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# Fuzzy Uncertainty Evaluation for Fault Tree Analysis

Ki Beom Kim [a*], Hyung Jin Shim [a], Moo Sung Jae [b]
[a]*Seoul National University, 1 Gwanakro, Gwanak-gu, Seoul 151-744, Republic of Korea*
[b]*Hanyang University, 17 Haengdang, Sungdong-gu, Seoul 133-791, Republic of Korea*
[*]*Corresponding author: kibum926@snu.ac.kr*

## 1. Introduction

The probabilistic uncertainty propagation using the Monte Carlo (MC) method [1,2] is generally performed to quantify uncertainties of occurrence probabilities of top events in fault tree analysis. This traditional probabilistic approach can calculate relatively accurate results. However it requires a long time because of repetitive computation due to the MC method. In addition, when informative data for statistical analysis are not sufficient or some events are mainly caused by human error, the probabilistic approach may not be possible because uncertainties of these events are difficult to be expressed by probabilistic distributions.

In order to reduce the computation time and quantify uncertainties of top events when basic events whose uncertainties are difficult to be expressed by probabilistic distributions exist, the fuzzy uncertainty propagation based on fuzzy set theory [3] can be applied. In this paper, we develop a fuzzy uncertainty propagation code and apply the fault tree of the core damage accident after the large loss of coolant accident (LLOCA).

## 2. Fuzzy Propagation

### *2.1 Fuzzy Set Theory*

All elements possess a binary degree of belonging to a set, that is, either an element belongs to a set or not in classical set theory. On the other hand, a fuzzy set is a set whose elements have a degree of belonging to the set where the degree called a membership function is given by a value in the interval [0, 1]. The membership function of element $x$ is expressed by $\mu(x)$ . If $x$ belongs to the fuzzy set completely, the membership function of $x$ is 1, whereas if $x$ does not belong to the fuzzy set at all, the membership function of $x$ is 0.

The alpha cut ( $\alpha$ -cut) [4] has a value in the interval [0, 1] in the same with a membership function. The set $A_\alpha$ corresponding to the alpha cut $\alpha$ is defined as

$$A_\alpha = \{x | \mu(x) \geq \alpha, x \in A\} . \quad (1)$$

A possibilistic distribution expressed by fuzzy set theory is obtained by connecting minimum values and maximum values of $A_\alpha$ corresponding to each $\alpha$ -cut and fuzzy numbers [5] which quantify uncertainties in possibilistic distributions are defined by next four conditions:

① $\mu(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$

② $\mu(x)$ is strictly increasing on $[a, b]$

③ $\mu(x)$ is strictly decreasing on $[c, d]$

④ $\mu(x) = 1$ for all $x \in [b, c]$

where $a$ , $b$ , $c$ , $d$ are called fuzzy numbers and satisfy $a \leq b \leq c \leq d$ .

The possibility measure and the necessity measure correspond to the concept of the cumulative distribution function of the probabilistic distribution. Uncertainty in the possibilistic distribution is defined by the interval between the possibility measure and the necessity measure.

### *2.2 Transformation of probabilistic distributions into possibilistic distributions*

It is necessary that occurrence probabilities of all basic events in the fault tree are described by possibilistic distributions in order to perform the fuzzy uncertainty propagation. Actually, possibilistic distributions are estimated by experts who decide fuzzy numbers. However in this paper, possibilistic distributions are obtained by transforming probabilistic distributions. There are several methods to transform probabilistic distributions into possibilistic distributions and in this paper, two methods below are used.

One of the methods is to simplify the possibilistic distribution which is triangular fuzzy numbers with $a$ , $b$ ( $b = c$ ), $d$ , positioned in correspondence of the 5% percentile, the mean value, the 95% percentile of the probabilistic distribution respectively [6]. Another method is the ratio scale transformation which is to extract the possibilistic distribution by dividing the probabilistic distribution by the greatest probability value [7].

### *2.3 Fuzzy Propagation*

The $\alpha$ -cut method [8] is one of the methods which calculate the uncertainty of the top event in fault tree analysis by means of using fuzzy theory when the uncertainties of all basic events are given. When occurrence probabilities of all basic events are expressed by possibilistic distributions, the steps of the $\alpha$ -cut method are:

① Cut the $\alpha$ -cut at regular intervals.
② Sample minimum values and maximum values in $A_\alpha$ corresponding to each $\alpha$ -cut in possibilistic distributions of all basic events.
③ Calculate occurrence probabilities of the top event using values sampled in ② and compute the possibilistic distribution of the top event.
④ Transform the possibilistic distribution into the possibility/necessity measure.

## 3. Validation

In order to verify the fuzzy uncertainty propagation code, we apply the fault tree of the radiation release accident in the thesis written by Sanjay [5]. After that, results from the fuzzy uncertainty propagation code are compared with theoretical results calculated by Sanjay. Table 1 is fuzzy numbers of the radiation release accident from the fuzzy uncertainty propagation code and the thesis written by Sanjay, respectively. From the table, the fuzzy uncertainty propagation code is well designed by observing that results from the fuzzy uncertainty propagation code agree well with theoretical results.

