A Point Dynamic Model for Stability Analysis of the PGSFR

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1. Introduction

The Korea Atomic Energy Research Institute (KAERI) and Argonne National Laboratory are jointly carrying out a broad R&D programme in support of the 150 MWe Prototype Generation-IV Sodium-cooled Fast Reactor (PGSFR) to test and demonstrate the performance of metal fuel containing transuranics (TRU) for commercial SFRs and the TRU transmutation capability of a burner reactor as a part of an advanced fuel cycle system [1].

To ensure the enhanced safety criteria for an advanced reactor system, the PGSFR design is highly based on the inherent safety mechanisms, i.e., passive responses to abnormal and emergency conditions, and thereby minimizes the need for active or engineered safety systems. In this regard, various inherent reactivity feedbacks in the PGSFR including thermal expansion of the sodium coolant, fuel temperature change, thermal bowing of the fuel, thermal expansion of the core and structure, and thermal expansion of the control rod driveline should be carefully evaluated in the design process. Of primary importance is to clarify the influence of the inherent reactivity feedbacks on the reactor dynamics and stability against small reactivity disturbances under power operating conditions. The reactor response to such small reactivity disturbances is determined by the interaction of the various reactivity coefficients, magnitude of the initial reactivity insertion, and nature of the heat removal system [2-6]. Correspondingly, the need of developing prediction methods and tools able to provide a high level knowledge about the reactor dynamics and stability in relation to inherent reactivity feedback mechanisms has been recognized.

In this work, a point dynamic model was proposed for stability analysis of the PGSFR. First, the linearized point-kinetics equations were coupled with thermal-hydraulic feedbacks via a lumped heat transfer model in the fuel, cladding, and coolant. Second, the relationship between the core outlet and inlet coolant temperatures was determined through an energy balance for the PHTS (Primary Heat Transport System) IHX (Intermediate Heat Exchanger) primary side. Finally, the Laplace transform and the frequency domain approach were applied to derive the reactor transfer functions for stability analysis. In addition, impacts of the sodium density coefficient, initial core power level, and fuel bowing on reactor stability were also examined. The results reveal the conditions under which the PGSFR can become unstable and therefore provide core designers useful information for further improvement of the reactor stability under power operating conditions.

2. Methods and Results

2.1 Point Dynamic Model for Stability Analysis

A point dynamic model based on the point kinetics and lumped heat transfer equations was proposed for stability analysis of the PGSFR against the forcing functions such as the external reactivity perturbation, core inlet coolant temperature perturbation, and primary coolant mass flow rate perturbation.

First, the point kinetics equations are given by:

$$\frac{dP(t)}{dt} = \frac{\rho(t)-\beta}{\Lambda} P(t) + \sum_j \lambda_j c_j(t) \quad (1)$$

$$\frac{dc_j(t)}{dt} = \frac{\beta_j}{\Lambda} P(t) - \lambda_j c_j(t) \quad (2)$$

Second, the energy balance equations for the fuel, cladding, and coolant are given by:

$$m_F c_{pF} \frac{dT_F}{dt} = P - h_{FC}(T_F - T_C) \quad (3)$$

$$m_C c_{pC} \frac{dT_C}{dt} = h_{FC}(T_F - T_C) - h_{CM}(T_C - T_M) \quad (4)$$

$$m_M c_{pM} \frac{dT_M}{dt} = h_{CM}(T_C - T_M) - W_M c_{pM}(T_{Mout} - T_{Min}) \quad (5)$$

Finally, the relation between the core outlet and inlet coolant temperatures is obtained via an energy balance for the PHTS IHX primary side as follows.

$$m_X c_{pM} \frac{dT_X}{dt} = W_X c_{pM}(T_{Xin} - T_{Xout}) - h_X(T_X - T_S) \quad (6)$$

Assuming small perturbations from the steady state in Eqs. (1)-(6) yields the following equation system.

