# Estimation of Peaking Factor Uncertainty due to Manufacturing Tolerance using Statistical Sampling Method

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# 1. Introduction

In a nuclear design and analysis, the lattice physics calculations are usually employed to generate lattice parameters for the nodal core simulation and pin power reconstruction. These lattice parameters which consist of homogenized few-group cross-sections, assembly discontinuity factors, and form-functions can be affected by input uncertainties which arise from three different sources: 1) multi-group cross-section uncertainties, 2) the uncertainties associated with methods and modeling approximations utilized in lattice physics codes, and 3) fuel/assembly manufacturing uncertainties.

The purpose of this paper is to study the effect on output parameters in the lattice physics calculation due to the last input uncertainty such as manufacturing deviations from nominal value for material composition and geometric dimensions. In this paper, data [1] provided by the light water reactor (LWR) uncertainty analysis in modeling (UAM) benchmark has been used as the manufacturing uncertainties. First, the effect of each input parameter has been investigated through sensitivity calculations at the fuel assembly level. Then, uncertainty in prediction of peaking factor due to the most sensitive input parameter has been estimated using the statistical sampling method, often called the brute force method.

For our analysis, the two-dimensional transport lattice code DeCART2D [2] and its ENDF/B-VII.1 based 47-group library were used to perform the lattice physics calculation.

#### 2. Methods and Results

#### 2.1 Manufacturing Uncertainties

Manufacturing uncertainties for three different LWR assembly-level models are available in the LWR UAM benchmark: PWR TMI-1, BWR PB-2, and VVER-1000 Kozloduy-6. For this work, TMI-1 test case has been selected to examine on a PWR assembly. TMI-1 fuel assembly consists of a 15x15 array of 208 fuel rods, 16 guide tubes, and an instrumentation tube. Fig. 1 shows the assembly configuration for TMI-1 used in this work. There are no burnable absorber (BA) rods such as the Gd fuel pin as well as a discrete BA rod located in guide tube. Table I gives the information [3] for TMI-1 assembly model.



Fig. 1. TMI-1 assembly configuration.

Table I: TMI-1 assembly design data

Parameter	Value				
Fuel assembly pitch, cm	21.811				
Fuel rod pitch, cm	1.4427				
Fuel pellet material	$UO_2$				
Fuel density, g/cm <sup>3</sup>	10.2083				
Fuel enrichment, w/o	4.85				
Fuel pellet diameter, cm	0.9391				
Cladding material	Zircaloy-4				
Cladding density, g/cm <sup>3</sup>	6.56				
Cladding outer diameter, cm	1.0928				
Cladding thickness, cm	0.0673				

Table II provides the manufacturing uncertainties in terms of  $3\sigma$  for TMI-1 test case. The notation " $3\sigma$ " is used to represent three standard deviations from the nominal value for that parameter. Normal distribution was assumed as the probability density function (PDF) for each parameter.

Table II: Manufacturing tolerances for TMI-1

Parameter	3σ				
Fuel density, g/cm <sup>3</sup>	±0.17				
Fuel pellet diameter, cm	±0.0013				
Gap thickness, cm	±0.0024				
Clad thickness, cm	±0.0025				
<sup>235</sup> U concentration, w/o	±0.00224				

# 2.2 Manufacturing Tolerance Sensitivity

Sensitivity of lattice parameters to manufacturing tolerances has been studied for the TMI-1 assembly. The sensitivities were calculated by increasing the tolerance of the parameters listed in Table II (i.e., using the upper limits) under all rods out (ARO) condition at hot full power (HFP). It should be noted that variations in gap and clad thickness can be considered in two ways.

1) The gap thickness can increase because the fuel pellet diameter decreases or the clad inner diameter increases (decrease of the clad thickness);

2) The clad thickness can increase because the clad inner diameter decreases (decrease of the gap thickness) while the clad outer diameter remains the same or the clad outer diameter increases while the clad inner diameter remains the same.

Parameter	$\Delta \mathbf{k}$
Fuel density	-0.00167
Fuel pellet diameter	-0.00036
Gap thickness	
Change of fuel pellet diameter	0.00134
Change of clad inner diameter	0.00048
Clad thickness	
Change of clad inner diameter	-0.00049
Change of clad outer diameter	-0.00158
<sup>235</sup> U concentration	0.00007

Table III: Manufacturing sensitivities for TMI-1

\* Reference k-inf = 1.44289

Table III shows sensitivity due to the manufacturing tolerances for TMI-1 test case. It should be noted that in this paper the term "sensitivity" has not been used in the usual way but as k-inf differences due to a parameter variation. It can be seen that the largest sensitivity occurs for the manufacturing tolerance of fuel density and clad thickness (by changing the outer diameter). The highest variation is an absolute value of about  $0.0017\Delta k$ . Further calculations show that  $\Delta k$  values change linearly with manufacturing tolerances for all the parameters considered.