Table 1. Comparison of fuzzy numbers between the fuzzy uncertainty propagation code and the thesis by Sanjay

| | Thesis written by Sanjay (Ref.) | Fuzzy uncertainty propagation code |
|---|---|---|
| $a$ | 0.046 | 0.046 |
| $b$ | 0.056 | 0.056 |
| $c$ | 0.057 | 0.058 |
| $d$ | 0.069 | 0.069 |

## 4. Application Results

The fault tree of the core damage accident after the LLOCA is composed of 125 events and 35 basic events whose mean values of occurrence probabilities are given.

The probabilistic uncertainty propagation using the MC method is applied to this fault tree to obtain reference results. It is assumed that probabilistic distributions of all basic events are lognormal distributions whose standard deviations are 1% of mean values of occurrence probabilities. The process of computing the probability of the top event (core damage accident) is repeated $10^5$ times. Table 2 is the mean value and the standard deviation of the probability of the top event obtained by the probabilistic approach.

Table 2. The mean value and the standard deviation of the probability of the core damage accident

| | Core damage accident |
|---|---|
| Mean value [/yr] | $8.255\times10^{-7}$ |
| Standard deviation [/yr] | $7.577\times10^{-9}$ |

In order to perform the fuzzy uncertainty propagation, probabilistic distributions of all basic events should be transformed into possibilistic distributions. We perform the fuzzy uncertainty propagation using the $\alpha$ -cut method in three cases below.

Case 1: All probabilistic distributions of basic events are transformed into triangular fuzzy numbers.
Case 2: All probabilistic distributions of basic events are transformed into possibilistic distributions using the ratio scale transformation.
Case 3: Some probabilistic distributions of basic events (12 basic events) are transformed into triangular fuzzy numbers and the others (23 basic events) are transformed by using the ratio scale transformation.

Table 3 shows the fuzzy numbers of the occurrence probability of the core damage accident obtained by the fuzzy uncertainty propagation with the $\alpha$ -cut interval set to 0.05 from 0 to 1 in Case 1. In Case 2 and Case 3, the fuzzy uncertainty propagation is performed with the $\alpha$ -cut interval set to 0.05 from 0.05 to 1. The fuzzy number $b$ ( $b=c$ ) are $8.253\times10^{-7}$ , $8.254\times10^{-7}$ in Case 2 and Case 3, respectively.

Table 3. Fuzzy numbers of the occurrence probability of the core damage accident (Case 1)

| | Core damage accident |
|---|---|
| $a$ [/yr] | $8.014\times10^{-7}$ |
| $b$ [/yr] | $8.255\times10^{-7}$ |
| $c$ [/yr] | $8.255\times10^{-7}$ |
| $d$ [/yr] | $8.502\times10^{-7}$ |

Figure 1 compares the possibility measure and the necessity measure of the top event in each case obtained by the fuzzy uncertainty propagation with the cumulative distribution function obtained by the probabilistic uncertainty propagation. Table 4 shows that the one-sided 95% upper limit obtained by the probabilistic approach and the fuzzy approach in each case whose confidence intervals are taken from the intervals between the possibility measure and the necessity measure. The differences between the probabilities of the cumulative distribution function and the possibility/necessity measure are the smallest in Case 1 followed by Case 3 and Case 2.

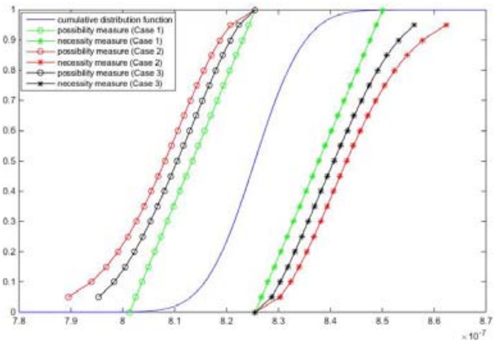


Fig. 1. Comparison of the possibility/necessity measure and the cumulative distribution function

Table 4. The one-sided 95% upper limit for the probability of the core damage accident

| | Probability [/yr] |
|---|---|
| Probabilistic approach | $8.380\times10^{-7}$ |
| Case 1 | $(8.242\times10^{-7}, 8.490\times10^{-7})$ |
| Case 2 | $(8.206\times10^{-7}, 8.626\times10^{-7})$ |
| Case 3 | $(8.222\times10^{-7}, 8.563\times10^{-7})$ |

The process to calculate the occurrence probability of the top event is performed $10^5$ times in the probabilistic uncertainty propagation, whereas 42 times (Case 1) or 40 times (Case 2 and Case 3) in the fuzzy uncertainty propagation. Therefore uncertainty quantification computed by the fuzzy uncertainty propagation is about 2400 times faster than that computed by the probabilistic uncertainty propagation.

## 5. Conclusion

The fuzzy uncertainty propagation code is implemented and tested for the fault tree of the radiation release accident. We apply this code to the fault tree of the core damage accident after the LLOCA in three cases and compare the results with those computed by the probabilistic uncertainty propagation using the MC method. The results obtained by the fuzzy uncertainty propagation can be calculated in relatively short time, covering the results obtained by the probabilistic uncertainty propagation

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