

## A Derivation of Source-based Kinetics Equation with Time Dependent Fission Kernel for Reactor Transient Analyses

Song Hyun KIM<sup>a,\*</sup>, Myeong Hyun Woo<sup>a</sup>, Chang Ho SHIN<sup>a</sup>, and Cheol Ho Pyeon<sup>b</sup>

<sup>a</sup>Department of Nuclear Engineering, Hanyang University, 222 Wangsimni-ro, Seoungdong-gu, Seoul 133-791, Korea

<sup>b</sup>Nuclear Engineering Science Division, Research Reactor Institute, Kyoto University, Asashiro-nishi, Kumatori-cho, Sennan-gun, Osaka 590-0494, Japan

\*Corresponding author: nucleon@nural.hanyang.ac.kr

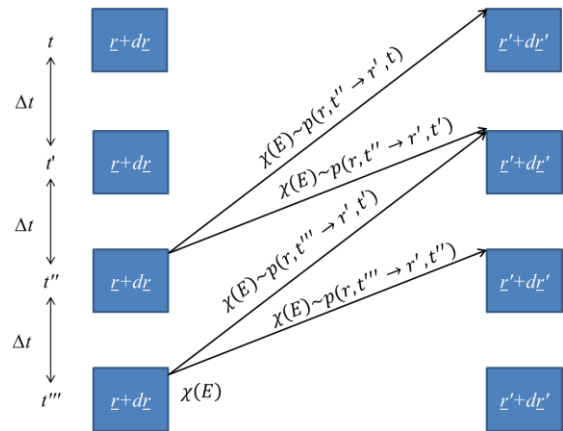
### 1. Introduction

Analyses on reactor kinetics have been pursued to solve lots of reactor transient problems. At the beginning of the kinetics theory, to analyze transient phenomenon of total reactor characteristics, point kinetics model was introduced and has been utilized. However, the point kinetics method cannot analyze local fluctuation of the reactor characteristics. To analyze the partial variations of reactor characteristics, two representative methods were introduced in previous studies; (1) quasi-statics method [1-3] and (2) multipoint technique [4-6]. The main idea of quasi-statics method is to use a low-order approximation for large integration times. To realize the quasi-statics method, first, time dependent flux is separated into the shape and amplitude functions, and shape function is calculated. It is noted that the method has a good accuracy; however, it can be expensive as a calculation cost aspect because the shape function should be fully recalculated to obtain accurate results. To improve the calculation efficiency, multipoint method was proposed. The multipoint method is based on the classic kinetics equation with using Green's function to analyze the flight probability from region  $r'$  to  $r$ . Those previous methods have been used to analyze the reactor kinetics analysis; however, the previous methods can have some limitations. First, three group variables ( $r_g, E_g, t_g$ ) should be considered to solve the time dependent balance equation. This leads a big limitation to apply large system problem with good accuracy. Second, the energy group neutrons should be used to analyze reactor kinetics problems. In time dependent problem, neutron energy distribution can be changed at different time. It can affect the change of the group cross section; therefore, it can lead the accuracy problem. Third, the neutrons in a space-time region continually affect the other space-time regions; however, it is not properly considered in the previous method. In this study, a new balance equation to overcome the problems generated by the previous methods is proposed using source-based balance equation. And then, a simple problem is analyzed with the proposed method.

### 2. Method and Result

#### 2.1 Derivation of Source-based Kinetics Equation

The key idea of proposed method is to use a property which is the time independency of the source energy distribution. As shown in Fig. 1, the source generated at location  $r+d_r$  and time  $t''$  will contribute the fission reactions at location  $r'+d_{r'}$  in the other times with probability  $p(r, t'' \rightarrow r', t')$ . If a source has a specific energy distribution at location  $r$ , the neutron sources will have same probability to contribute fission reaction in the other space-time regions. Interestingly, the energy distributions of neutron sources are only depend on the birth histories which are defined to fission source, delayed source, and external source. In this study, the energy distribution property of the sources is used to remove the energy variables.



$$p(r, t'' \rightarrow r', t) = p(r, t''' \rightarrow r', t')$$

$$p(r, t'' \rightarrow r', t') = p(r, t''' \rightarrow r', t'')$$

Fig. 1. Main Principle of Proposed Method

The source balance equation for  $(r', E', t')$  with having isotropic angular distribution can be written to Eq. (1).

