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# Quantitative Study of Geometrical Effect on CANDU Channel Power by Estimating Real Variance

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## 1. Introduction

A CANDU core analysis which is mainly conducted with the RFSP code is based on the assumption that the boundary of the CANDU core has a rectangular shape despite the cylindrical shape in a real core because of calculation convenience. It is based on the belief that the geometrical shape does not have an influence on the major results such as the multiplication factor and powers. However it is difficult to find quantitative studies that support this belief, and it should be justified with studies including various attempts and simulations.

In this study, the difference between the results using different core boundaries will be reported in which for the reliability and accuracy, the McCARD code will be used for the numerical simulations.

However, because the McCARD code is based on the Monte Carlo method, which is a probabilistic theory, it accompanies probabilistic errors, or a so-called variance. Although the McCARD code provides us with the diverse analyses regarding a real variance such as the Hystoy-based Batch(HB) method and Fission Source Distribution(FSD) method, the most fundamental and accurate way is to simulate a certain problem infinitely and calculate its real variance[1].

Because the difference between different boundaries can lie along the interval of the standard deviation, an accurate estimation of variance is required and thus 100 simulations are conducted in this study for channel power error estimation.

Originally, more realistic problems and larger number of problems should be solved. However, in this research, only two problems, which are a mathematical initial core problem and a practical initial core problem are solved due to a limitation of the study scope.

## 2. Methods and Problems

The Central Limit Theorem provides a basic philosophy about the Monte Carlo simulation, and the unbiased standard deviation for sampling with a theory of error propagation are used in this study. All of these three theories will be described briefly in this study. Finally, descriptions about both simulation conditions and problem definition will be presented.

### *2.1 Central Limit Theorem*

If there is a sequence of n independent and identically distributed(i.i.d) random variables each having finite values of expectation $\mu$ and variance $\sigma^2$ which is larger than zero, the distribution of the sample average of these random variables approaches the normal distribution with a mean $\mu$ and variance $\sigma^2/n$ irrespective of the shape of the original distribution depending on the size of n.

Because we have an i.i.d condition, the covariance term can be neglected and can be explained by the following equations

$$\begin{aligned}\sigma^2\left[\overline{Q}\right] &= \frac{1}{n}\sigma^2\left[Q_1+Q_2+\cdots+Q_n\right] \\ &= \frac{1}{n^2}\left\{E\left[\left(Q_1+Q_2+\cdots+Q_n\right)^2\right]-n^2\left(E\left[Q\right]^2\right)\right\} \\ &= \frac{1}{n^2}\left\{nE[Q_i^2]-n\left(E\left[Q\right]^2\right)\right\}+\frac{1}{n^2}\sum_i\sum_{j\neq i}\left\{E\left[Q_iQ_j\right]-\left(E\left[Q\right]^2\right)\right\} \\ &= \frac{1}{n}\sigma^2\left[Q\right]+\frac{1}{n^2}\sum_i\sum_{j\neq i}\left\{\mathrm{cov}\left[Q_i,Q_j\right]\right\}\end{aligned} \tag{1}$$

$$\sigma^2\left[\overline{Q}\right]=\frac{1}{n}\sigma^2\left[Q\right] \tag{2}$$

### *2.2 Biased and Unbiased Estimation of Standard Deviation*

The sample average and variance $\mu$ and $\sigma^2/n$ are unbiased and biased estimations respectively. The variance of the sample is the same as the following equation by considering the freedom to make the biased estimation unbiased.

$$\sigma^2\left[\overline{Q}\right]=\frac{1}{N-1}\sum_{i=1}^{N}\left(Q_i-\overline{Q}\right)^2;\overline{Q}=\frac{1}{N}\sum_{i=1}^{N}Q_i \tag{3}$$

In equation (3), Q means the assembly power in this study and N is the number of simulations.

### *2.3 First Order Error Propagation on Root Mean Square Error(RMSE)*

The root mean square error of two results from different problems can be calculated using equation (4), where $nc_{qc}$ is the number of channels in the quarter core, and $P_i^C$ and $P_i^R$ are the assembly fission powers of the core using a cylindrical and rectangular geometry for the i-th channel, respectively.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{nc_{qc}} \left(P_i^C - P_i^R\right)^2}{nc_{qc}}} \tag{4}$$

In addition, for a multi-variable function, the error can be estimated by the following equations.

$$\sigma^2 f = \lim_{n\to\infty} \frac{1}{n} \cdot \sum_{k=1}^{n} \left( f_k - \overline{f} \right) \tag{5}$$

