Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 7-8, 2015

X-ray Beam Spectral Reconstruction Using Laplace Transform and Attenuation Curves

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1. Introduction

As the use of X-ray tubes is widely spread mainly for medical diagnostic purposes or industrial applications, there is increasing demand for accurate and convenient way getting of X-ray beam spectral information. While measurement methods may provide quite accurate spectral information, these methods still require expensive detectors (example: HPGe, High Purity Germanium detector) and some conversion of measurement information into real spectrum.

Archer et. al.(1988)[1] have provided a useful spectral reconstruction technique based on Laplace transform and attenuation curves. In this paper we studied the applicability of the method to a 50 kV X-ray source spectrum and MCNPX 2.7.0 simulated attenuation curves, instead of ionization chamber measurements. After confirmation of the method in this paper, this method will be applied to accurately evaluate an unknown X-ray beam spectrum. Also similar technique may be applied easily to other particle beam lines which require accurate spectral information.

2. Methods and Results

The spectral reconstruction procedure of Archer's method is briefly presented in Fig. 1. First, we get the attenuation curve from Monte Carlo simulation(MCNPX) of the source-absorber(shield)-detector(ion chamber) set-up. Secondly, we find out three fitting parameters through nonlinear least squares fitting of the attenuation curve. Finally, source X-ray spectrum is calculated using three obtained parameters and Laplace transform-based reconstruction equation.

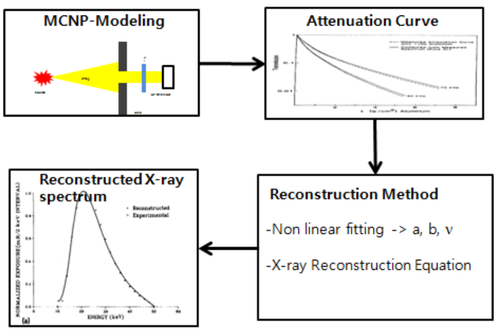
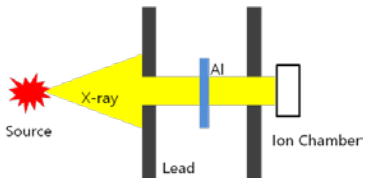


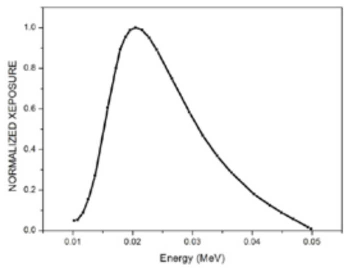
Fig. 1. Spectral Reconstruction Precedure

2.1 Attenuation Curve Calculation Using MCNPX 2.7.0

In this paper, the attenuation curve (ionization chamber responses as a function of Al shield thickness) was obtained through MCNPX simulations. Ionization chamber detection set-up used in the MCNPX calculations is described in Fig. 2. The source is taken from 50 kV X-ray spectrum provided by Archer. The shield material is Al (Aluminum) and its thickness ranges from 0.0 to 5.0 g/cm^2. The ionization chamber is assumed to be Model 10X5-6[1], with window area of 4.337 cm^2, and length of 2 cm. The distance between the source and the ion chamber window is 3.0 m and two collimators made with lead(Pb) are also used to reduce unwanted scattering contributions to the detector[3].



(a)



(b)

Fig. 2. (a) Source-shield-detector Set-up[3] for MCNPX Simulations, (b) 50 kV X-ray Source Spectrum[1]

Even though in the course of this study, various tallies and various particles(photon-electron mode) have been tried in MCNPX simulations, the results provided in the paper are from sum counts of pulse height tally(tally-8) output of photon-only mode calculations of MCNPX. The number of histories is set to 2x10^8 (NPT = 2x10^8) for all MCNPX cases.

2.2 Reconstruction Method

The following equation (1), the transmission model, is fitted to the attenuation curve (or transmission curve) obtained from measurements or simulations. Three parameters a, b and nu are found by a nonlinear least-squares fit. In the equation (1), x is absorber thickness in g/cm^2, and 'μ^0_m' is the mass attenuation coefficient of the absorber in unit of cm^2/g and corresponds to the maximum energy of the spectrum. The parameters a and b have the same units as x and ν is unit-less.

$$g(x) = \left(\frac{ab}{(a+x)(b+x)} \right)^{\upsilon} \exp(-\mu_m^0 x) \quad \text{- - - (1)}$$

Using three fitting parameters above, source X-ray spectrum can be calculated by the equation (2) [1,5]. In the equation (2), 'Γ(ν)' is the gamma function, and '$I_{\nu-1/2}$' is a modified Bessel function. 'μ_m' is the mass attenuation coefficient as a function of energy provided by XCOM[2], and '$d\mu_m/dE$' is determined by differentiating 'μ_m' numerically.

$$f_r(E) = \frac{\pi^{1/2}(ab)^{\upsilon}}{\Gamma(\upsilon)} \left[\frac{\mu_m - \mu_m^0}{a-b} \right]^{\upsilon-1/2} \exp\left[-\frac{a+b}{2}(\mu_m - \mu_m^0) \right] \quad \text{- - - (2)}$$
$$\times \; I_{\upsilon-1/2}\left[\frac{1}{2}(a-b)(\mu_m - \mu_m^0) \right]$$
$$\times \left(-\frac{d\mu_m}{dE} \right)$$