$$S_T(r', E', t') = S_f(r', E', t') + S_d(r', E', t') + S_e(r', E', t') \quad (1)$$

where  $S_T(r', E', t')$  is total source generation rate per unit time at location  $r'$ , energy  $E'$ , and time  $t'$ ; and  $S_f(r', E', t')$ ,  $S_d(r', E', t')$ , and  $S_e(r', E', t')$  are fission source, delayed source, and external source generation rates, respectively. The fission neutron sources at  $(r', E', t')$  are generated from fission reactions with the neutrons

produced in previous sources for times  $t'' < t'$ . Therefore, the fission source generation rate can be expressed with following equation:

$$s_f(\underline{r}', E', t') = (1-\beta) \int_0^{r'} \int_0^{E_{max}} dE'' dr'' dt'' s_f(r'', E'', t'') f(r'', E'', t'' \rightarrow r', E', t') \quad (2)$$

where  $\beta$  is delayed fraction,  $V_T$  is a total region of the calculation system,  $E_{max}$  is a maximum energy of the sources, and  $f(r'', E'', t'' \rightarrow r', E', t')$  a time dependent fission kernel which is defined as a probability that a neutron sources at  $(r'', E'', t'')$  will generate new fission neutrons having  $E'$  at  $r'$  and  $t'$ . The  $S_T$  is a time dependent source; thus, the energy distribution can be changed as different times. However, if the total source  $S_T$  is separated to the source birth histories  $S_f$ ,  $S_d$  and  $S_e$ , each source has a specific energy distribution without considering time. Hence, to simplify energy dependent terms, the fission source is rewritten to Eq. (3) as each birth history.

$$\begin{aligned} s_f(\underline{r}', E', t') = & (1-\beta) \int_0^{r'} \int_0^{E_{max}} dE'' dr'' dt'' \chi_a(E'') s_f(r'', E'', t'') f_j(r'', E'', t'' \rightarrow r', E', t') \\ & + (1-\beta) \sum_{m=0}^M \int_0^{r'} \int_0^{E_{max}} dE'' dr'' dt'' \chi_{d,m}(E'') s_{d,m}(r'', E'', t'') f_{d,m}(r'', E'', t'' \rightarrow r', E', t') \\ & + (1-\beta) \int_0^{r'} \int_0^{E_{max}} dE'' dr'' dt'' \chi_e(E'') s_e(r'', E'', t'') f_e(r'', E'', t'' \rightarrow r', E', t') \end{aligned} \quad (3)$$

where  $\chi_a(E'')$  is source energy spectrum for 'a' birth history,  $m$  is delayed neutron type,  $M$  is the total number of delay neutron types, and  $f_a(r'', E'', t'' \rightarrow r', E', t')$  is the time dependent fission kernel for the source having 'a' birth history.

Also, the delayed neutrons can be expressed to Eq. (4) using conventional kinetic equation [7].

$$s_{d,m}(\underline{r}', E', t') = \chi_m(E') \lambda_m c_m(\underline{r}', t') \quad (4)$$

where  $\lambda_m$  is the decay constant of  $m$  delayed neutron precursor, and  $c_m(\underline{r}', t')$  is the concentration of  $m$  delayed neutron precursor. Eqs. (3) and (4) is substituted to Eq. (1), and then, following equation is derived:

$$\begin{aligned} s_f(\underline{r}', E', t') = & (1-\beta) \int_0^{r'} \int_0^{E_{max}} dE'' dr'' dt'' s_f(r'', E'', t'') \int_0^{E_{max}} dE' \chi_a(E') f_j(r'', E'', t'' \rightarrow r', E', t') \\ & + (1-\beta) \sum_{m=0}^M \int_0^{r'} \int_0^{E_{max}} dE'' dr'' dt'' s_{d,m}(r'', E'', t'') \int_0^{E_{max}} dE' \chi_{d,m}(E') f_{d,m}(r'', E'', t'' \rightarrow r', E', t') \\ & + (1-\beta) \int_0^{r'} \int_0^{E_{max}} dE'' dr'' dt'' s_e(r'', E'', t'') \int_0^{E_{max}} dE' \chi_e(E') f_e(r'', E'', t'' \rightarrow r', E', t') \\ & + \sum_{m=0}^M \chi_m(E') \lambda_m c_m(\underline{r}', t') \\ & + s_e(\underline{r}', E', t') \end{aligned} \quad (5)$$

$dE'$  is multiplied into both sides of Eq. (5), and it is integrated over all energy boundaries ( $0 < E' < E_{max}$ ). Then, the energy independent source balance equation is given as shown in Eq. (6).

