an integral operator of the fission kernel [7]. In this study, to use the position information in the FSD diagnostics, *S(r)* is divided into *x*, *y*, *z* coordinates as following equation [6].

$$S^t(\mathrm{r}) \equiv S^t(x)\cdot\vec{e}_x + S^t(y)\cdot\vec{e}_y + S^t(z)\cdot\vec{e}_z \tag{2}$$

where $S^t(x)$, $S^t(y)$, and $S^t(z)$ are the source number density in *x*, *y*, and *z* coordinates at cycle *t*. Using the FSDs, if we can verify whether the distributions at each cycle are identical for each other, the FSD convergence can be judged. Therefore, with the proposed method, the judgments of the FSD convergence are performed as the following steps.

i) To save computational memory, the fission source number densities are converted to an empirical cumulative probability distribution functions (ECPDF) for each cycle *t*.

ii) The identity of the ECPDFs for i^{th} cycle is estimated by comparing cycles from i+p^{th} to i+p+q^{th} using Kolmogorov-Smirnov two sample (KS) test [8]. p and q should be decided by considering the dependency of the FSD.

The ECPDF $\hat{F}_X(t)$ is defined as the following equation [9].

$$\hat{F}_\mathrm{x}(t) \equiv \frac{1}{m}\sum_{i=1}^{m} I(X_i \le t) \tag{3}$$

where m is the number of samples from absolutely continuous distribution function F_x. X_i is variable of F_x division number i. $\boldsymbol{I}(A)$ is the indicator function of event A, which returns 1 when A is true; otherwise, it returns 0.

After the fission source density for each cycle is converted to the ECPDF, the KS tests are pursued. KS test is a kind of nonparametric statistical tests. It is used to test the null hypothesis $H_0\colon F_X(x) = F_Y(x)$. $F_X(x)$ and $F_Y(x)$ are two absolutely continuous distribution functions and x is real numbers. If the sample sizes in each distribution functions are large enough, KS test compares the discrete ECPDF. For the discrete case, the null hypothesis of this test is $H_0\colon \hat{F}_X(x) = \hat{F}_Y(x)$. $\hat{F}_X(x)$ is the ECPDF of variable X, and $\hat{F}_Y(x)$ is the ECPDF of variable Y.

In KS test, $D_{m,n}$ is used to test null hypothesis H_0. $D_{m,n}$ is defined as

$$D_{m,n} = sup\left|\hat{F}_\mathrm{x}(x) - \hat{F}_\mathrm{y}(x)\right| \tag{4}$$

where *sup* is the supremum function, m is the number of samples of $\hat{F}_X$, and n is the number of samples of $\hat{F}_Y$. The test rejects H_0 for a significant level α when Eq. (5) is satisfied.

$$D_{m,n} \ge D_{m,n,\alpha} \tag{5}$$

where $D_{m,n,\alpha}$ is a critical value depending on α. KS statistic is illustrated on Fig. 1. Black arrow in Fig. 1 describes the statistic of KS test.

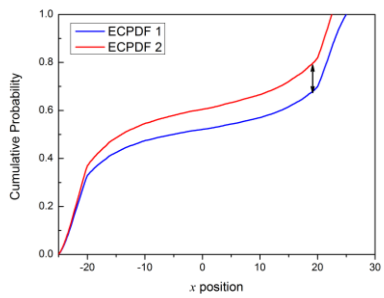


Fig.1 Overview of Kolmogorov-Smirnov Two Sample Test Using ECPDFs

2.2 Verification of Proposed Diagnostic Method.

To verify the proposed method, a benchmark problem is designed as shown in Fig. 2. The details of the benchmark problem is given in Table I. Using the MC code which is being developed in our research group, the MC simulation was performed with ENDF-VI cross section library. For the diagnostics, it is assumed that p and q are 50 and 50, respectively. For the test, the division number i in Eq. (3) was set to 2,000. The pass rate is the number of FSDs passing the KS test per total comparing cycles. The significant level (α) was set to 0.01. This means that the criteria of the pass rate for the diagnostic of the FSD convergence was 99 %.

