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Study of Simplification of Markov Model for Analyzing System Dependability

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1. Introduction

In general, the Markov model can accurately describe the stats of system for analyzing a system dependability such as reliability, availability however the number of system state to consider are dramatically increased as a complexity of system is increased thus it is difficult to use the Markov model for analyzing the system dependability [1].

In this paper, we introduce the simplification methodology of the Markov model for analyzing system dependability using system failure rate concept. This system failure rate is the probability that the system is failed or unavailable given that the system was as good as at this time. Using this parameter, the Markov model of sub system can be replaced to the system failure rate and then this parameter just is considered in the Markov model of whole system.

2. Simplification Methodology of Markov Model

2.1 System failure rate for the reliability model

In the reliability model, system failure rate is defined as follows [2]:

$$\text{System Failure rate} = \frac{f(t)}{R(t)} \qquad (1)$$

R(t) is system reliability function and f(t) is failure density function. This parameter is the probability of system failure per unit time at age t for the individual in this population. This means that the number of failures during the differential time duration over the number of survivals at age t. Fig.1 shows this concept.

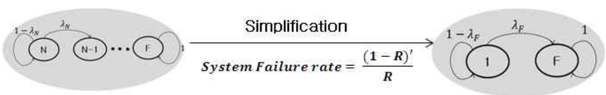

Fig. 1. Simplification of reliability model.

Using this concept, we simplify the triple redundancy modules.Fig.2 shows the Markov model of TMR. The state equation of this Markov model is express as

$$\frac{d}{dt}\begin{bmatrix} P_3(t) \\ P_{UD}(t) \\ P_2(t) \\ P_1(t) \\ P_F(t) \end{bmatrix} = \begin{bmatrix} -3\lambda & 0 & 0 & 0 & 0 \\ 3\lambda(1-C) & -2\lambda & 0 & 0 & 0 \\ 3\lambda C & 0 & -2\lambda & 0 & 0 \\ 0 & 0 & 2\lambda C & -\lambda & 0 \\ 0 & 2\lambda & 2\lambda(1-C) & \lambda & 0 \end{bmatrix} \begin{bmatrix} P_3(t) \\ P_{UD}(t) \\ P_2(t) \\ P_1(t) \\ P_F(t) \end{bmatrix} \qquad (2)$$

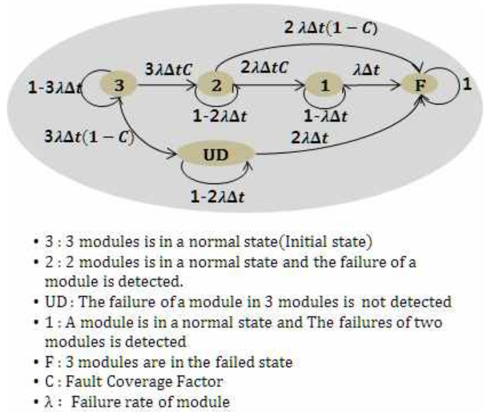

Fig. 2. Markov model of TMR without repair process[3]

The reliability function of the TMR is obtained from (2) as follow:

$$R_{TMR}(\mathrm{t}) = P_3(t) + P_{UD}(t) + P_2(t) + P_1(t) = \frac{3e^{\lambda t} + 3C^2 - 6C^2e^{\lambda t} + 3C^2e^{2\lambda t} - 2}{e^{3\lambda t}} \qquad (3)$$

And using (1), system failure rate is express as

$$\Lambda_{TMR}(\mathrm{t}) = -\frac{\frac{dR_{TMR}}{dt}}{R_{TMR}} = \frac{3\lambda\left(e^{\lambda t}-1\right)\left(C^2e^{\lambda t} - 3C^2 + 2\right)}{3e^{\lambda t} + 3C^2 - 6C^2e^{\lambda t} + 3C^2e^{2\lambda t} - 2} \qquad (4)$$

2.2 System failure rate for the availability model

The system failure rate of availability model can be defined as similar to that of reliability model.

$$\text{System Failure rate} = \frac{-dA(t)/dt}{A(t)} \qquad (5)$$

A(t) is system availability function. This parameter is the probability that the system is unavailable per unit time at age t for the individual in this population. This means that the number of unavailable system during the differential time duration over the number of available systems at age t. Fig. 3 shows this concept.

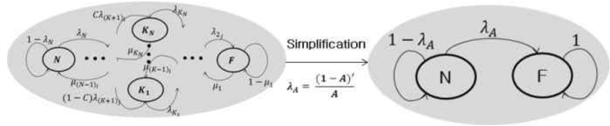

Fig. 3. Simplification of Availability model.