$$\frac{d\delta P}{dt} = \frac{P_0}{\Lambda}\delta\rho - \frac{\beta}{\Lambda}\delta P + \sum_j \lambda_j \delta c_j \quad (7)$$

$$\frac{d\delta c_j}{dt} = \frac{\beta_j}{\Lambda}\delta P - \lambda_j \delta c_j \quad (8)$$

$$m_F c_{pF} \frac{d\delta T_F}{dt} = \delta P - h_{FC}(\delta T_F - \delta T_C) \quad (9)$$

$$m_C c_{pC} \frac{d\delta T_C}{dt} = h_{FC}(\delta T_F - \delta T_C) - h_{CM}(\delta T_C - \delta T_M) \quad (10)$$

$$m_M c_{pM} \frac{d\delta T_M}{dt} = h_{CM}(\delta T_C - \delta T_M) - W_{M0} c_{pM}(\delta T_{Mout} - \delta T_{Min}) - c_{pM}(T_{Mout0} - T_{Min0})\delta W_M \quad (11)$$

$$m_X c_{pM} \frac{d\delta T_X}{dt} = W_{X0} c_{pM}(\delta T_{Xin} - \delta T_{Xout}) + c_{pM}(T_{Xin0} - T_{Xout0})\delta W_X - h_X \delta T_X \quad (12)$$

where

$$\delta\rho = \delta\rho_{ex} + r_D \delta T_F + r_Z \delta T_C + (r_M + r_R)\delta T_M + r_{Min}\delta T_{Min} \quad (13)$$

Given an assumption that the temperature decrease through the IHX primary side is exponential, the following approximation can be obtained.

$$\delta T_{Xout} = \delta T_{Xin0} e^{-\gamma t} = \frac{T_{Xout0}}{T_{Xin0}} \delta T_{Xin} \quad (14)$$

Consider the relationship between T_X, T_{Xin}, and T_{Xout} as follows.

$$\delta T_X \cong \frac{\delta T_{Xin} + \delta T_{Xout}}{2} = \frac{T_{Xin0}+T_{Xout0}}{2T_{Xout0}}\delta T_{Xout} = a_1 \delta T_{Xout} \quad (15)$$

Now expressing δT_X in Eq. (12) in terms of δT_{Xout} gives:

$$m_X c_{pM} a_1 \frac{d\delta T_{Xout}}{dt} = W_{X0} c_{pM}(\delta T_{Xin} - \delta T_{Xout}) + c_{pM}(T_{Xin0} - T_{Xout0})\delta W_X - h_X a_1 \delta T_{Xout} \quad (16)$$

Expressing δT_{Xin}, δT_{Xout}, and δW_X in Eq. (16) in terms of δT_{Min}, δT_{Mout}, and δW_M yields:

$$4m_X c_{pM} a_1 \frac{d\delta T_{Min}}{dt} = 4W_{X0} c_{pM}(\delta T_{Mout} - \delta T_{Min}) + c_{pM}(T_{Xin0} - T_{Xout0})\delta W_M - 4h_X a_1 \delta T_{Min} \quad (17)$$

Laplace transformation of Eqs. (7)-(11), (13), (17) gives the following equation system.

$$s\delta P(s) = \frac{P_0}{\Lambda}\delta\rho(s) - \frac{\beta}{\Lambda}\delta P(s) + \sum_j \lambda_j \delta c_j(s) \quad (18)$$

$$s\delta c_j(s) = \frac{\beta_j}{\Lambda}\delta P(s) - \lambda_j \delta c_j(s) \quad (19)$$

$$\delta\rho(s) = \delta\rho_{ex}(s) + r_D\delta T_F(s) + r_Z\delta T_C(s) + (r_M + r_R)\delta T_M(s) + r_{Min}\delta T_{Min}(s) \quad (20)$$

$$s\delta T_F(s) = b_1\delta P(s) + b_2\delta T_F(s) + b_3\delta T_C(s) \quad (21)$$

$$s\delta T_C(s) = b_4\delta T_F(s) + b_5\delta T_C(s) + b_6\delta T_M(s) \quad (22)$$

$$s\delta T_M(s) = b_7\delta T_C(s) + b_8\delta T_M(s) + b_9\delta T_{Mout}(s) + b_{10}\delta T_{Min}(s) + b_{11}\delta W_M(s) \quad (23)$$

$$\delta T_{Mout}(s) = (b_{12} + b_{13}s)\delta T_{Min}(s) + b_{14}\delta W_M(s) \quad (24)$$

From Eqs. (18)-(19), we obtain the zero-power transfer function as follows.

$$G(s) = \frac{\delta P(s)}{P_0\delta\rho(s)} = \frac{1}{s\left(\Lambda + \sum_{j=1}^{6}\frac{\beta_j}{s+\lambda_j}\right)} \quad (25)$$