$$\begin{aligned} s_f(\underline{r}', t') = & (1-\beta) \int_0^{r'} \int_0^{V_T} dr'' dt'' s_f(r'', t'') f_j(r'', t'' \rightarrow r', t') \\ & + (1-\beta) \sum_{m=0}^M \int_0^{r'} \int_0^{V_T} dr'' dt'' s_{d,m}(r'', t'') f_{d,m}(r'', t'' \rightarrow r', t') \\ & + (1-\beta) \int_0^{r'} \int_0^{V_T} dr'' dt'' s_e(r'', t'') f_e(r'', t'' \rightarrow r', t') \\ & + \sum_{m=0}^M \lambda_m c_m(\underline{r}', t') \\ & + s_e(\underline{r}', t') \end{aligned} \quad (6)$$

$$\begin{aligned} \text{where } s_a(\underline{r}', t') \equiv & \int_0^{E_{max}} dE' s_a(\underline{r}', E', t') \\ f_a(r'', t'' \rightarrow r', t') \equiv & \int_0^{E_{max}} \int_0^{E_{max}} dE'' dE' \chi_a(E'') f_a(r'', E'', t'' \rightarrow r', E', t') \\ c_{d,m}(\underline{r}', t') \equiv & \int_0^{E_{max}} dE' \chi_{d,m}(E') c_{d,m}(\underline{r}', t') \end{aligned}$$

If a neutron generated at  $r''$  and  $t''$  makes a fission neutrons at  $r'$ , the neutron generation time from  $r''$  to  $r'$  will have time-dependent distributions with the source energies. As referred in Eq. (6), the time dependent fission kernel was separated as the birth history; therefore, the sources for each birth history has a specific energy distribution  $\chi_a(E)$ . Therefore, the source having same birth history will have a specific time distribution at any  $t''$  because it always has a same energy distribution. This means that the time dependent fission kernel with the birth history is energy independent and it can be separated to shape and amplitude functions. From here, the time dependent fission kernel is expressed to Eq. (7).

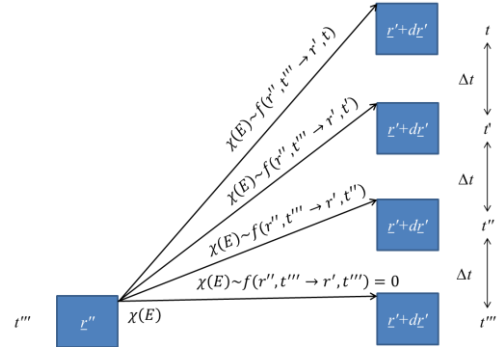


Fig. 2. Distribution Property of Time Dependent Fission Kernel

$$f_a(r'', t'' \rightarrow r', t') \equiv T_a(t'' \rightarrow t' | r'' \rightarrow r') f'_a(r'' \rightarrow r') \quad (7)$$

where  $f'_a(r'' \rightarrow r')$  is the amplitude function of the time dependent fission kernel, and  $T_a(t'' \rightarrow t' | r'' \rightarrow r')$  a conditional distribution function (time shape function) which is defined as a time probability distribution from  $t''$  to  $t'$  for birth history 'a' when a neutron generated from  $r''$  makes fission neutron at  $r'$ . Substituting Eq. (7) into Eq. (6), following equation is derived.

$$\begin{aligned}
 s_T(\underline{r}', t') &= (1-\beta) \int_{V_f} dr'' f'_{f_j}(r'' \rightarrow r') \int_0^{t'} dt'' s_f(r'', t'') T_f(t'' \rightarrow t' | r'' \rightarrow r') \\
 &+ (1-\beta) \sum_m^M \int_{V_f} dr'' f'_{d,m}(r'' \rightarrow r') \int_0^{t'} dt'' s_{d,m}(r'', t'') T_{d,m}(t'' \rightarrow t' | r'' \rightarrow r') \quad (8) \\
 &+ (1-\beta) \int_{V_f} dr'' f'_{e'}(r'' \rightarrow r') \int_0^{t'} dt'' s_e(r'', t'') T_e(t'' \rightarrow t' | r'' \rightarrow r') \\
 &+ \sum_m^M \lambda_m c_m(\underline{r}', t') \\
 &+ s_e(\underline{r}', t')
 \end{aligned}$$

Also, Eq. (8) can be expressed to Eq. (9) for each source type.

