An Implicit Numerical Method for the Simulation of Two-phase Flow

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1. Introduction

An implicit numerical method is presented for the analysis of two-phase flows in PWRs. Numerical stability and efficiency are improved by decoupling energy equations from the pressure equation. All the convection and diffusion terms are calculated implicitly. Thus time step size can be larger than the Courant limit. The proposed numerical method is verified against conceptual two-phase flow problems.

2. Numerical Method

A two-fluid single pressure model has been adopted in this study for the simulation of transient two-phase flows where the two fluids are gas and liquid. The governing equations are discretized using the finite volume method. The computational cells can be unstructured allowing application to complicated geometries [1].

2.1 Coupled Method (PME-explicit)

As a first step of the solution scheme, phasic momentum equations are calculated explicitly.

$$\vec{u}^{*}_{k,i} = \vec{\gamma}_{k,i} + \beta_{k,i}\nabla P_i \quad (1)$$

Then the new time velocity, $\vec{u}^{n+1}_{k,i}$, is as follows:

$$\vec{u}^{n+1}_{k,i} = \vec{u}^{*}_{k,i} + \beta_{k,i}\nabla \delta P_i \quad (2)$$

where $\delta P_i (= P_i^{n+1} - P_i^{n})$ is determined to satisfy the mass and energy conservation equations. Applying divergence operator $(\nabla \cdot)$ to Eq. (2) and integrating the equation, we obtain

$$\sum_f \Psi^{n+1}_{k,f} = \sum_f \left[\Psi^{*}_{k,f} + \beta_{k,f} \left|\vec{S}_f\right| \left(\delta P_j - \delta P_i\right) / \left|\delta \vec{r}_{ij}\right| \right] \quad (3)$$

where $\Psi_{k,f}$ is the k-phase volume flow rate at surface f defined as the product of velocity and surface vector, $\Psi_{k,f} = \vec{u}_{k,f} \cdot \vec{S}_f$. And $\left|\delta \vec{r}_{ij}\right|$ is the distance between two cell centers. The new time pressure is calculated by solving a pressure equation where Eq. (3) is substituted into scalar conservation equations to eliminate the new time volume flux, $\Psi^{n+1}_{k,f}$.

$$\delta P_i + \sum_f \sum_k C_f \left(\delta P_i - \delta P_j\right) = B_i \quad (4)$$

An iterative method is used to get a solution from the linear equation. After solving Eq. (4), the new time phasic velocity and volume flow rate are determined from Eqs. (2) and (3). Then the scalar values at new time step are calculated. Finally, the remaining dependent variables such as temperature, density, and other physical properties are updated using the equations of state.

2.2 Decoupled Method (PM-explicit)

The same procedures are applied for the prediction of velocity vectors as described in section 2.1. However, in this method, only mass conservation equation is used for setting up the pressure equation. Discretization of combined gas and liquid mass equations gives the following equation.

$$\frac{1}{V_i}\sum_k \left[\frac{\alpha_k}{\rho_k}\frac{\delta \rho_k}{\delta t} + \frac{1}{\rho_k}\sum_f \left(\alpha_k \rho_k\right)_f \Psi^{n+1}_{k,f} \right] = -\frac{1}{\left(h_g - h_l\right)} \left[\frac{P_v}{P} H_{ig}\left(T_s^{n+1} - T_g^{n+1}\right) + H_{il}\left(T_s^{n+1} - T_l^{n+1}\right) \right] \left(\frac{1}{\rho_g} - \frac{1}{\rho_l} \right) \quad (5)$$

After linearizing T_k^{n+1} and $\delta \rho_k$, it can be re-arranged in a simple form as follows.

$$A\delta P = S + \sum_f \sum_k C_{k,f} \Psi^{n+1}_{k,f} \quad (6)$$

Eq. (3) is substituted into Eq. (6) to set up the pressure equation.

$$\left[1 + \sum_f L_{ij}\right] \delta P_i - \sum_f L_{ij} \delta P_j = M \quad (7)$$

where, $L_f = -A^{-1} \sum_k C_{k,f} \beta_{k,f} \left|\vec{S}_f\right| / \left|\delta \vec{r}_{ij}\right|$, and

$$M = -A^{-1} S + A^{-1} \sum_k C_{k,f} \Psi^{*}_{k,f}$$

Then the phase fraction is calculated by solving the combined mass conservation equation. Finally, non-condensable gas and phasic energy are calculated.

2.3 Implicit Method (PM-implicit)

The solution scheme described in the above sections is a semi-implicit scheme since all the convection and

diffusion terms are calculated explicitly where time step size is usually limited by the Courant number. A numerical scheme is presented in this section for an implicit calculation of the governing equations.