Using the first order Taylor series expansion of the function and assumption about the average of the multi variable function which is the same as the following equation, the variance of the function can be calculated.

$$\overline{f} \equiv f\left(\overline{x}, \overline{y}\right) \tag{6}$$

$$f \equiv f\left(\overline{x}, \overline{y}\right) + \left(x - \overline{x}\right) \frac{\partial f}{\partial x}\bigg|_{\overline{x}} + \left(y - \overline{y}\right) \frac{\partial f}{\partial y}\bigg|_{\overline{y}} \tag{7}$$

$$\sigma^2 f = \lim_{n\to\infty} \frac{1}{n} \cdot \sum_{k=1}^{n} \left\{ f\left(\overline{x}, \overline{y}\right) + \left(x_k - \overline{x}\right) \frac{\partial f}{\partial x}\bigg|_{\overline{x}} + \left(y_k - \overline{y}\right) \frac{\partial f}{\partial y}\bigg|_{\overline{y}} - \overline{f} \right\}^2 \tag{8}$$

$$\sigma^2 f = \sigma_x^2 \left( \frac{\partial f}{\partial x}\bigg|_{\overline{x}} \right)^2 + \sigma_y^2 \left( \frac{\partial f}{\partial y}\bigg|_{\overline{y}} \right)^2 + 2 \cdot \frac{\partial f}{\partial x}\bigg|_{\overline{x}} \cdot \frac{\partial f}{\partial y}\bigg|_{\overline{y}} \cdot \mathrm{cov}(x, y) \tag{9}$$

In equation (9), it is assumed that the covariance between parameters can be neglected.

*2.4 Problem Description and Simulation Condition*

The simulated problem is the mathematical initial core and the practical initial core in the CANDU. The imaginary core is filled with fresh fuels and the practical core uses two types of fuels which are fresh fuel and depleted fuel.
For a reliable simulation for the reduced real variance, a total of 100 simulations are done for each type of problems and problems using different boundaries. The number of inactive cycles is 200 and the number of active cycles is 800. The random seed option is used to provide different sequences for each case.
The number of particles is one million, and the size of the batch is 5,000[2].

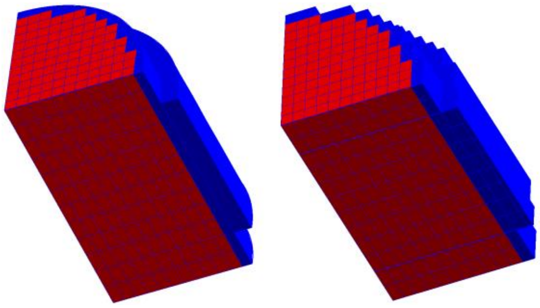

Fig. 1. Mathematical Initial Core Problem

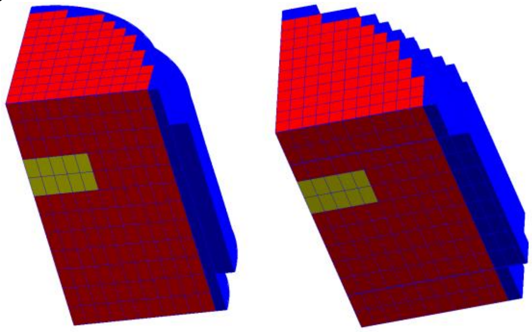

Fig. 2. Practical Initial Core Problem

In Figs. 1 and 2, two problems are described and the red, yellow and blue colors mean fresh fuel, depleted fuel, and the reflector, respectively. The directions of the x,y, and z axes are right, down, and front, and bundle position 8, 9 has depleted fuels. The cross section is depicted in Fig. 3.

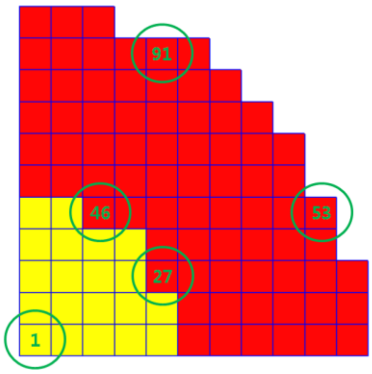


Fig. 3. Position of Depleted Fuels and Channel Index

**3. Simulation Results**

To see the difference between the two types of boundaries, the RMSE is calculated as in Table I, and we can see the boundary shape does affect the power results. This difference is because the amount of leakage is different with each other due to the geometrical shape. Namely, the shape of the neutron flux and power shape differ with each other. In addition, its amount can be measured using RMSE which is 1%, and would be more than 1% for more difficult problems. Following the first-order error propagation, the standard deviation of the RMSE is also calculated and it indicates that the RMSE result can be quite trustable.