$$s_T(\underline{r}', t') = s_f(\underline{r}', t') + s_d(\underline{r}', t') + s_e(\underline{r}', t') \quad (9-1)$$

$$s_f(\underline{r}', t') = (1-\beta) \int_{V_f} dr'' f'_{f_j}(r'' \rightarrow r') \int_0^{t'} dt'' s_f(r'', t'') T_f(t'' \rightarrow t' | r'' \rightarrow r') \quad (9-2)$$

$$+ (1-\beta) \sum_m^M \int_{V_f} dr'' f'_{d,m}(r'' \rightarrow r') \int_0^{t'} dt'' s_{d,m}(r'', t'') T_{d,m}(t'' \rightarrow t' | r'' \rightarrow r')$$

$$+ (1-\beta) \int_{V_f} dr'' f'_{e'}(r'' \rightarrow r') \int_0^{t'} dt'' s_e(r'', t'') T_e(t'' \rightarrow t' | r'' \rightarrow r')$$

$$s_d(\underline{r}', t') = \sum_m^M \lambda_m c_m(\underline{r}', t') \quad (9-3)$$

$$s_e(\underline{r}', t') = s_e(\underline{r}', t') \quad (9-4)$$

To calculate the concentration rate of the delay neutron precursor, the precursor balance equation with the time dependent fission kernels is used as given in Eq. (10).

$$\frac{c_m(\underline{r}', t')}{dt'} = \beta_m \int_{V_f} dr'' f'_{f_j}(r'' \rightarrow r') \int_0^{t'} dt'' s_f(r'', t'') T_f(t'' \rightarrow t' | r'' \rightarrow r') \quad (10)$$

$$+ \beta_m \sum_m^M \int_{V_f} dr'' f'_{d,m}(r'' \rightarrow r') \int_0^{t'} dt'' s_{d,m}(r'', t'') T_{d,m}(t'' \rightarrow t' | r'' \rightarrow r')$$

$$+ \beta_m \int_{V_f} dr'' f'_{e'}(r'' \rightarrow r') \int_0^{t'} dt'' s_e(r'', t'') T_e(t'' \rightarrow t' | r'' \rightarrow r')$$

$$- \lambda_m c_m(\underline{r}', t')$$

To simplify  $T_a(t'' \rightarrow t' | r'' \rightarrow r')$ , some test calculations were pursued using MCNPX code with Tn tally option [8]. In these test calculations, a specific feature of  $T_a(t'' \rightarrow t' | r'' \rightarrow r')$  was found that the distribution shape follows exponential distribution. Thus, it can be approximately expressed to following equation:

$$T_a(t'' \rightarrow t' | \underline{r}'' \rightarrow \underline{r}') \approx \begin{cases} \theta_{a,r'' \rightarrow r'} e^{-\theta_{a,r'' \rightarrow r'}(t'-t'')} & \underline{r}'' \rightarrow \underline{r}', \\ 0 & \text{otherwise.} \end{cases} \quad (11)$$

where  $\theta_{a,r'' \rightarrow r'}$  is a time shape parameter for birth history 'a'.

The source balance equation given in Eq. (9) can only solve the source distribution in a system. To get responses, adjoint flux relationship is utilized as given in Eq. (12).

$$R = \langle \langle \psi^+, s \rangle \rangle = \langle \langle \psi, s^+ \rangle \rangle \quad (12)$$

where  $R$  is response,  $\langle \langle \rangle \rangle$  is integration operator for phase-space,  $\psi^+$  is the adjoint flux, and  $s^+$  is the adjoint source. Conveniently, we already know the source energy spectrum for each birth history-based source. Therefore, Eq. (12) can be rewritten to Eq. (13).

$$\begin{aligned}
 R(r, E, t) &= \int_{V_f} \int_{E_{min}}^t dt' dE' dr' \psi^+(r', E', t') \cdot s_T(r', E', t') \\
 &= \int_{V_f} \int_{E_{min}}^t dt' dE' dr' \psi^+(r', E', t') \\
 &\quad \cdot \{ \chi_f(E') \cdot s_f(r', t') + \sum_m^M \chi_{d,m}(E') \cdot s_{d,m}(r', t') + \chi_e(E') \cdot s_e(r', t') \} \\
 &= \sum_a \int_{V_f} \int_0^t dt' dr' \psi_a^+(r', t') s_a(r', t')
 \end{aligned} \quad (13)$$

$$\text{where } \psi_a^+(r', t') \equiv \int_{E_{min}} dE' \psi^+(r', E', t') \chi_a(E')$$

It is really difficult to directly solve Eqs. (9) and (10). In this study, the numerical integration method is used to solve the equations. For a short time interval  $\Delta t$  with discrete region  $\underline{r}'_i$  and time group  $t'_g$ , the number of sources can be approximately calculated with following equation:

$$S_T(\underline{r}_j, t_g) \Delta t \Delta V \approx S_f(\underline{r}_j, t_g) \Delta t \Delta V + S_d(\underline{r}_j, t_g) \Delta t \Delta V + S_e(\underline{r}_j, t_g) \Delta t \Delta V \quad (14)$$

$$\text{where } S_a(\underline{r}_j, t_g) = \int_{V_f, t_g - \Delta t/2}^{t_g + \Delta t/2} dt' dr' s_a(\underline{r}', t') / \int_{V_f, t_g - \Delta t/2}^{t_g + \Delta t/2} dt' dr'$$

