

## Modeling of Wall Effects on Drag and Lift Acting on Bubbles in Turbulent Bubbly Pipe Flows

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### 1. Introduction

The two-fluid model based on Eulerian-Eulerian approach has been widely used for simulating two-phase flows in many industrial applications. However, the two-fluid approach needs accurate modeling for interfacial momentum exchange such as drag, shear-induced lift, and wall-induced lift. In particular, it is important to accurately model the wall effect in order to predict ‘wall peaking’ or ‘core peaking’ phenomena observed in bubbly pipe flow. Those phenomena characterized by the radial distribution of void fraction are mainly determined by the balance between shear-induced lift and wall-induced lift usually called ‘wall-lubrication force’. However, the wall effect is not fully understood yet and the wall force coefficient in previous studies has a wide range of values, probably tuned to the experimental results. Therefore, we propose a new model considering the wall effect on drag and lift forces and evaluate its accuracy by simulating turbulent bubbly flows with available data for comparison.

### 2. Numerical Method and Results

Our numerical method is based on two-fluid model with momentum exchange terms accounting for drag, shear-induced lift, wall-induced lift and turbulent dispersion force. Models for each force will be explained one by one below. Bubble-induced turbulent viscosity is also considered according to Sato et al. [3].

#### 2.1 Drag (Without Wall Effect)

The drag model proposed by Ishii and Zuber [1] is used by most researchers for a spherical bubble. Therefore, their model is adopted in this study.

$$\bar{M}_G^{drag} = -\frac{3}{4} \frac{\alpha_G}{d_b} C_D \rho_L (\bar{u}_G - \bar{u}_L) |\bar{u}_G - \bar{u}_L|$$

$$C_D = \frac{24}{Re_b} (1 + 0.1 Re_b^{0.75})$$

Here  $C_D$  is the drag coefficient and  $Re_b$  is the bubble Reynolds number.

#### 2.2 Shear-Induced Lift

When a bubble exists in unbounded shear flow, it experiences a force in the direction perpendicular to the bubble motion relative to liquid. This force is called

‘shear-induced lift’, and it is caused by asymmetric pressure distribution on the bubble surface. The shear-induced lift can be expressed as

$$\bar{M}_G^{lift, shear} = C_L \alpha_G \rho_L (\bar{u}_L - \bar{u}_G) \times (\nabla \times \bar{u}_L)$$

Here  $C_L$  is the lift factor whose value has been changed (from about 0.01 to 0.5) depending on the researcher.

Recent numerical and experimental studies showed that the direction of the lift changes if a relatively large bubble experiences deformation to some extent. The model with constant lift factor, commonly used in two-fluid models, does not take into account the change in the lift direction. On the other hand, the model proposed by Tomiyama et al. [2] allows for the change in the lift direction with respect to the bubble size. Therefore, their model is adopted in this study. In their model, the lift factor is given as a function of Reynolds number and Eotvos number.

#### 2.3 Wall Effects on Lift and Drag

To develop the present model for wall effect, we perform a series of separate simulations where a sphere is moving parallel to a nearby wall in still fluid as shown in Fig. 1. A sphere moving near a wall experiences the wall-normal repulsive force which is called ‘wall-induced lift’ in this study. The magnitude of this force depends on the distance from the wall. So, we accumulate the data of lift forces acting on the sphere for various wall distances and then make a correlation as follows.

$$\bar{M}_G^{lift, wall} = \frac{3}{4} \frac{\alpha_G}{d_b} \rho_L U_R^2 \{C'_{w1} + C'_{w2} \frac{d_b}{y_w} + C'_{w3} (\frac{d_b}{y_w})^2\} \bar{n}_w$$

Here, the coefficients  $C'_w$ , for  $Re_b = 100$  as an example, are 0.0140, -0.1037, 0.1465 in increasing order.

