Bayesian Model on Fatigue Crack Growth Rate of Type 304 Stainless Steel

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1. Introduction

▷ The fatigue crack growth rate is typically estimated by deterministic methods in accordance with the ASME Boiler and Pressure Vessel Code Sec. XI
→ Deterministic model include uncertainties in the constant of the model

▷ Uncertainty problems can be overcome by probabilistic methods that estimate the degradation of materials even if the additional data scarcity of the fatigue model

▷ The unknown constants of Paris' law were updated probabilistically by Bayesian inference
▷ This methods can be used for the probabilistic structural integrity assessment of nuclear materials in the lab scale

2. Methods and Results

2.1 Materials

▷ Polished Type 304 stainless steel was used

Table I. Chemical composition of Type 304 stainless steel

Chemical composition (%)	C	Si	Mn	P	S	Ni	Cr	Mo	N	Cu	Fe
Type 304 stainless steel	0.044	0.47	1.15	0.038	0.002	8.00	18.14	0.22	0.023	0.34	Bal.

Table II. Mechanical properties of Type 304 stainless steel (STS 304) measured at 25℃ in air condition in accordance with ASTM E8/E8M-15a (Straining rate=0.75mm/min)

Materials	STS 304
0.2 % offset yield strength (MPa)	264.4
Ultimate tensile strength (MPa)	757.2
Elastic modulus (GPa)	178.9
Elongation (%)	66.93

2.2 Fatigue Test Procedure

▷ The test was controlled by a servo-hydraulic testing control machine named Instron® Model 8516 with a load capacity of 100kN

▷ SEN (Single Edge Notch) specimens were made by electrical-discharge machining (EDM) wire cutting

▷ The specimens were made in accordance with ASTM E647-13ae1, Gary S. Was et al., a dissertation from Il Soon Hwang, and a thesis from Jae Young Yoon [1-3]

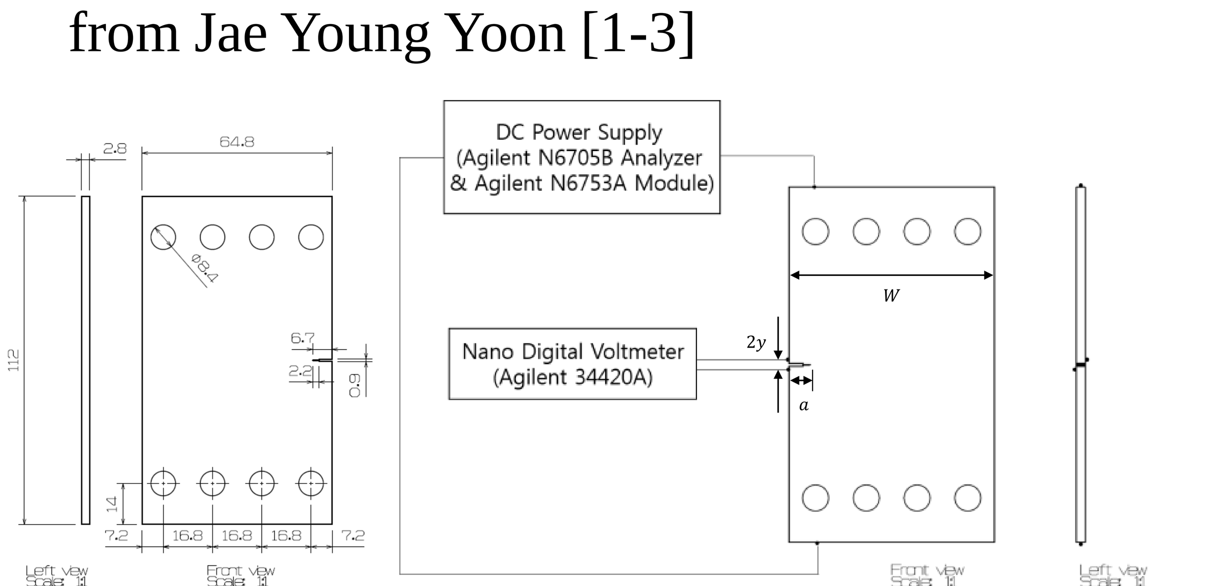

Fig 1. SEN specimens for fatigue testing and wire attachment positions on specimens (unit : mm)