As a same manner, Eqs. (9-2), (9-3), and (9-4) with discrete region  $\underline{r}'_i$  and time group  $t'_g$  are respectively expressed to following equations:

$$S_f(\underline{r}_j, t_g) \approx (1-\beta) \sum_{r_i}^{r_{N_i}} F_f(r_i, r_j) \sum_{k=1}^{g-1} S_f(r_i, t_{g-k}) T_f(t_{g-k} \rightarrow t_g | r_i \rightarrow r_j) \quad (15)$$

$$+ (1-\beta) \sum_m^M \sum_{r_i}^{r_{N_i}} F_{d,m}(r_i, r_j) \sum_{k=0}^{g-1} S_{d,m}(r_i, t_{g-k}) T_{d,m}(t_{g-k} \rightarrow t_g | r_i \rightarrow r_j)$$

$$+ (1-\beta) \sum_{r_i}^{r_{N_i}} F_e(r_i, r_j) \sum_{k=1}^{g-1} S_e(r_i, t_{g-k}) T_e(t_{g-k} \rightarrow t_g | r_i \rightarrow r_j)$$

$$S_d(\underline{r}_j, t_g) \approx \sum_m^M \lambda_m C_m(\underline{r}_j, t_g) \quad (16)$$

$$S_e(\underline{r}_j, t_g) \approx S_e(\underline{r}_j, t_g) \quad (17)$$

where  $F_f(r_i, r_j)$  is a fission matrix used in conventional power iteration method [9]. Also, the precursor balance equation given in Eq. (10) can be expressed to follows:

$$\begin{aligned}
 C_m(r_i, t_{g+1}) &\approx C_m(\underline{r}_j, t_g) [1 - \lambda_m \Delta t] \\
 &+ \beta_m \sum_{r_j}^{r_{N_j}} F_f(r_i, r_j) \sum_{k=1}^{g-1} S_f(r_i, t_{g-k}) T_f(t_{g-k} \rightarrow t_g | r_i \rightarrow r_j) \quad (18)
 \end{aligned}$$

$$+ \beta_m \sum_m^M \sum_{r_j}^{r_{N_j}} F_{d,m}(r_i, r_j) \sum_{k=0}^{g-1} S_{d,m}(r_i, t_{g-k}) T_{d,m}(t_{g-k} \rightarrow t_g | r_i \rightarrow r_j)$$

$$+ \beta_m \sum_{r_j}^{r_{N_j}} F_e(r_i, r_j) \sum_{k=1}^{g-1} S_e(r_i, t_{g-k}) T_e(t_{g-k} \rightarrow t_g | r_i \rightarrow r_j)$$

In Eqs. (15) and (18), the time shape matrix  $T_a(t_{g-k} \rightarrow t_g | r_i \rightarrow r_j)$  is defined to Eq. (19).

$$\begin{aligned}
 T_a(t_{g-k} \rightarrow t_g | r_i \rightarrow r_j) &= \begin{cases} \int_{t_i - t_{i-1} - \Delta t}^{t_i - t_{i-1} + \Delta t} \theta_a[r_i, r_j] e^{-\theta_a[r_i, r_j] t} dt = e^{-\theta_a[r_i, r_j] t_{g-k}} - e^{-\theta_a[r_i, r_j] t_g} & \text{for } r_i \rightarrow r_j, \\ 0 & \text{otherwise.} \end{cases} \quad (19)
 \end{aligned}$$

where  $t$  is a time domain based on  $t_0=t_{g,k}+\Delta t/2$  and  $\theta_a[r_i,r_j]$  is a time shape parameter matrix which is a matrix type expression of  $\theta_{a,r''\rightarrow r'}$ .

For the perturbed cases, which are occurred by sudden changes of the reactor parameters (i.e. control rod insertion or remove of external source), the perturbed external source  $S_e'(r_j,t_g)$  and fission matrix  $F_a'(r_i,r_j)$  can be written as follows:

$$S_e'(r_j,t_g) \approx S_e(r_j,t_g) + \Delta S_e(r_j,t_g) \quad (20)$$

$$F_a'(r_i,r_j) \approx F_a(r_i,r_j) + \Delta F_a(r_i,r_j) \quad (21)$$

Applying Eq. (20) and (21) into Eqs. (15) - (18) in a specific time, the transient analysis can be performed.

