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Development of a cutset factorization method to minimize cutset structure

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1. Introduction

In Probabilistic Safety Assessment (PSA) of nuclear power plants, (1) event trees and fault trees are modelled for the accident scenarios, (2) Minimal Cut Sets (MCSs) are calculated from the integrated fault trees, (3) MCS post-processing is performed to delete impossible MCSs and manipulate human errors, and (4) core damage frequency is finally calculated from these post-processed MCSs.

Regular fault tree solvers restructure a fault tree, convert it into a modularized one, generate modularized MCSs from the modularized fault tree, and expands modules in MCSs. These modularized MCSs should be expanded for performing MCS post-processing. It is well known that post-processed MCSs cannot be easily modularized.

2. Objective of this study

Since the final MCSs are so big and complex that it is not easy to calculate accurate top event probability (or core damage frequency) from the MCSs. In order to overcome this difficulty, a new method to efficiently detect independent modules in MCSs, and factorize MCSs by using independent modules was developed in this study.

In a PSA, top event probability p_{MCUB} or p_{REA}[1-3] of a coherent Boolean logic f is calculated from the post-processed MCSs. They are frequently overestimated compared to the exact top event probability $p(f)$ as

$$p(f) \ll p_{MCUB} \leq p_{REA} \tag{1}$$

In order to reduce or eliminate this high level of overestimation, current PSA industry converts post-processed MCSs into a Binary Decision Diagram (BDD) [3,4], and calculates accurate top event probability p_{BDD}[5] as

$$p(f) = p_{BDD} \ll p_{MCUB} \leq p_{REA} \tag{2}$$

However, since the MCS conversion into a BDD frequently fails due to complex MCSs, major MCSs that have higher probabilities are converted in a BDD and its probability p_{BDD}^{major} is calculated. Then, it is combined with the conventional probability p_{MCUB}^{minor} of minor MCSs [5].

$$p(f) \leq 1 - \left(1 - p_{BDD}^{major}\right)\left(1 - p_{MCUB}^{minor}\right) \ll p_{MCUB} \tag{3}$$

If the post-processed MCSs can be modularized, more MCSs can be converted into a BDD, and more accurate top event probability can be calculated. The size of MCSs can be drastically reduced proportionally to the number of independent modules.

The objective of this study is to develop an efficient algorithm of a MCS factorization by identifying independent modules. This development was performed for calculating more accurate top event probabilities.

3. Cutset factorization method

In order to explain a new method developed in this study, a Boolean function f is factored for the two variables x and y. If a Boolean function f is factored for the first variable x,

$$\begin{aligned} f &= x f_x + f_{\bar{x}} \\ f_x &= f(x=1) = f(x=\text{true}) \\ f_{\bar{x}} &= f(x=0) = f(x=\text{false}) \end{aligned} \tag{4}$$

Then, the sub-functions f_x and $f_{\bar{x}}$ can be further factored for the next variable y as

$$f_x = y f_{xy} + f_{x\bar{y}} \tag{5}$$

$$f_{\bar{x}} = y f_{\bar{x}y} + f_{\bar{x}\bar{y}} \tag{6}$$

When these two equations are cast into Eq. (4), it becomes

$$f = xy f_{xy} + x f_{x\bar{y}} + y f_{\bar{x}y} + f_{\bar{x}\bar{y}} \tag{7}$$

Here,

$$\begin{aligned} f_{xy} &= f(x=1, y=1) \\ f_{x\bar{y}} &= f(x=1, y=0) \\ f_{\bar{x}y} &= f(x=0, y=1) \\ f_{\bar{x}\bar{y}} &= f(x=0, y=0) \end{aligned} \tag{8}$$

If a Boolean function f has an independent module $x + y$ or xy, the conditions below should be satisfied.

$$x + y \text{ is a module iff } f_{xy} = 0, f_{x\bar{y}} = f_{\bar{x}y} = g \tag{9}$$

$$xy \text{ is a module iff } f_{xy} = g, f_{x\bar{y}} = f_{\bar{x}y} = 0 \tag{10}$$

(Proof) If $x + y$ is an independent module, a Boolean function f should be

$$f = (x + y)g + f_{\bar{x}\bar{y}} \tag{11}$$

Since Eq. (6) has to have the form in Eq. (11),

$$f_{xy} = 0, f_{x\bar{y}} = f_{\bar{x}y} = g \tag{12}$$

Similarly, if xy is an independent module, a Boolean function f should be

$$f = xyg + f_{\bar{x}\bar{y}} \tag{13}$$

Since Eqs. (6) and (13) should be identical,

$$f_{xy} = g, f_{x\bar{y}} = f_{\bar{x}y} = 0 \tag{14}$$

By testing Eq. (9) and (10), independent modules can be found, and a Boolean function of MCSs can be factored with modules.

