Statistical Analysis for Criticality Uncertainty in Manufacturing Random Parameters

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1. Introduction

The criticality analyses have uncertainty due to uncertainty in manufacturing parameters, nuclear data, geometric approximation, calculation method, and so on. Each uncertainty source should be evaluated and used conservatively.

The Expert Group on Uncertainty Analysis for Criticality Safety Assessment (UACSA) was organized for quantification of these uncertainties.[1,2] The PHASE II work of UACSA was focused on the uncertainty from manufacturing tolerance. [3] The benchmark problem of PHASE II assumes all manufacturing parameters change systematically, which means every fuel rods in fuel assembly has same parameters. But this assumption is a bit unrealistic. Each fuel rod can have random manufacturing parameters.

The benchmark problem of UACSA PHASE IV assumes randomness of manufacturing parameters.[4] This paper presents the results of criticality uncertainty from randomness of manufacturing parameters and compares with results of different assumption in parameter randomness.

2. Methods and Results

2.1 Uncertainty Quantification Method

The uncertainty quantification method in the Ref.[3] is based on the binomial probability formula. The probability p(N,k) that k events happened and (N-k) events did not happened when the probability of event is 95% can be written as Eq.(1).

$$p(N,k) = \frac{N!}{k!(N-k)!} 0.95^{N-k} 0.05^k \tag{1}$$

The probability p(N,k) in Eq.(1) represents the probability that *k* resulting multiplication factors of N calculations are greater than 95% upper limit. If *L*'th largest multiplication factor represents the upper limit of 95%/95% confidence level, the results except (L-1) results are within confidence interval. Thus, k in Eq.(1) is less than L. To satisfy confidence level, the probability that k in Eq.(1) is less than L should be less than 5% as written in Eq.(2).

$$P = \sum_{k=0}^{L-1} p(N,k) \le 0.05.$$
(2)

Thus, the multiplication factor greater than L'th largest value can be conservatively used as bounding value of 95%/95% confidence interval. The number L is determined by the number of calculations N, and the summation of probability in Eq.(2) for various N is shown in Fig.1.



Fig.1 The probability that multiplication factors great than L'th largest value are less than 95% upper limit

The bounding multiplication factor with large number of calculations (N) would give closer value of best estimation of 95%/95% confidence level. From the results of the benchmark problem in Ref.[3], The bounding value obtained from L=5 with 181 calculations was very close to the bounding value with higher L with more calculations up to L=10 with 311 calculations.

In this paper, due to computation burden, only 60 calculations per each case were performed and bounding value when L=1 with 60 calculations was selected.

2.2 Benchmark Problem

The benchmark problems of UACSA PHASE IV work are based on the critical experiments in the International Criticality Safety Benchmark Evaluation Project(ICEBEP) handbook.[5] Experiments of LEU-COMP-THERM-007 (LCT-007) and LEU-COMP-THERM-039 were taken for consisting benchmark problem.

This paper provides calculation results for LCT-007 case 2 which is defined by a single water-moderated array of 16x17 fuel rods in square pitch(1.6cm) arrangement. The fuel rod pitch is determined by the position of holes in the grids which are located at the top and bottom of fuel array and criticality was achieved by filling water to critical height. Overview of benchmark problem is shown in Fig.2.

The purpose of this paper with the benchmark problem is to determine the uncertainty from manufacturing parameters. It is assumed that there are uncertainties in manufacturing parameters such as fuel rod cladding inner diameter, clad thickness, fuel pellet diameter, hole positions in grids, rod positions in holes, hole diameter, height of fissile column, fuel density, fuel impurities, U234~U238 content in U, and critical height.

To check the effect in method for treating, benchmark problem provides five scenarios in Table 1.



Fig. 2 Geometric overview of the benchmark problem

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manufa	actur	ing parame	ters					
Scenar	io		Δ	B	С	D	F	٦

Scenario	А	В	С	D	E
Grid hole position	Ν	D	D	D	D
Rod position	Ν	RL	RL	RL	RL
Grid hole diameter	S	S	D	D	D
Clad inner diameter	S	S	S	D	D
Clad thickness	S	S	S	S	D
Other parameters	S	S	S	S	S

N : nominal value

- D : Different random parameters for each fuel rod
- S : Same random parameters for every fuel rod
- RL : Randomly placed to be leaned to grid

2.3 Numerical Results

The criticality calculations to determine uncertainty from manufacturing parameters were performed by KENO VI in the SCALE6.1 code[6]. The continuous energy nuclear data library based on ENDF-B/VII was used and standard deviation of each result was about 0.0004~0.0005. Fig. 3 shows the results of each 60 calculations per scenario, total 300 calculations.

The error bars in Fig.3 represent 2-sigma range. From Fig.1, the largest value among 60 calculations was selected 95/95 bounding value for each scenario and is shown in Table 2. Statistical uncertainty was not considered in Table 2.

Table 2. Bounding values for 95/95 confidence level

	k-eff	Uncertainty $(k - k_{nominal})$
Nominal	0.99330	-
A	1.00025	0.00695
В	0.99813	0.00483
С	0.99877	0.00547
D	0.99701	0.00371
Е	0.99533	0.00203



Fig.3 Criticality calculation results for scenarios A~E

From the Fig. 3 and Table 2, low uncertainty in scenario D and E was observed, comparing with others. This behavior is expected because configuration with different manufacturing parameters for different fuel rods shows less reactivity comparing by configuration with same limiting parameters for all of fuel rods.

3. Conclusions

Criticality uncertainty from manufacturing parameters is assessed with statistic method and assumption of parameter randomness via benchmark problem based on the criticality experiment. The resulting uncertainty seems to decrease as randomness increases.

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