Implementation of Adjoint-weighted Kinetics Parameter Calculation in MCS

Yunki Jo, and Deokjung Lee*
Ulsan National Institute of Science and Technology 50,UNIST-gil, Eonyang-eup, Ulju-gun, Ulsan, 689-798, Korea
**Corresponding author: deokjung@unist.ac.kr*

1. Introduction

There have been several studies to calculate the adjoint-weighted quantities such as the effective delayed neutron fraction, β_{eff}, and the prompt neutron generation time, Λ, in the point kinetics equation. Kiedrowski and Brown [1] developed an adjoint-weighted kinetics parameters calculation method using the iterated fission probability (IFP), a quantity proportional to the adjoint function. Adjoint-weighted kinetics parameters calculation method was implemented in MCS. MCS is Monte Carlo neutron transport code that has been developed in UNIST [2].

2. Kinetics Parameters Tally Method Weighted by Self-Consistent Adjoint Function

Two adjoint-weighted kinetics parameters, β_{eff} and Λ, are defined by [3]:

$$\beta_{eff} = \frac{\langle \phi_0^*, B\phi \rangle}{\langle \phi_0^*, F\phi \rangle} , \quad (1)$$

$$\Lambda = \frac{\left\langle \phi_0^*, \frac{1}{v}\phi \right\rangle}{\langle \phi_0^*, F\phi \rangle} . \quad (2)$$

The brackets denote an integration over all phase space. ϕ is the forward angular flux and ϕ_0^* is the fundamental-mode solution of the adjoint angular flux. v is the neutron speed, and **B** and **F** are the delayed and total fission operator respectively.

A neutron called progenitor is introduced at a point in phase space and original generation. The progenitor will produce more neutrons through fission reaction, and these progeny neutrons will produce more neutrons in successive latent generations. After sufficient number of latent generations, the neutron population from this progenitor will reach an asymptotic value, which is IFP:

$$. \ \phi_0^* \cong \phi_{0,n}^*; n \gg 1 \quad (3)$$

If $\phi_{0,n}^*$ is converged after sufficient n latent generations, β_{eff} in the i-th cycle weighted by $\phi_{0,n}^*$ can be calculated as [4]:

$$\overline{\beta_{eff}^i} = \frac{\overline{k_d^{i,-n} \beta_0^{i-n}}}{\overline{k^i}} , \quad (4)$$

$$\overline{k_d^{i,-n} \beta_0^{i-n}} = \frac{1}{M^i} \sum_{j \in D^{i-n+1}} \sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}} , \quad (5)$$

$$\overline{k^i} = \frac{1}{M^i} \sum_{j=1}^{M^i} \sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}} . \quad (6)$$

where $k_d^{i,-n}$ and β_0^{i-n} are the number of n-th next generation fission neutrons from a delayed fission source generated at the n-th previous cycle and the ratio of the number of delayed neutron to the total number of fission neutron at cycle i-n. $\bar{k}^i$ is the fundamental mode eigenvalue for cycle i.

Λ in the i-th cycle weighted by $\phi_{0,n}^*$ also can be calculated as:

$$\overline{\Lambda^i} = \frac{\frac{1}{M^i} \sum_{j=1}^{M^i} \left(\sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}} \right) \left(w^{(i-n+1)j'1} \frac{\Delta l^{(i-n+1)j'k'}}{v^{(i-n+1)j'k'}} \right)}{\frac{1}{M^i} \sum_{j=1}^{M^i} \left(\sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}} \right) \left(w^{(i-n+1)j'1} \nu\Sigma_f^{(i-n+1)j'k'} \Delta l^{(i-n+1)j'k'} \right)} , \quad (7)$$

$$\overline{\Lambda^i} = \frac{\overline{\Lambda'^i}}{\overline{k^i k^{i-1}}} , \quad (8)$$