## 2.2 Results of Simple Test Problem

To verify the reconstruction ability of the kinetics phenomena of the proposed method, a benchmark problem was assumed as shown in Fig. 3. Using reflective boundary condition, an infinite arrangement was used. The details of the benchmark problem are given in Table I (The benchmark problem is a system having  $k_{eff} \sim 0.903$ ). To generate fission matrix, the fuel region was divided into 20 sub-regions having a unit volume  $V_m$ . And then, the fission matrix  $F_a(r_i,r_j)$  as the source birth histories and time shape parameter matrix  $\theta_a[r_i,r_j]$  were produced using MCNPX 2.7.0 code with Fmn and Tn cards [8]. In the calculations, the watt fission energy spectrum of U-235 thermal fission was used. Also, 0.396 MeV (average energy of delayed neutrons [10]) and 3 MeV were used for the delayed neutrons and external source, respectively. In these calculations, ENDF-VI cross section library was used for the estimations.

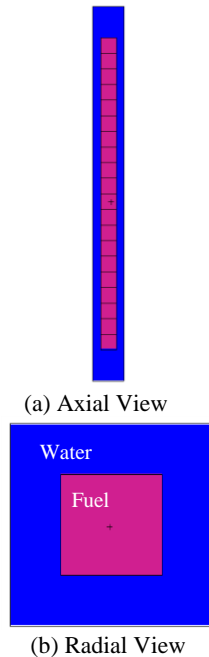


Fig. 3. Axial and Radial Views of the Benchmark Problem

Table I: Details of the Benchmark Problem

| Group | Classification | Value   |
|-------|----------------|---|
| Fuel  | Geometry       | Parallelepipedon<br>(5 cm x 5 cm x 100 cm)        |
|       | Density        | 10.96 g/cm <sup>3</sup>                           |
|       | Mass Fraction  | O-16: 0.11852<br>U-235: 0.01523<br>U-238: 0.86625 |
| Water | Geometry       | Parallelepipedon<br>(10 cm x 10 cm x 120 cm)      |
|       | Density        | 1 g/cm <sup>3</sup>                               |
|       | Atom Fraction  | H-1: 0.66667<br>O-16: 0.33333                     |

For the kinetics analyses with the proposed method, the decay constants and delayed fractions given in Table II were used that was estimated in previous study [11]. The external sources were assumed that they are uniformly distributed in the fuel region having  $2 \times 10^7$  #/sec total strength. The time step  $\Delta t$  was chosen to 0.00005 sec. The total transient time, which is the time range to estimate it in this study, is 0.5 sec. With the given information, following transient calculations were pursued with the C++ program language;

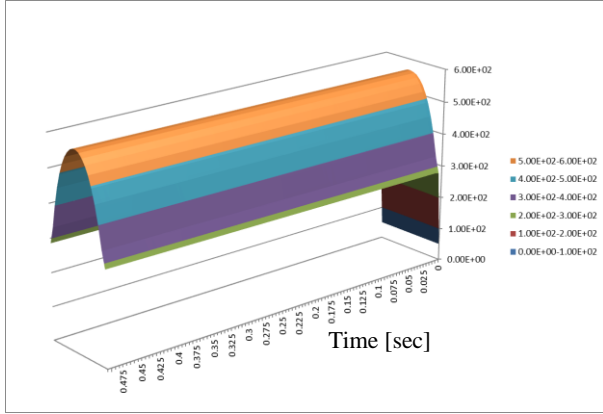
- Case I) Time-dependent source distributions in the given condition;
- Case II) Time-dependent source distributions in a super-criticality condition by applying  $\Delta F_a(r_i,r_j) = 0.11 F_a(r_i,r_j)$ ;
- Case III) Time-dependent source distributions of Case II condition with sudden removal of the external sources  $\Delta S_e(r_j,t_g) = -S_e(r_j,t_g)$  at the half of the total transient time.

Table II: Delay Neutron Data for U-235 Thermal Fission [11]