Table I: RMSE and Its SD for Problems

| | MIC | PIC |
|---|---|---|
| RMSE(%) | 1.038 | 1.142 |
| SD(%) | 0.016 | 0.015 |

Although the main point in this study is to see the difference between results using different shapes of boundaries, the quantitative result of the real variance compared to the apparent variance and variances of other methods is also important. In Tables II through V, variances are compared with each other, and it is verified that the real variance of the channel/assembly power is much (about 5-7 time) higher than the apparent variance.

Table II: Comparison of Estimated Real Variance of Channel-wise Fission Power for MIC with Cylindrical Boundary

| Channel Index | $\sigma_{REF}$ (%) | $\sigma_{APP}$ (%) | $\sigma_{BAT}$ (%) |
|---|---|---|---|
| 1 | 0.141 | 0.021 | 2.554 |
| 27 | 0.061 | 0.018 | 2.538 |
| 46 | 0.059 | 0.018 | 2.535 |
| 53 | 0.143 | 0.033 | 2.511 |
| 91 | 0.142 | 0.032 | 2.513 |

Table III: Comparison of Estimated Real Variance of Channel-wise Fission Power for MIC with Rectangular Boundary

| Channel Index | $\sigma_{REF}$ (%) | $\sigma_{APP}$ (%) | $\sigma_{BAT}$ (%) |
|---|---|---|---|
| 1 | 0.124 | 0.021 | 2.585 |
| 27 | 0.068 | 0.017 | 2.567 |
| 46 | 0.056 | 0.017 | 2.569 |
| 53 | 0.152 | 0.033 | 2.549 |
| 91 | 0.136 | 0.032 | 2.548 |

Table IV: Comparison of Estimated Real Variance of Channel-wise Fission Power for PIC with Cylindrical Boundary

| Channel Index | $\sigma_{REF}$ (%) | $\sigma_{APP}$ (%) | $\sigma_{BAT}$ (%) |
|---|---|---|---|
| 1 | 0.159 | 0.023 | 2.526 |
| 27 | 0.069 | 0.018 | 2.509 |
| 46 | 0.070 | 0.018 | 2.509 |
| 53 | 0.146 | 0.031 | 2.494 |
| 91 | 0.156 | 0.030 | 2.490 |

Table V: Comparison of Estimated Real Variance of Channel-wise Fission Power for PIC with Rectangular Boundary

| Channel Index | $\sigma_{REF}$ (%) | $\sigma_{APP}$ (%) | $\sigma_{BAT}$ (%) |
|---|---|---|---|
| 1 | 0.141 | 0.023 | 2.642 |
| 27 | 0.073 | 0.018 | 2.622 |
| 46 | 0.065 | 0.018 | 2.622 |
| 53 | 0.133 | 0.031 | 2.604 |
| 91 | 0.132 | 0.031 | 2.600 |

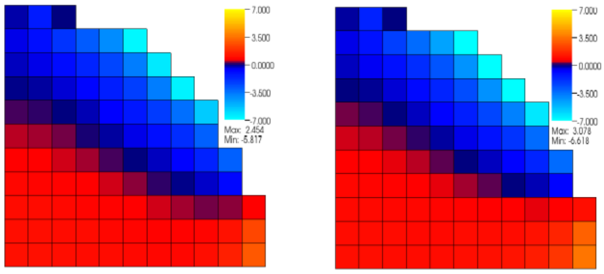

Fig. 4. Power Error Map for MIC and PIC Problems

In Fig. 4, it can be verified that using the rectangular boundary increased fission powers of channels in center region and decreased fission power of channel in outer region nearby the boundary.

## 4. Conclusions

In this study, a comparison of the results using different shapes on the boundary is done and the real variance of the channel in CANDU core is also calculated.

The results indicate that the boundary shape effect cannot be ignored and the real variance of the channel power in CANDU core is 5-7 times larger than that of the apparent variance.

This results supports the rationales for the development of finite element method (FEM) based code such as the DEFENS code [3],[4], because the FEM does not have limitations about the geometry. Thus, the accuracy enhancement originating from its inherent characteristics can be achieved using the FEM.

Although, the covariance between real and apparent variance for channels, pin, and core level are not calculated in this study, it is necessary to obtain those values from a probabilistic point of view.

Finally, the use of the FEM is justified because of its geometrical freedom and the bonus accuracy increase is shown to be acquired by simply using the FEM, which are presumed to be about 1% of the power results.

## 5. Acknowledgement

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