$$\overline{\Lambda'^i} = \frac{1}{M^i} \sum_{j=1}^{M^i} \sum_{k=1}^{K^{ij}} w^{ijk} \frac{\nu\Sigma_f^{ijk}}{\Sigma_t^{ijk}} \sum_{k'=1}^{k} \Delta t^{ijk} . \quad (9)$$

where i, j and k are cycle, particle, and collision indices, respectively. j' and k' are particle and collision indices of the (i-n+1)-th cycle from which the j-th fission source of cycle i is generated. w^{ijk}, Σ_f^{ijk} and Σ_{t}^{ijk} are the neutron weight, fission and total cross section for k-th collision of j-th particle at cycle i. M^i, K^i and D^i are the number of particles, the total number of collisions and the domain of delayed fission sources for cycle i, respectively. Δl^{ijk} is the track length between position at (k-1)-th collision of the particle j and k-th collision of particle j at cycle i, and v^{ijk} is the velocity corresponding to E for Δl^{ijk}. Δt^{ijk} is the time between the (k-1)-th collision and k-th collision of the particle j at cycle i.

3. Numerical Results

The MC forward eigenvalue calculation with continuous energy cross-section library was conducted for three problems. The first one is a bare one-dimensional (1-D) slab problem. A slab model with a half-thickness of 10 cm is used. The second one is a

PWR pin cell problem. PWR pin cell model was designed as shown in Fig. 1. The pin cell is composed of four regions from the center region: UO2 fuel, Air, Zirconium for cladding, and water for coolant. Table I indicates the pin cell model geometry. r1~r4 in the Table I were assigned from inner circle radius to outer geometry of pin cell. The third one is a bare Godiva [5] problem which has the radius of 8.741 cm sphere geometry and fuel material with 94.73 wt% U^{235}. These three problems are calculated to verify implemented adjoint-weighted kinetics parameters calculation method. The results are compared with results from McCARD [6]. The self-consistent adjoint function was assumed to be converged after 10 power iterations. The MC calculation was performed for 200 inactive cycles and 700 active cycles with 50,000 histories per cycle.

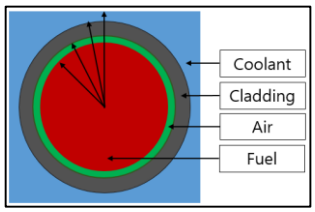

Fig.1. 2-D PWR pin cell model

Table I: PWR Pin Cell Model Geometry

	Radius (cm)
r1	0.98
r2	1.10
r3	1.34
r4	1.47

3.1 Adjoint-weighted flux distribution

The forward scalar flux and the adjoint-weighted flux were calculated by MCS for verification of adjoint method. Exceptively, adjoint flux was calculated instead of adjoint-weighted flux in 1-D slab problem. The results are given in Fig. 2, 3 and 4. Forward scalar flux and adjoint flux show coincidence in 1-D slab problem. In PWR pin cell problem and Godiva problem, adjoint-weighted flux is concentrated on center region. The calculation of PWR pin cell problem was performed for 50 inactive and 3,000 active cycles and 100,000 histories for the convergence of the graph.

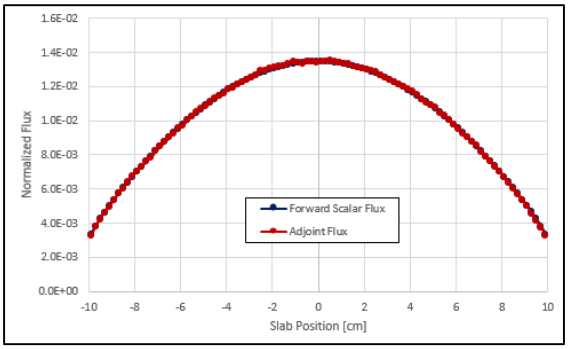

Fig. 2. Forward scalar flux and adjoint flux calculated by MCS in 1-D slab problem for 50 inactive cycles, 500 active cycles and 50,000 histories.