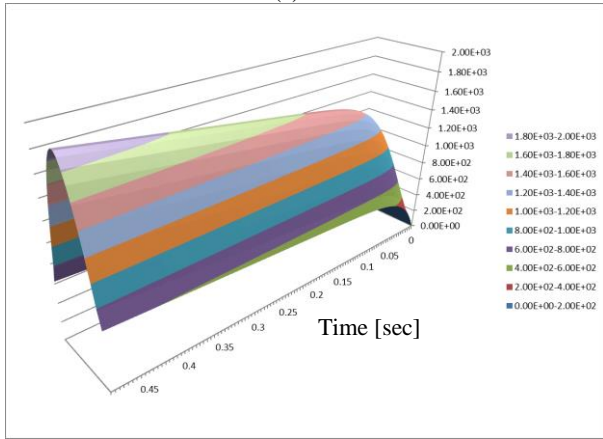
| Group | Decay Constant, $\lambda_k$ (s <sup>-1</sup> ) | Delayed Yield, $\nu_{dk}$ (n/fiss.) | Delayed Fraction, $\beta_k$ |
|-------|--|-------------------------------------|-----------------------------|
| 1     | 0.01334  | 0.000585                            | 0.000240                    |
| 2     | 0.03274  | 0.003018                            | 0.001238                    |
| 3     | 0.1208   | 0.002881                            | 0.001182                    |
| 4     | 0.3028   | 0.006459                            | 0.002651                    |
| 5     | 0.8495   | 0.002648                            | 0.001087                    |
| 6     | 2.853  | 0.001109                            | 0.000455                    |
| Total | -  | 0.016700                            | 0.006854                    |

Fig. 4 shows the results of the total source distributions  $S(r_j,t_g)$  for each transient case. For the Case I which has the sub-criticality condition, the result shows that the number of the total sources is slightly increased due to the increase of the number of delay neutrons with external source. However, it seems to have a stable value during the transient time whereas the total sources for Cases II and III are significantly increased. Analysis shows that the increases for Cases II and III are due to the super-criticality conditions of their systems. Also, at  $t=0.25$  sec in Case III, the number of total sources are gradually decreased until  $t=0.42$  sec; however, the tendency changed to be increased from

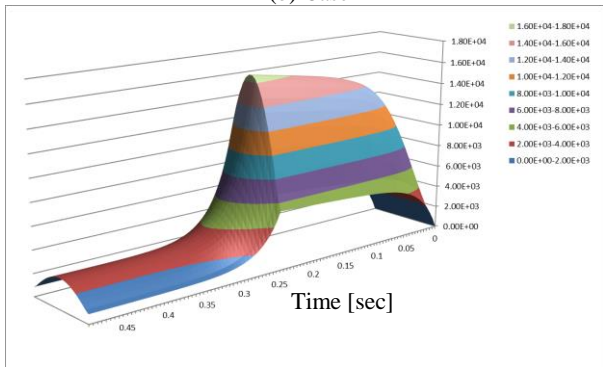
$t=0.425$  sec. The reason was analyzed that the system is a super-criticality condition; therefore, the number of sources at a specific time keeps to be increased although the external sources were removed.



(a) Case I



(b) Case II

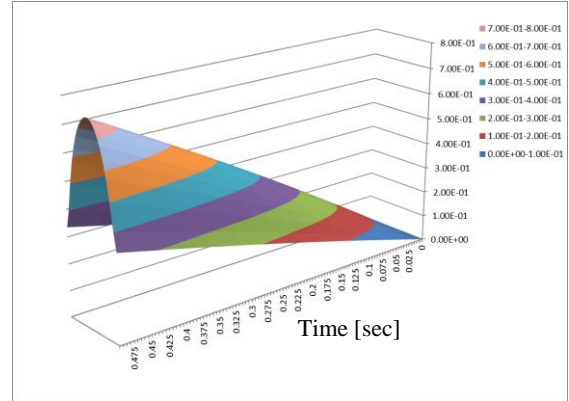


(c) Case III

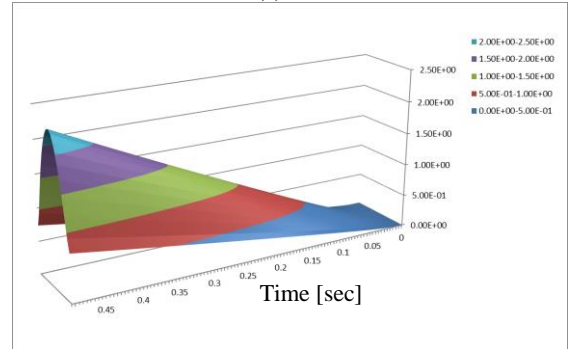
Fig. 4. Results on the Number of Total Sources  $S_T(r_j, t_g)\Delta t\Delta V$  in Each Region and Unit Time  $\Delta t$

Fig. 5 shows the results of the sum of the delayed neutrons for each time step and region. As shown in Fig. 5 (a), the delayed neutrons are getting increased until the total transient time; however, the increase rate during the time is small. It is analyzed that the system has a sub-criticality condition; therefore, the delayed neutrons are slightly increased by the non-propagated neutron sources. For Case III, the external sources were removed at the half of the transient time. As the results,