The implicit method for the momentum equation is implemented in three steps. The first step accounts for the implicit coupling of vapor and liquid velocities. This step is called implicit "phase link" is shown by.

$$\alpha_{k,i}\rho_{k,i}\frac{\vec{u}_{k,i}^{(1)}-\vec{u}_{k,i}^{n}}{\delta t}=-\alpha_{k,i}\nabla P_i^n \pm C_{int}\left(\vec{u}_{l,i}^{(1)}-\vec{u}_{g,i}^{(1)}\right)+SRC_{k,i}^n \quad (8)$$

where $\vec{u}_{k,i}^{(1)}$ is a phasic velocity at the first step and $SRC_{k,i}^n$ includes all the explicit source terms except convection and diffusion terms. As a second step, the convection and diffusion terms are calculated implicitly for gas- and liquid-phase.

$$\alpha_{k,i}\rho_{k,i}\frac{\vec{u}_{k,i}^{(2)}-\vec{u}_{k,i}^{(1)}}{\delta t}+\frac{1}{V_i}\sum_f\left[\left(\alpha_k\rho_k\right)_f\left(\vec{u}_{k,f}^{(2)}-\vec{u}_{k,i}^{(2)}\right)\right]\Psi_{k,f}^n= \quad (9)$$
$$\frac{1}{V_i}\sum_f\left(\alpha_k\mu_k\right)_f\frac{\vec{u}_{k,j}^{(2)}-\vec{u}_{k,i}^{(2)}}{\left|\delta\vec{r}_{ij}\right|}\left|\vec{S}_f\right|$$

In the next, an additional implicit "phase link" step is carried out.

$$\alpha_{k,i}\rho_{k,i}\frac{\vec{u}_{k,i}^{(3)}-\vec{u}_{k,i}^{(2)}}{\delta t}=\pm C_{int}\left[\left(\vec{u}_{l,i}^{(3)}-\vec{u}_{g,i}^{(1)}\right)-\left(\vec{u}_{l,i}^{(3)}-\vec{u}_{g,i}^{(1)}\right)\right] \quad (10)$$

where $\vec{u}_{k,i}^{(3)}$ means a velocity at third step.

The same implicit method for momentum conservation equation is applied where the calculations are divided into "phase link" and "space link" steps. The first step accounts for the implicit coupling of gas energy, liquid energy, and non-condensable gas fraction. And as the second step, the convection and diffusion terms are calculated implicitly for gas energy, liquid energy, and non-condensable gas fraction respectively

3. Verifications

The numerical schemes are verified against two-phase flow test problems which includes boiling and flashing. Each test problems are simulated using PME-explicit, PM-explicit, PM-implicit, and PM-implicit-c. PM-implicit-c is identical to PM-implicit except the time step size. It has been has been increased in PM-implicit-c beyond Courant limit.

A two-dimensional boiling flow in a vertical upward pipe is simulated where the length and height of the pipe are 0.1m and 2.0m. A structured grid with 250(5x50) uniform computing cells are used. Initially the pipe is filled with subcooled liquid of 441.8K at 1.0MPa. Inlet flow boundary condition is given to the bottom of the pipe with a speed of 0.1m/s and temperature of 441.8K. Outlet pressure is kept constant at 1.0MPa. A uniform volumetric heat source which linearly increases up to 23.0MW/m^3 in 10 seconds is applied to the liquid column to induce bulk boiling. Then boiling occurs in the liquid column when the liquid temperature reaches the saturation point. The calculation is carried out for 20 seconds to get a steady state solution. In Fig.1, void fractions averaged on x-direction are compared along the vertical direction (y-direction) at 20 second. Subcooled liquid from the bottom starts boiling at y=1.62 and increase up to 0.97 at the exit.

A two-dimensional flashing flow in a horizontal pipe is simulated where the length and height of the pipe are 2.0m and 0.1m. A structured grid with 250(50x5) uniform computing cells are used. Initially the pipe is filled with liquid of 450.0K at 1.0MPa which is slightly lower than saturation temperature of 453.0K. Inlet flow boundary condition is given to the left side of the pipe with a speed of 4.0m/s and temperature of 450.0K. Outlet pressure boundary condition is given to the right side of the pipe which is linearly decreased from 1.0MPa to 0.864MPa during the first 10 seconds and then kept constant. The calculation reached a steady state at 15 second. In Fig.2, void fractions averaged on y-direction are compared along the horizontal direction (x-direction) at 15 second.

All the four calculations are almost identical to each other where Courant number is 1.0 for PME-explicit, PM-explicit, PM-implicit and 5.0 for PM-implicit-c.

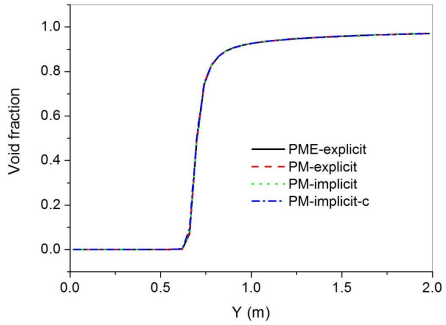

Figure 1. Steady state void fraction (boiling)

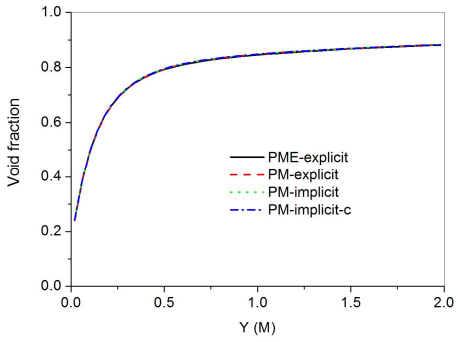

Figure 2. Steady state void fraction (flashing)

4. Conclusions

An implicit numerical method has been proposed for two-phase calculation where energy equations are decoupled from the pressure equation. Convection and diffusion terms are calculated implicitly. The calculation results are the same for PME-explicit, PM-

explicit, and PM-implicit. Large time step size has been tested with PM-implicit-c and the results are also the same.

REFERENCES

[1] H. Y. Yoon et al., Recent Improvements in the CUPID Code for a Multi-dimensional Two-phase Flow Analysis of Nuclear Reactor Components, Nuclear Engineering and Technology, Vol.46, No.5, p.655, 2014.