2.3 Results

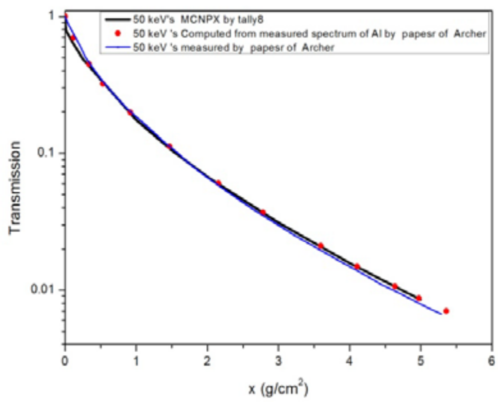


Fig. 3. Comparison of Attenuation Curves by MCNPX and Other Methods

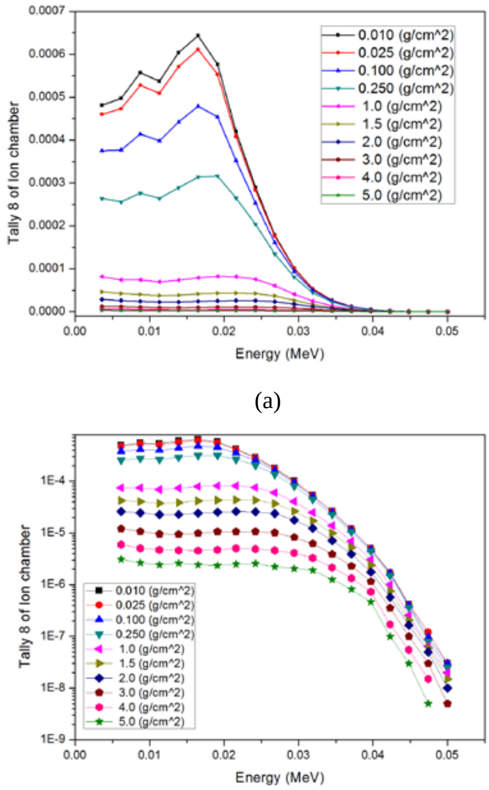
In Fig. 3 the attenuation curve obtained by MCNPX simulations was compared with original 50 kV source X-ray attenuation curve of Archer[1] and the results are very similar. In our MCNPX model we used a pure Al shield, while Archer's paper suggested inclusion of some impurities in the absorber composition [1].

Table 1. Comparison of fitting Parameters

	a	b	ν	R-squared
MCNPX1)	1.2921	0.2342	0.6190	0.9930
Archer2)	1.0515	0.3494	0.6661	0.9999

1) fitting parameters from the attenuation curve by MCNPX
2) fitting parameters from transmission data of Arhcer's paper

In Table 1 three parameters of Fig. 3's nonlinear fit are presented and compared with the parameters from Archer's 50 keV attenuation curve. Even though some differences are noticed in the parameters, both cases provide good fitting results, which is confirmed by the R-squared values.



(a)

(b)

Fig. 4. Tally 8 Responses of Ionization Chamber 9a) in Linear Scale and (b) in Log Scale

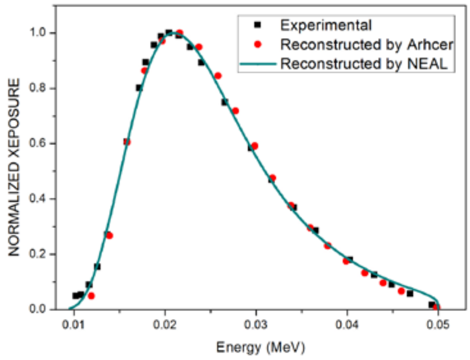


Fig. 5. Comparison of Spectrum Reconstruction Results

In the course of this study, we tried various tallies to generate ion chamber responses and after comparing the responses of tally-6 and tally-8 with original transmission curves, it is concluded that sum counts from tally-8 output give the closest results to the original responses and, in addition, it provides some insights in real responses as a function of energy. Even though the actual outcomes used in the fitting are the sum counts of tally-8 output, the results in Fig. 4 give us the image of detailed spectral responses of ionization chamber (if the detector could provide the spectral information). Finally, in Fig. 5 the reconstructed spectrum is compared with Archer's experimental spectrum and his reconstructed spectrum, which are quite close to each other.

3. Conclusions

It is concluded that Laplace transform-based spectral reconstruction technique given in equations (1) and (2) works well for a 50-kV X-ray source. In this paper we obtained the attenuation curve by the use of MCNPX simulations. We were able to rebuild the X-ray spectrum of 50 kV through this research by Monte Carlo simulation (fitting parameters, a: 1.2921, b: 0.2342, ν: 0.6190, R-squared: 0.9930). After confirmation of the method in this paper, this method may be applied in evaluating an unknown X-ray beam spectrum. Also similar technique may be applied easily to other particle beam lines whose accurate spectral information is desired.

Acknowledgments

This research was financially supported by the Ministry of Education(MOE) and National Research Foundation of Korea(NRF) through the Human resource Training Project for Regional Innovation(2012H1B8A2026280, by Kyungpook National University Research(2014).

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