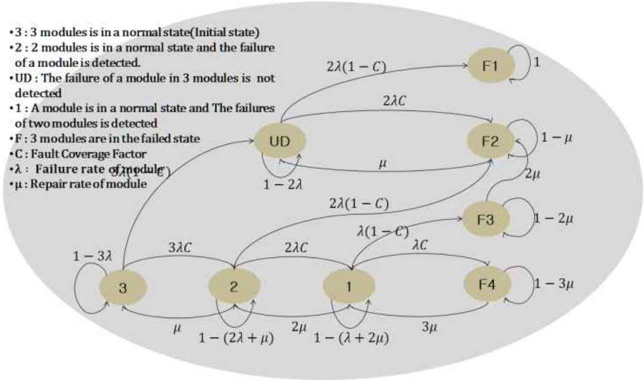

Fig. 4. Markov model of TMR with repair process

Fig.4 shows the Markov model of TMR considering the repair rate. Based on the Fig.4, the state equation is expressed as

$$\begin{bmatrix} P_3' \\ P_2' \\ P_{UD}' \\ P_1' \\ P_{f1}' \\ P_{f2}' \\ P_{f3}' \\ P_{f4}' \end{bmatrix} = \begin{bmatrix} -3\lambda & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 3\lambda C & -(2\lambda+\mu) & 0 & 2\mu & 0 & 0 & 0 & 0 \\ 3\lambda(1-C) & 0 & -2\lambda & 0 & 0 & \mu & 0 & 0 \\ 0 & 2\lambda C & 0 & -(\lambda+2\mu) & 0 & 0 & 0 & 3\mu \\ 0 & 0 & 2\lambda(1-C) & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\lambda(1-C) & 2\lambda C & 0 & 0 & -\mu & 2\mu & 0 \\ 0 & 0 & 0 & \lambda(1-C) & 0 & 0 & -2\mu & 0 \\ 0 & 0 & 0 & \lambda C & 0 & 0 & 0 & -3\mu \end{bmatrix} \begin{bmatrix} P_3 \\ P_2 \\ P_{UD} \\ P_1 \\ P_{f1} \\ P_{f2} \\ P_{f3} \\ P_{f4} \end{bmatrix} \quad (6)$$

The solution of (6) is expressed as

$$P = \sum_{n=1}^{8} C_n \Phi_n e^{\theta_n t}$$

θ_n is the eigen value of transition matrix, Φ_n is the eigen vector (1×8 matrix) and C_n is the constant which can be obtained from initial condition. The availability of TMR is the sum of from row1 to row 4 in P matrix. This is

$$A_{TM\,R} = P(1) + P(2) + P(3) + P(4) \quad (7)$$

Also the system failure rate considering repair rate is expressed as

$$\Lambda_{TM\,R}(\mathrm{t}) = -\frac{\frac{dA_{TM\,R}}{dt}}{A_{TM\,R}} \quad (8)$$

3. Application to RPS architecture

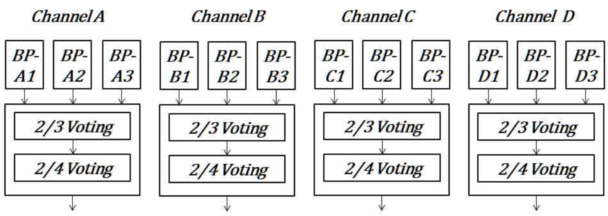

Fig. 5. RPS architecture[4]

When the Reactor Protection System (RPS) composed of 3 Bistable logic modules (BP) and 1 Coincidence module is considered, the CP performs 2/3 voting logic about 3 BPs in the same channel and performs 2/4 voting logic about 4 channels as shown Fig.5.

Using the system failure rate considering the repair rate as discussed in section 2.2, we can obtain the Markov model about the one channel as shown in Fig. 6

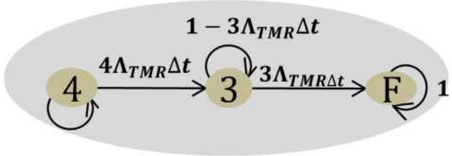

Fig. 6. Markov model of RPS using system failure rate

$\Lambda_{TM\,R}$ is the system failure rate in (8). The state equation of the Markov model in Fig.6 can be obtained and the solution can be solved in the similar manner in section 2.2.

3. Conclusions

In this paper, we proposed the method to simplify the Markov model in complex system architecture. we define the system failure rate and using this parameter, the Markov model of system could be simplified.

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