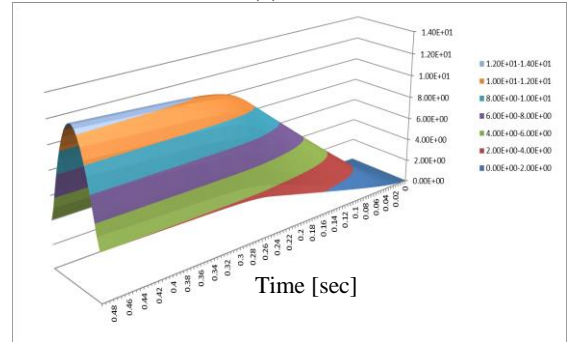
the increase rate of the delayed neutrons is decreased at that time. However, it shows that the number of delayed neutrons is getting increased even though the external source was removed. The analysis shows that it is because the system of Case III has the super-criticality condition.



(a) Case I



(b) Case II



(c) Case III

Fig. 5. Results on the Number of Delayed Sources  $S_d(r_j, t_g)\Delta t\Delta V$  in Each Region and Unit Time  $\Delta t$

From the results given in Figs. 4 and 5, it shows that the proposed method can apply to effectively solve the transient problems without considering the energy terms used in the other kinetics methods.

### 3. Conclusions and Future Work

In this study, a source-based balance equation with the time dependent fission kernel was derived to simplify the kinetics equation. First, using birth history

of the neutron sources, the energy independent kinetic equation was derived. Also, to realize the proposed method, numerical approximation was applied. Using the proposed method, the possibility to analyze the kinetics was verified by solving a simple problem. To get the parameters used in the proposed method, MCNPX code was utilized, and then the kinetics analyses were pursued by using C++ program language. The analysis results showed that the proposed method can effectively solve the kinetics phenomena which are notified in those evaluated by the other methods. The source history-based kinematic method can easily solve the kinetics problem without considering energy variable; therefore, it will contribute to increase the calculation efficiency on the analysis of the time-dependent problems. As a future work, the proposed method will be verified with the realistic time dependent problems.

### **Acknowledgement**

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (No.2012M2A8A2025679) and Innovative Technology Center for Radiation Safety (iTRS).

### **REFERENCES**

- [1] K. O. Ott and D. A. Meneley, Accuracy of the Quasistatic Treatment of Spatial Reactor Kinetics, Nuclear Science and Engineering, Vol. 36, p. 402, 1969.
- [2] J. Devooght and E. Mund, Generalized Quasi-Static Method for Nuclear Reactor Space-Time Kinetics, Nuclear Science and Engineering, Vol. 76, p. 10, 1980.
- [3] M. Dahmani, A. M. Baudron, J. J. Lautard, and L. Erradi, A 3D Nodal Mixed Dual Method for Nuclear Reactor Kinetics with Improved Quasistatic Model and a Semi-implicit Scheme to Solve the Precursor Equation, Annals of Nuclear Energy, Vol. 28, p. 805, 2001.
- [4] Keisuke Kobayashi, Rigorous Derivation of Multi-Point Reactor Kinetics Equations with Explicit Dependence on Perturbation, Journal of Nuclear Science and Technology, Vol. 29(2), p. 110, 1992.
- [5] Y. Nagaya and K. Kobayashi, Solution of 1-D Multigroup Time-Dependent Diffusion Equations Using the Coupled Reactor Theory, Annals of Nuclear Energy, Vol. 22, p. 421, 1995.
- [6] P. Ravetto, M. M. Rostagno, G. Bianchini, M. Carta, and A. D'Angelo, Application of the Multipoint Method to the Kinetics of Accelerator-Driven Systems, Nuclear Science and Engineering, Vol. 148, p. 79, 2004.
- [7] W. M. Stacey JR, Space-time Nuclear Reactor Kinetics, Academic Press, New York and London, 1969.
- [8] D.B Pelowitz, editor, MCNPX<sup>TM</sup> User's Manual, Version 2.7.0, LA-CP -11-00438, Los Alamos National Laboratory, 2011.
- [9] Song Hyun Kim, Myeong Hyun Woo, Chang Ho Shin, Hyun Chul Lee, and Jea Man Noh, A New Strategy on the Monte Carlo Eigenvalue Estimation Method Based on the Fission Matrix Using Fine-Coarse Mesh Division, Transactions of the Korean Nuclear Society Spring Meeting Jeju, Korea, May 7-8, 2015.
- [10] D. J. Hughes, J. Dabbs, A. Chan, and D. Hall, Delayed Neutrons from Fission of <sup>235</sup>U, Physics Review, Vol. 73, p. 111, 1948.
- [11] P. F. Rose, ENDF/B-VI Summary Documentation, Cross Section Evaluation Working Group, Report BNL-NCS-17541 (ENDF-201), 1991.