▷ Stress intensity factor of fatigue specimens

$$K = \frac{P\sqrt{a}}{BW}\left[1.986 + 1.782\left(\frac{a}{W}\right) + 6.998\left(\frac{a}{W}\right)^2 - 21.505\left(\frac{a}{W}\right)^3 + 45.351\left(\frac{a}{W}\right)^4\right]$$

until $\frac{a}{W} = 0.621$ (it shows a maximum difference of 6%)

Where K is the stress intensity factor ($MPa\sqrt{m}$), P is the applied load (N), a is the crack length (mm), B is the thickness of specimens (mm), and W is the width of specimens (mm)

▷ Crack length was measured by direct current potential drop (DCPD) method using voltmeter and optical method using travelling microscope

▷ The relation for crack length and voltage is followed by Johnson's equation [4]

$$a = \frac{2W}{\pi}\cos^{-1}\left\{\frac{\cosh\left(\frac{\pi y}{2W}\right)}{\cosh\left[\frac{U}{U_0}\cosh^{-1}\left\{\frac{\cosh\left(\frac{\pi y}{2W}\right)}{\cos\left(\frac{\pi a_0}{2W}\right)}\right\}\right]}\right\}$$

Where U is the potential drop (V), a is the crack length (mm), a_o is the reference crack length (mm), y is the length between notch centerline and the voltage measurement point shown in Fig. 1

▷ Constant load and constant ΔK test conditions

Table III. Fatigue test conditions (left) and pre-cracking conditions (right)

Fatigue Test No.	1	2
Mode	Const. load	Const. ΔK
ΔP (kN)	18	-
ΔK ($MPa\sqrt{m}$)	-	30
R ratio (σ_min/σ_max)	0.1	
Frequency (Hz)	5	
Environment	Air	
Temperature (℃)	25	

Mode	Const. load
ΔP (kN)	18
R (σ_min/σ_max)	0.1
Frequency (Hz)	10
Environment	Air
Temperature (℃)	25
Crack length (mm)	1