Using one precursor group approximation gives:

$$G(s) = \frac{1}{s\left(\Lambda + \frac{\beta}{s+\lambda}\right)} \quad (26)$$

where

$$\frac{1}{\lambda} = \frac{1}{\beta}\sum_{j=1}^{6}\frac{\beta_j}{\lambda_j} \quad (27)$$

From Eqs. (21)-(24), we can express the images of the fuel, cladding, and core average coolant temperatures in terms of the images of the reactor power and forcing functions (core inlet coolant temperature and primary coolant mass flow rate) and substitute them into Eq. (20) to obtain the following equation for the total reactivity change.

$$\delta\rho(s) = \delta\rho_{ex}(s) + r_D A_1(s)\delta P(s) + r_Z A_4(s)\delta P(s) + (r_M + r_R)A_7(s)\delta P(s) + \begin{bmatrix} r_D A_2(s) + r_Z A_5(s) + \\ (r_M + r_R)A_8(s) + r_{Min} \end{bmatrix}\delta T_{Min}(s) + \begin{bmatrix} r_D A_3(s) + r_Z A_6(s) + \\ (r_M + r_R)A_9(s) \end{bmatrix}\delta W_M(s) \quad (28)$$

Now we consider the total reactivity change in Eq. (28) is contributed from the following six terms:

(1) external reactivity perturbation (e.g. control rods): $\delta\rho_{ex}(s)$

(2) feedback from the fuel temperature:

$$\delta\rho_F(s) = r_D A_1(s)\delta P(s) \quad (29)$$

(3) feedback from the cladding temperature:

$$\delta\rho_C(s) = r_Z A_4(s)\delta P(s) \quad (30)$$

(4) feedback from the coolant temperature:

$$\delta\rho_M(s) = (r_M + r_R)A_7(s)\delta P(s) \quad (31)$$

(5) core inlet coolant temperature perturbation:

$$\delta\rho_u(s) = \begin{bmatrix} r_D A_2(s) + r_Z A_5(s) + \\ (r_M + r_R)A_8(s) + r_{Min} \end{bmatrix}\delta T_{Min}(s) \quad (32)$$

(6) primary coolant mass flow rate perturbation:

$$\delta\rho_w(s) = \begin{bmatrix} r_D A_3(s) + r_Z A_6(s) + \\ (r_M + r_R)A_9(s) \end{bmatrix}\delta W_M(s) \quad (33)$$

Thus, Eq. (28) can be rewritten in a short form as:

$$\delta\rho(s) = \delta\rho_{ex}(s) + \delta\rho_F(s) + \delta\rho_C(s) + \delta\rho_M(s) + \delta\rho_u(s) + \delta\rho_w(s) \quad (34)$$

And the following transfer functions are defined.

$$H_F(s) = \frac{\delta\rho_F(s)}{\delta P(s)} = r_D A_1(s) \quad (35)$$

$$H_C(s) = \frac{\delta\rho_C(s)}{\delta P(s)} = r_Z A_4(s) \quad (36)$$

$$H_M(s) = \frac{\delta\rho_M(s)}{\delta P(s)} = (r_M + r_R)A_7(s) \quad (37)$$

$$K_u(s) = \frac{\delta\rho_u(s)}{\delta T_{Min}(s)} = r_D A_2(s) + r_Z A_5(s) + (r_M + r_R)A_8(s) + r_{Min} \quad (38)$$

$$K_w(s) = \frac{\delta\rho_w(s)}{\delta W_M(s)} = r_D A_3(s) + r_Z A_6(s) + (r_M + r_R)A_9(s) \quad (39)$$

The block diagram of the reactor dynamics representing these transfer functions is shown in Fig. 1 and the reactivity summation equation, Eq. (34), can be rewritten as follows.

$$\frac{\delta P(s)}{P_0 G(s)} = \delta\rho_{ex}(s) + H_F(s)\delta P(s) + H_C(s)\delta P(s) + H_M(s)\delta P(s) + K_u(s)\delta T_{Min}(s) + K_w(s)\delta W_M(s) \quad (40)$$

We will consider one perturbation at a time, assuming the other perturbations equal to zero (the system is linear, thus superposition of perturbations can be used), to find the following reactor transfer functions. The external-reactivity-to-power transfer function is obtained by assuming that $\delta T_{Min} = \delta W_M = 0$ (they are unperturbed):

$$H(s) = \frac{\delta P(s)}{\delta\rho_{ex}(s)} = \frac{P_0 G(s)}{1 - P_0 G(s)[H_F(s) + H_C(s) + H_M(s)]} \quad (41)$$