4. Acceleration of cutset factorization

Depending on the size of MCSs, huge number of comparisons may be required to find modules by Eqs. (9) and (10). In this study, an acceleration method was developed to minimize this comparison number.

If a Boolean function f that consists of MCSs has independent modules $x + y$ or xy, the conditions below are satisfied.

$$\text{If } x + y \text{ is a module, } MIF(f,x) = MIF(f,y) \quad (15)$$

$$\text{If } xy \text{ is a module, } CIF(f,x) = CIF(f,y) \quad (16)$$

Here, $MIF(f,x)$ is a Marginal Importance Factor (Birnbaum importance measure), and $CIF(f,x)$ is a Critical Importance Factor (Fussell-Veseley importance measure) [1,2,4]. The two importance measures play key roles in PSA of nuclear power plants.

(Proof) These two importance measures for x and y are calculated with Eq. (7) as

$$\begin{aligned} MIF(f,x) &= \frac{\partial p(f)}{\partial p(x)} = p\left(y f_{xy} + f_{x\bar{y}}\right) \\ MIF(f,y) &= \frac{\partial p(f)}{\partial p(y)} = p\left(x f_{xy} + f_{\bar{x}y}\right) \end{aligned} \quad (17)$$

$$\begin{aligned} CIF(f,x) &= \frac{p(x)}{p(f)} MIF(f,x) = \frac{p(x)}{p(f)} p\left(y f_{xy} + f_{x\bar{y}}\right) \\ CIF(f,y) &= \frac{p(y)}{p(f)} MIF(f,y) = \frac{p(y)}{p(f)} p\left(x f_{xy} + f_{\bar{x}y}\right) \end{aligned} \quad (18)$$

If the conditions in Eqs. (9) and (10) for the modules of $x + y$ and xy are substituted into Eqs. (17) and (18),

$$MIF(f,x) = MIF(f,y) = p(g) \quad (19)$$

$$CIF(f,x) = CIF(f,y) = \frac{p(x)p(y)p(g)}{p(f)} \quad (20)$$

As proved in Eqs. (19) and (20), Eqs. (15) and (16) are satisfied for the modules of $x + y$ and xy.

Candidates of independent modules are selected by Eqs. (15) and (16), and independent modules among candidates are identified by Eqs. (8) and (9). In this way, by testing the conditions in Eqs. (15) and (16), the number of module candidates can be drastically reduced, and independent modules are easily identified with a little efforts.

5. Applications

In order to show the effectiveness of this study, benchmark tests were performed with MCSs that are generated from the fault trees [6]. Table 1 shows that complex MCSs can be converted into a very small structure, when they have reasonable number of independent modules.

The correctness of factorization was confirmed by the comparison of top event probabilities before and after the factorization.

6. Conclusions

This paper proposes a new method to factorize MCSs. Importance measures are employed for the acceleration of this factorization. The benchmarks tests showed the effectiveness of this algorithm.

The algorithm in this paper minimizes computational memory and quickly detects modules in MCSs. Furthermore, it can be easily implemented into industry PSA tools. It is recommended that this method be implemented into MCS analysis tools for calculating more accurate core damage frequency in PSA of nuclear power plants.

Furthermore, it is recommended that more efficient factorization method be developed.

Table 1. Factorized MCSs by modules

Problem	MCSs	Factorized MCSs	Modules	Run time (sec)
baobab1	46,188	46,188	0	0.1
baobab2	4,805	4,805	0	0.0
baobab3	24,386	15,537	6	0.8
chinese	392	11	10	0.0
das9201	14,217	60	30	0.1
das9202	27,778	1	38	3.6
das9203	16,200	5	10	0.4
das9204	16,704	6	14	0.9
das9205	17,280	1	6	0.8
das9206	19,518	400	30	0.2
das9207	25,988	1,402	52	0.2
das9208	8,060	1,374	21	0.0
edf9202	130,112	144	39	63.4
edf9205	21,308	340	25	0.3
edfpa15p	27,870	26,304	6	1.3
edfpa15r	26,549	26,350	2	1.4
elf9601	151,348	811	57	85.1
ftr10	305	37	26	0.0
isp9601	276,785	42	45	148.9
isp9603	3,434	152	31	0.0
isp9605	5,630	5,630	0	0.0
isp9606	1,776	31	22	0.0
isp9607	150,436	119	25	112.4
jbd9601	14,007	902	78	0.1

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