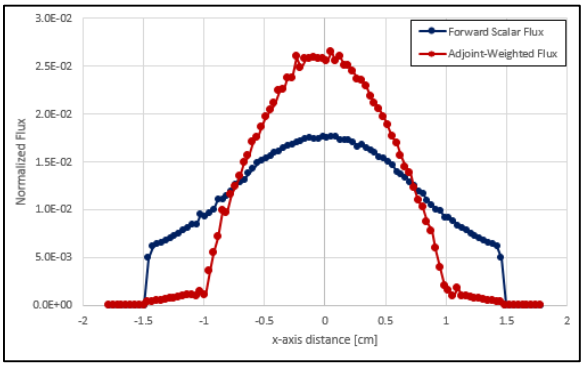

Fig. 3. Forward scalar flux and adjoint-weighted flux calculated by MCS in PWR pin cell problem for 50 inactive cycles, 3,000 active cycles and 100,000 histories.

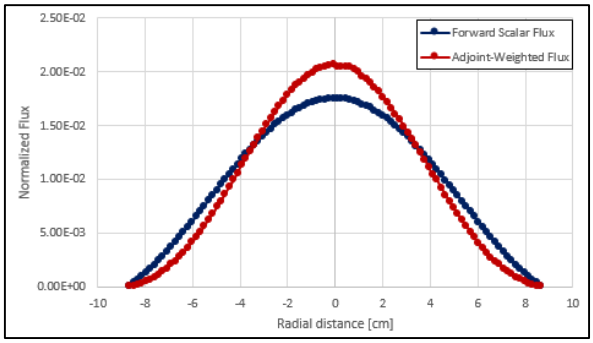

Fig. 4. Forward scalar flux and adjoint-weighted flux calculated by MCS in Godiva problem for 50 inactive cycles, 500 active cycles and 50,000 histories.

3.2 Results

Calculation results from the 1-D slab problem, PWR pin cell problem, and Godiva problem are in the Table II, Table III, and Table IV, respectively. β_{eff} and Λ are kinetics parameters weighted by self-consistent adjoint function. In the tables, the data having $\pm$ sign mean the relative standard deviation. The results show good agreement.

Table II: Comparison of Kinetics Parameter Results from MCS and McCARD in 1-D slab model

	MCS	McCARD	Error [%]
k_{eff}	0.31016 $\pm$ 0.00005	0.31020 $\pm$ 0.00005	-0.01289
β_{eff}	6.15906E-03 $\pm$ 0.00907	6.18944E-03 $\pm$ 0.00807	-0.49084
Λ	1.91163E-07 $\pm$ 0.00253	1.90135E-07 $\pm$ 0.00236	0.54067

Table III: Comparison of Kinetics Parameter Results from MCS and McCARD in PWR pin cell model

	MCS	McCARD	Error [%]
k_{eff}	0.02571 $\pm$ 0.00001	0.02569 $\pm$ 0.00001	0.07785
β_{eff}	2.89378E-03 $\pm$ 0.02562	3.04991E-03 $\pm$ 0.02437	-5.11917
Λ	5.28301E-08 $\pm$ 0.10659	5.16594E-08 $\pm$ 0.08759	2.26619

Table IV: Comparison of Kinetics Parameter Results from MCS and McCARD in Godiva model

	MCS	McCARD	Error [%]
k_{eff}	1.00148 $\pm$ 0.00012	1.00113 $\pm$ 0.00013	0.03496
β_{eff}	6.61615E-03 $\pm$ 0.00764	6.55364E-03 $\pm$ 0.00745	0.95382
Λ	5.72795E-09 $\pm$ 0.00107	5.68750E-09 $\pm$ 0.00103	0.71121

4. Conclusions

Adjoint-weighted kinetics parameters calculation method was implemented in MCS, which is Monte Carlo neutron transport code has been developed in UNIST. Three test problems are calculated for verification of adjoint-weighted kinetics parameter calculation methods by MCS. The results show good agreement.

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