The core-inlet-coolant-temperature-to-power transfer function is obtained by assuming that $\delta\rho_{ex} = \delta W_M = 0$ (they are unperturbed):

$$L(s) = \frac{\delta P(s)}{\delta T_{Min}(s)} = K_u(s)H(s) \quad (42)$$

The primary-coolant-mass-flow-rate-to-power transfer function is obtained by assuming that $\delta\rho_{ex} = \delta T_{Min} = 0$ (they are unperturbed):

$$M(s) = \frac{\delta P(s)}{\delta W_M(s)} = K_w(s)H(s) \quad (43)$$

It was found that the characteristic equations of *L(s)* and *M(s)* are identical to that of *H(s)*. Thus the reactor stability property is the same for all considered forcing functions and the stability can be judged based on the roots of the following characteristic equation of *H(s)*.

$$1 - P_0 G(s)[H_F(s) + H_C(s) + H_M(s)] = 0 \quad (44)$$

It is noteworthy to recall that the necessary and sufficient condition for any closed-loop system (i.e., the reactor with reactivity feedbacks) to be stable to small perturbations is that all the roots of the characteristic equation, Eq. (44), have negative real parts.

For solving Eq. (44), the PGSFR kinetics parameters, reactivity coefficients, and steady state thermal-hydraulic data are given in Tables I and II.

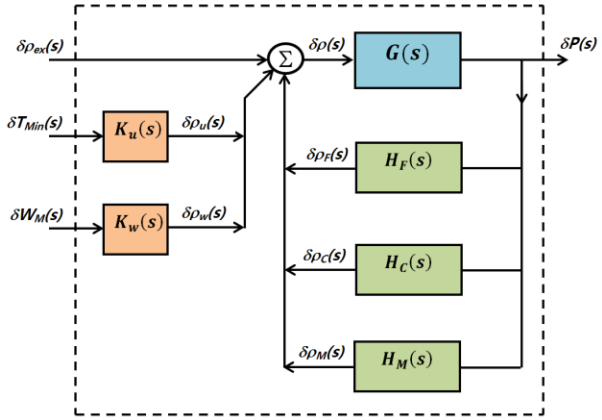

Fig. 1. Block diagram of the reactor dynamics

Table I: PGSFR kinetics parameters and reactivity coefficients

Parameter/ Coefficient	BOEC	EOEC
Λ, μsec	0.330	0.344
β, pcm	675	652
r_D, pcm/K (T in ^oC)	-1270 $T^{-1.19834}$	-1198 $T^{-1.18282}$
r_Z, pcm/K	-0.21876	-0.22633
r_R, pcm/K	-0.65654	-0.68027
r_{Min}, pcm/K	-1.10490	-1.14459
r_M, pcm/K	-0.21200	-0.19700

Table II: PGSFR steady state T-H data

m_F, kg	7330	W_{M0}, kg/sec	1991.2
c_{pF}, J/kg/K	500	T_{Min0}, ^oC	390
h_{FC}, W/K	1.14 E11	T_{Mout0}, ^oC	545
m_C, kg	1804	m_X, kg	2784.8
c_{pC}, J/kg/K	750	h_X, W/K	2334524
h_{CM}, W/K	1.14 E8	T_{Xin0}, ^oC	545
m_M, kg	1803.5	T_{Xout0}, ^oC	390
c_{pM}, J/kg/K	1269.5	-	

2.2 Stability Results and Discussion

In this Section, the stability of the reactor without and with reactivity feedbacks is evaluated based on the zero power transfer function, Eq. (26), and the roots of the characteristic equation, Eq. (44), respectively. Also, the influence of the sodium density coefficient, initial core power, and fuel bowing on reactor stability was investigated to figure out the conditions under which the reactor may become unstable in the presence of one or more possible positive reactivity coefficients.

The Bode diagram of the zero power transfer function $G(s)$ (i.e., the reactor without reactivity feedbacks) for one group and six groups of delayed neutron precursors was plotted in Fig. 2. It can be seen that one-group and six-group approximations exhibit the same behavior of the zero power transfer function. That is, as the frequency approaches zero, the magnitude becomes infinite. Thus, the PGSFR without inherent reactivity feedbacks was found to be intrinsically unstable.