Meanwhile, the nearby wall can also affect the drag on the sphere. The present simulation results show that the drag force is enhanced by up to about 30% very near

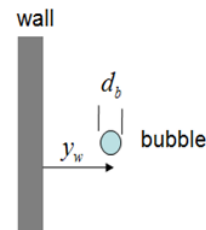


Fig. 1 Simulation of a moving sphere near a wall

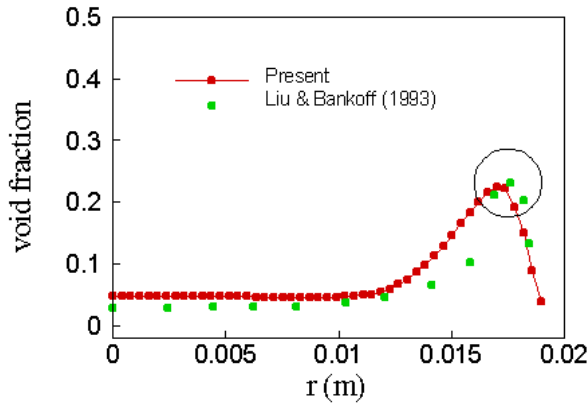


Fig. 2 Radial distribution of void fraction at exit

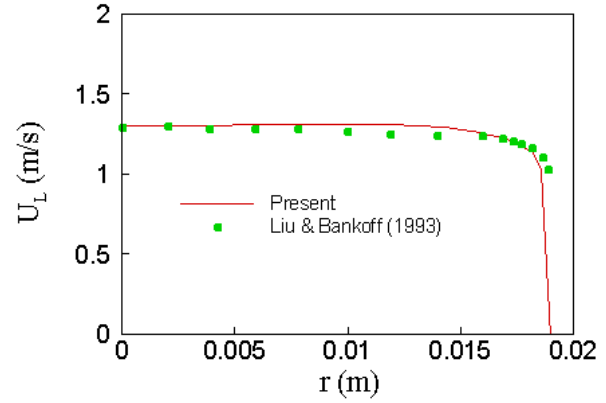


Fig. 3 Radial distribution of water velocity at exit

the wall as compared to that far away from the wall. Therefore, we use a new correlation taking into account the wall-enhanced drag.

#### 2.4 Turbulent dispersion force

To consider the turbulent bubbly flows, turbulent dispersion force is also introduced. In this study, the model proposed by Burns et al. [4] and Sato et al. [3] is used.

$$\bar{M}_G^{dispersion} = -\frac{3}{4} C_D \frac{\alpha_G}{d_b} |\bar{u}_G - \bar{u}_L| \frac{\mu_L^{urb}}{\sigma_{TD}} \left( \frac{1}{\alpha_L} + \frac{1}{\alpha_G} \right) \nabla \alpha_G$$

$$\mu_L^{urb} = \mu_L^{SIT} + \mu_L^{BIT}, \quad \mu_L^{BIT} = C_B \rho_L \alpha_G d_b |\bar{u}_G - \bar{u}_L|$$

Here, the typical model constants of  $\sigma_{TD} = 0.9$  and  $C_B = 0.6$  are used.

#### 2.5 Simulation results

In order to verify our model, we solve turbulent bubbly flow in a vertical pipe [5, 6]. The diameter and length of the pipe are 38mm and 2.8m, respectively. The bubble diameter is 3mm and the averaged void fraction is 10%. Under the assumption of 2D axisymmetric flow, developing flow is simulated. At inlet, single-phase turbulent velocity profiles (1/7-th power law) are given for both water and air, and the uniform void fraction is used. At the outlet, pressure boundary condition is prescribed.

Figs. 2 and 3, respectively, show the fully-developed void fraction and water velocity at exit as a function of radial position. The experimental results are also included for comparison. Fig. 2 includes a circle denoting the bubble touching the wall, in order to show the void peak location from the wall in terms of the bubble size. As you can see, the void fraction is almost flat in the interior, but it has a peak near the wall resulting in ‘wall-peaking’ distribution. The overall distribution of void fraction is in reasonably good agreement with the previous result. It is clear that the water velocity shows sharp variation near the wall, but insignificant one in the interior. This velocity profile also agrees well with the previous result.

### 3. Conclusions

In this study, we proposed a new model for interfacial momentum exchange for wall-bounded bubbly flow. In particular, to accurately consider the wall effects on drag and lift, separate simulations were performed for the flow around a moving sphere near the wall. The present model was verified by solving the turbulent bubbly flow in a vertical pipe and comparing the results with previous ones. The present void fraction and water velocity profiles showed good results.

### ACKNOWLEDGEMENT

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