#### 2.3 Estimation of Peaking Factor Uncertainty

To estimate the uncertainty in prediction of peaking factor due to the manufacturing tolerance of fuel density, a full assembly model is considered for TMI-1. In this study, 1,000 different DeCART2D input sets were obtained by the random sampling using the MIG code [4]. Since the fuel pellets are independent each other (i.e., there is no correlation), the random variate X normally distributed with a mean  $\mu$  and a standard deviation  $\sigma$  can be calculated using the Box-Muller method.

$$X = \mu + \sigma \sqrt{-2\ln(1 - u_1)} \cos(2\pi u_2)$$
(1)

where  $u_1$  and  $u_2$  are independent random numbers that are uniformly distributed in the interval (0,1).



Fig. 2. Normal Q-Q plot of peaking factors for TMI-1.

Table IV: Peaking factor results

Case	Peaking Factor	Difference (%)					
Reference	1.0770	-					
Mean	1.0832	0.58					
Maximum	1.0941	1.59					
* Difference $(X - X) / X \times 100$							

\* Difference= $(X - X_{ref}) / X_{ref} \times 100$ 

-0.04	-0.05	0.06	-0.02	-0.01	-0.05	0.00	0.01	0.07	0.09	-0.01	0.00	0.02	-0.01	0.03
0.00	-0.07	0.01	-0.09	0.02	0.07	-0.06	0.01	-0.03	-0.01	-0.13	0.07	-0.02	0.12	0.01
-0.10	0.01	-0.02	-0.15	-0.02	imes	0.06	0.02	-0.04	Х	-0.05	0.03	-0.02	0.00	0.04
-0.04	-0.03	-0.07	imes	-0.07	-0.03	-0.09	-0.01	-0.01	0.04	0.07	$\times$	-0.13	0.05	-0.01
-0.03	-0.07	-0.14	0.20	0.02	0.09	-0.03	-0.07	0.04	0.02	0.08	0.02	0.14	-0.06	-0.05
0.04	0.07	$\times$	-0.06	0.00	$\times$	-0.06	-0.02	-0.03	Х	-0.03	0.00	$\times$	0.01	-0.01
-0.11	-0.06	-0.01	-0.04	0.00	0.03	-0.07	0.01	-0.03	0.07	-0.08	-0.05	-0.06	-0.01	0.20
0.01	0.02	0.11	0.06	0.07	-0.01	0.03	imes	0.00	-0.01	-0.03	0.01	0.08	0.01	0.13
0.02	0.08	-0.03	-0.03	0.12	-0.09	0.00	0.07	-0.07	-0.10	-0.01	-0.01	-0.05	-0.09	0.07
0.02	0.00	imes	-0.05	-0.07	imes	-0.06	-0.04	-0.02	imes	-0.08	0.05	imes	-0.10	-0.03
0.09	0.07	-0.01	0.06	0.03	-0.01	-0.03	0.15	0.02	0.09	-0.01	-0.03	0.03	0.03	0.09
0.02	0.06	0.07	imes	-0.06	-0.02	0.06	0.01	0.05	0.06	0.13	imes	0.00	0.04	0.00
0.03	0.01	-0.01	-0.06	0.11	imes	-0.07	-0.03	0.03	imes	-0.16	-0.06	0.02	-0.04	0.00
-0.07	0.06	0.14	-0.07	0.06	-0.06	0.09	0.01	-0.10	-0.04	-0.07	0.10	0.00	-0.06	-0.02
-0.01	-0.04	-0.01	-0.04	-0.04	-0.08	-0.09	-0.02	0.00	0.07	0.03	0.04	0.08	0.06	-0.04

Fig. 3. Pin-by-pin fuel density deviations in g/cm<sup>3</sup> for the worst case.

Fig. 2 shows a normal Q-Q plot of peaking factors calculated with 1,000 randomly sampled inputs. The linearity of the points suggests that data are normally

distributed with the mean of 1.08321 and the standard deviation of 0.00266. Table IV shows the mean and maximum peaking factors as well as the reference value for TMI-1. The relative difference between the reference and mean values is about 0.6% while that between the reference and maximum values is about 1.6%. Fig. 3 shows the pin-wise fuel density deviations that yield the worst peaking factor. The peaking occurs in a pin with the largest deviation near the guide tube.

In thermal margin analyses of PWR, the effects on local heat flux and subchannel enthalpy rise due to deviations from nominal design values within tolerance are considered by certain factors called engineering factors of 1.03. Since the peaking factor uncertainty for TMI-1 is expected to be less than 3%, it can be seen that the engineering factor accounts for the effects on peaking factor due to the manufacturing tolerance is conservative well enough.

# 3. Conclusions

Sensitivity calculations have been performed in order to study the influence of manufacturing tolerances on the lattice parameters. The manufacturing tolerance that has the largest influence on the k-inf is the fuel density. The second most sensitive parameter is the outer clad diameter. Although the peaking factor uncertainty due to manufacturing deviations from nominal fuel density is estimated with the worst case, the uncertainty is expected to be less than the engineering factor of 3% that accounts for the effects of manufacturing deviations in PWR fuel fabrication.

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