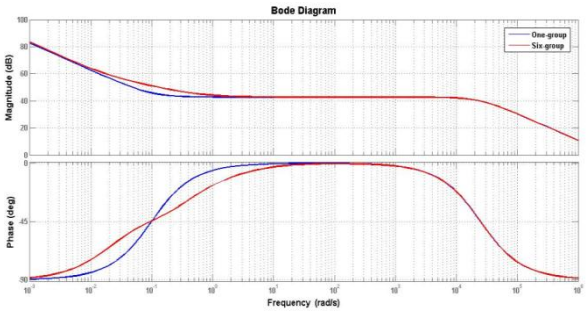

Fig. 2. Bode plot of the zero power transfer function

Under small reactivity disturbances, assuming the fuel temperature raised to ~900^{o}C gives the Doppler coefficient $r_D \cong$ -0.366 and -0.384 pcm/K (see Table I) at the beginning and end of the equilibrium cycle (BOEC and EOEC) respectively. Given the inherent reactivity feebacks of the PGSFR as shown in Table I, all the roots of the characteristic equation, Eq. (44), were found to have negative real parts at BOEC and EOEC as shown in Fig. 3. Hence, the current design of the PGSFR is inherently stable, regardless of the fuel burnup in the equilibrium cycle.

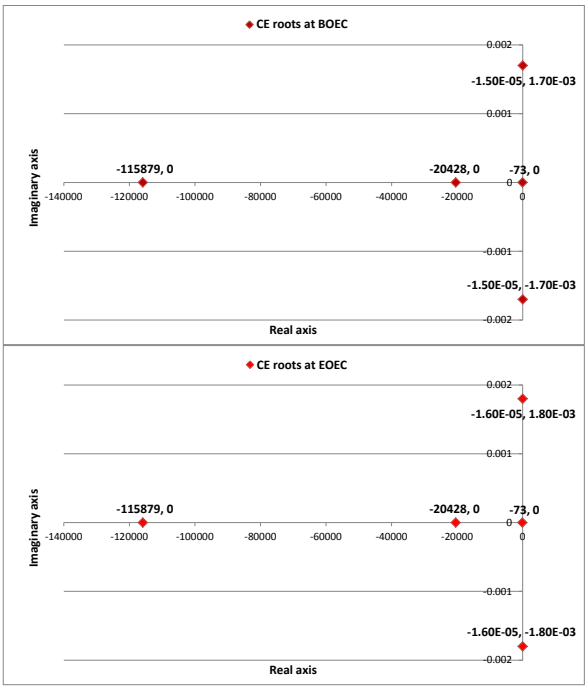

Fig. 3. Roots of the characteristic equation at BOEC (up) and EOEC (down)

Parametric studies on the initial core power level, fuel bowing, and sodium density coefficient show that the PGSFR is stable under the conditions: (a) overall reactivity coefficient contributed from fuel, cladding, and coolant is kept negative and (b) sodium density coefficient is kept lower than ~0.658/0.682 pcm/K at BOEC/EOEC respectively. Besides, it was found that the stability of the PGSFR is even more enhanced with fuel burnup in the equilibrium cycle. Also, the higher

the initial core power level is, the more unstable the reactor can be. Moreover, if there is one or more positive reactivity coefficients, it is recommended that they should be kept at least somewhat lower than the magnitude of the overall negative reactivity coefficient in order to keep the reactor stable.

3. Conclusions

In conclusion, it was shown that the stability property of the PGSFR is the same for all the three considered forcing functions. Furthermore, the PGSFR was found to be inherently stable thanks to the inherent negative reactivity coefficients and its stability is even more enhanced with fuel burnup in the equilibrium cycle. Especially, the conditions under which the PGSFR can become unstable in the presence of one or more positive reactivity coefficients were revealed. As a result, this study can provide designers useful information about the reactor dynamics along with the impacts of positive reactivity coefficients for further improvements of the reactor stability under power operating conditions.