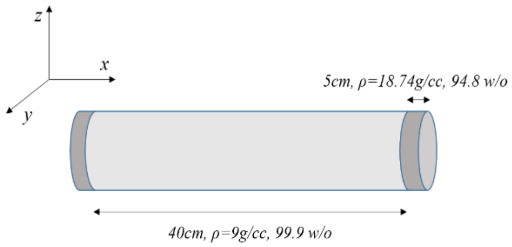
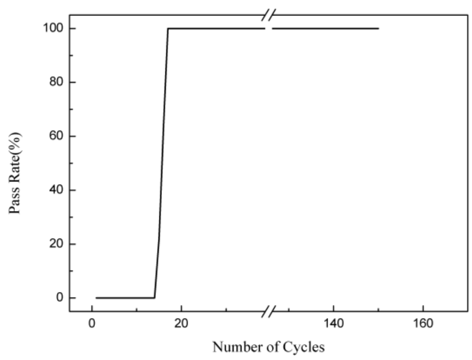


Fig. 2 Illustration of Simulated Uranium Rod

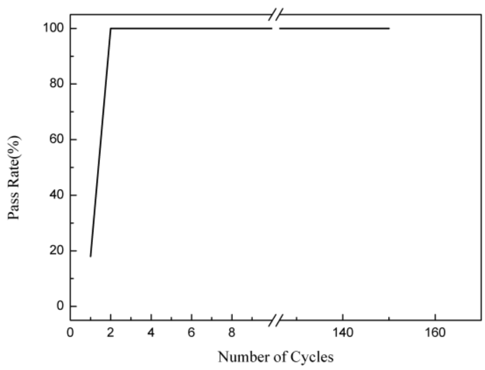
Table I: Details of the Benchmark Problem

Description	Value
Material	Uranium
Enrichment(w/o)	99.9, 94.8
Density(g/cc)	9, 18.74
Particle/Cycle	60000
Inactive Cycle	50
Active Cycle	150
k_{eff}(relative error)	0.56122(0.00094)

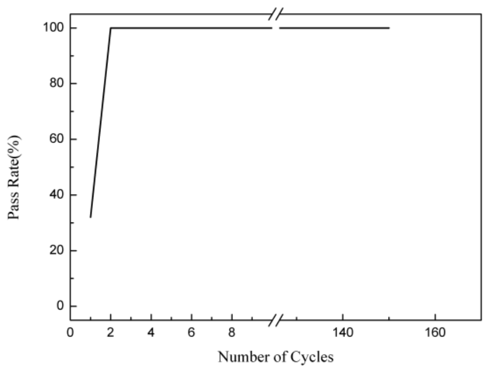
Fig. 3 shows the results of the pass rate of the benchmark problem using KS test. Due to the geometrical property, the FSDs of y and z direction are converged within 5th cycle. For the S(x), it was judged that the FSDs are converged about 20th cycle. To compare the results, the Shannon Entropy was evaluated as shown in Fig. 4. It shows that the converged cycle using Shannon Entropy method is about 25th cycle. Therefore, the diagnostic ability of the proposed method seems reasonable.



(a) x-axis



(b) y-axis



(c) z-axis

Fig. 3. Pass Rate versus Cycle in x-, y-, and z-axis

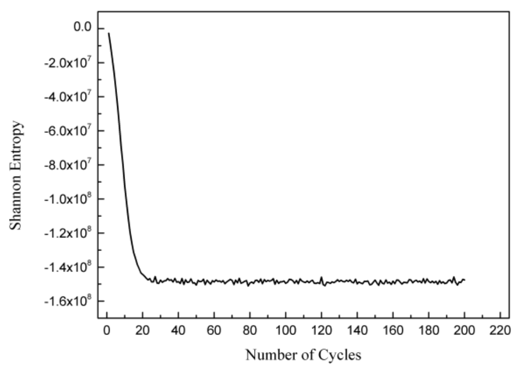


Fig. 4. Results of Shannon Entropy

3. Conclusions

In this study, a new strategy for diagnosing convergence of FSD was proposed to increase judge ability. The main idea of the proposed strategy is checking the distribution identity of the ECPDFs at each cycle. First, FSD of each cycles is expressed to ECPDFs on x, y, and z direction. Then, the degree of the consistency of ECPDF is tested using the statistical hypothesis test. For a benchmark problem, the FSDs were estimated by the proposed method, and the results were compared with results using Shannon Entropy method. It is analyzed that the proposed method has good judgment ability for the convergence of the FSD.

4. Acknowledgement

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