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Appendix

$$b_1 = \frac{1}{m_F c_{pF}};\ b_2 = -\frac{h_{FC}}{m_F c_{pF}} = -b_3;\ b_4 = \frac{h_{FC}}{m_C c_{pC}};$$
$$b_5 = -\frac{h_{FC}+h_{CM}}{m_C c_{pC}};\ b_6 = \frac{h_{CM}}{m_C c_{pC}};\ b_7 = \frac{h_{CM}}{m_M c_{pM}} = -b_8;$$
$$b_9 = -\frac{W_{M0}}{m_M} = -b_{10};\ b_{11} = \frac{T_{Min0}-T_{Mout0}}{m_M};$$
$$a_1 = \frac{T_{Xin0}+T_{Xout0}}{2T_{Xout0}};\ b_{12} = \frac{a_1 h_X + W_{X0} c_{pM}}{W_{X0} c_{pM}};\ b_{13} = \frac{a_1 m_X}{W_{X0}};$$
$$b_{14} = \frac{T_{Xout0}-T_{Xin0}}{4W_{X0}};\ b_{15} = b_9 b_{12} + b_{10};$$
$$b_{16} = b_9 b_{13};\ b_{17} = b_9 b_{14} + b_{11};$$
$$A_1(s) = \frac{b_1[(s-b_5)(s-b_8)-b_6 b_7]}{(s-b_2)(s-b_5)(s-b_8)-b_6 b_7(s-b_2)-b_3 b_4(s-b_8)};$$
$$A_2(s) = \frac{b_3 b_6 (b_{15}+b_{16} s)}{(s-b_2)(s-b_5)(s-b_8)-b_6 b_7(s-b_2)-b_3 b_4(s-b_8)};$$
$$A_3(s) = \frac{b_3 b_6 b_{17}}{(s-b_2)(s-b_5)(s-b_8)-b_6 b_7(s-b_2)-b_3 b_4(s-b_8)};$$
$$A_4(s) = \frac{(s-b_2)A_1(s)-b_1}{b_3};\ A_5(s) = \frac{(s-b_2)A_2(s)}{b_3};$$
$$A_6(s) = \frac{(s-b_2)A_3(s)}{b_3};$$
$$A_7(s) = \frac{[(s-b_2)(s-b_5)-b_3 b_4]A_1(s)-b_1(s-b_5)}{b_3 b_6};$$
$$A_8(s) = \frac{[(s-b_2)(s-b_5)-b_3 b_4]A_2(s)}{b_3 b_6};$$
$$A_9(s) = \frac{[(s-b_2)(s-b_5)-b_3 b_4]A_3(s)}{b_3 b_6}.$$

Nomenclature

$P(t)$ is the reactor power at time t
$c_j(t)$ is the concentration of precursor group j at time t
$\rho(t)$ is the total reactivity at time t
β and Λ are the total delayed neutron fraction and the mean neutron generation time
λ_j and β_j are the decay constant and delayed neutron fraction of precursor group j
r_F is the fuel Doppler coefficient (pcm/K)
r_B is the fuel bowing coefficient (pcm/K)
r_Z is the axial expansion coefficient (pcm/K)
r_M is the sodium density coefficient (pcm/K)
r_R is the sub-assembly radial expansion coefficient (pcm/K)
r_{Min} is the grid-plate radial expansion coefficient (pcm/K)
T_F and T_C are the fuel and cladding temperatures (K)
T_M and T_{Min} are the core averge coolant and core inlet coolant temperatures (K)
m_F is the fuel mass (kg)
c_{pF} is the fuel specific heat capacity (J/kg/K)
h_{FC} is the fuel-gap-cladding global heat transfer coefficient (W/K)
m_C is the cladding mass (kg)
c_{pC} is the cladding specific heat capacity (J/kg/K)
h_{CM} is the cladding-coolant global heat transfer coefficient (W/K)
m_M is the coolant mass in the active core (kg)
c_{pM} is the coolant specific heat capacity (J/kg/K)
W_M is the primary coolant mass flow rate (kg/sec)
T_{Mout} is the core outlet coolant temperature (K)
W_{M0} is the primary coolant mass flow rate at the steady state (kg/sec)
T_{Min0} and T_{Mout0} are the core inlet and outlet coolant temperatures at the steady state (K)
PHTS is the Primary Heat Transport System
IHX is the Intermediate Heat Exchanger
m_X is the IHX primary side mass (kg)
T_X is the IHX primary side average temperature (K)
W_X is the primary coolant mass flow rate through the IHX (kg/sec)
H_X is the IHX heat transfer coefficient (W/m^2/K)
A_X is the IHX heat transfer area (m^2)
$h_X = H_X A_X$ is the IHX global heat transfer coefficient (W/K)
T_{Xin} and T_{Xout} are the IHX primary side inlet and outlet temperatures (K)
T_S is the IHX secondary side average temperature (K)
W_{X0} is the IHX coolant mass flow rate at the steady state (kg/sec)
T_{Xin0} and T_{Xout0} are the IHX primary side inlet and outlet temperatures at the steady state (K)
δx is the small change in the quantity x from the steady state

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