2.3 Fatigue Test Results

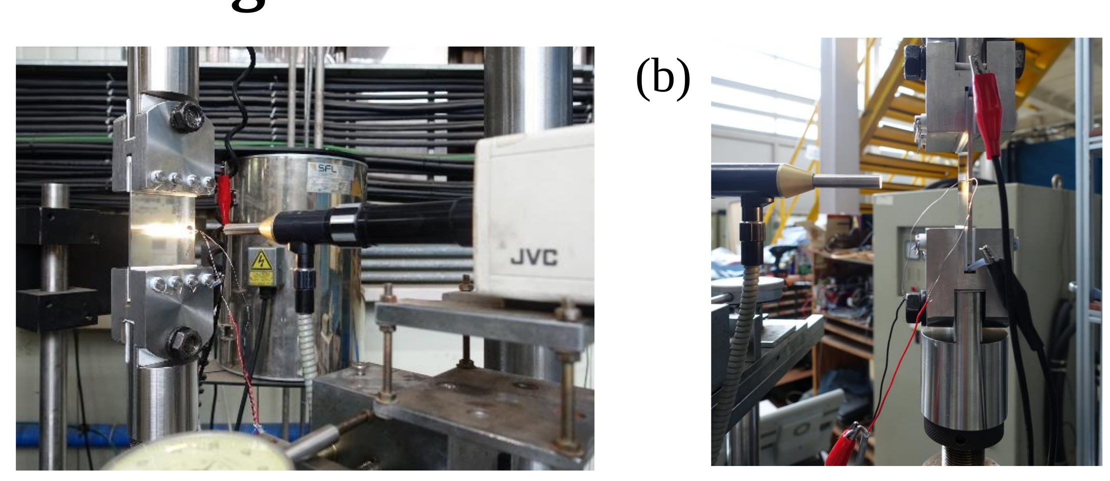

Fig 2. Fatigue test picture (a) left side view (b) right side view

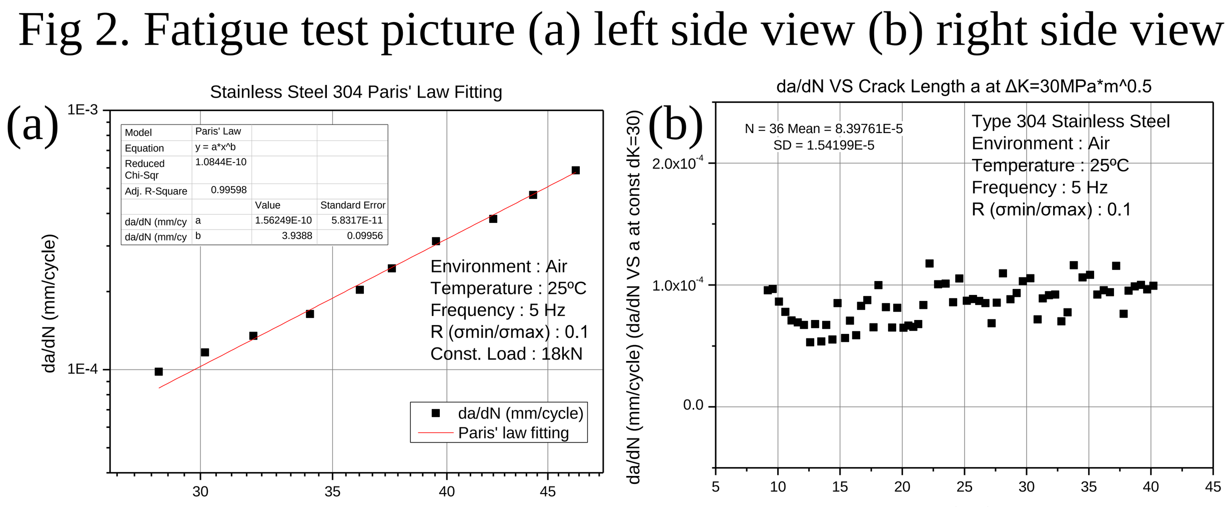

Fig 3. Fatigue test results (a) constant load (b) constant ΔK

▷ Paris' law model = $\frac{da}{dN} = C(\Delta K)^m = 1.56 \times 10^{-10} \times (\Delta K)^{3.94}$

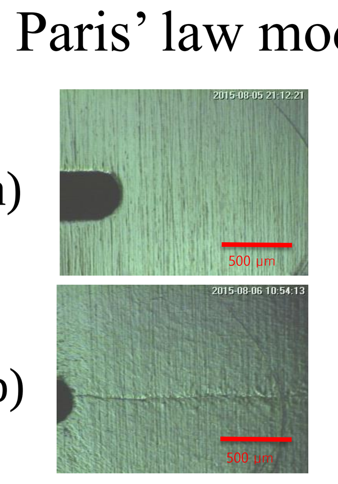

Fig 4. Picture of (a) notch (b) crack with microscope

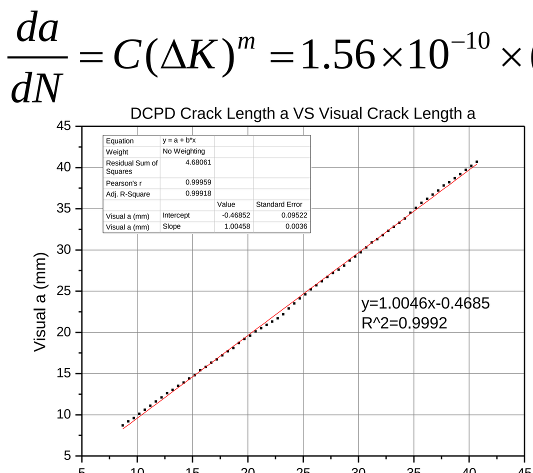

Fig 5. Crack length measured by DCPD VS by microscope

▷ DCPD methods measured crack length very well

2.4 Bayesian Updating

▷ Bayesian theorem [5]

$$f(C,m \mid \dot{a}) = kL(\mathrm{C,m} \mid \dot{a})\, f(\mathrm{C,m})$$

$f(\mathrm{C,m})$: Prior distribution of constant C and m
$f(\mathrm{C,m} \mid \dot{a})$: Posterior distribution of constant C and m
$L(\mathrm{C,m} \mid \dot{a})$: Likelihood
k : Normalizing constant

▷ Normal distribution [6]

- Probability density function (PDF)

$$f(C,m) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(C,m-\mu_{C,m})^2}{2\sigma_{C,m}^2}}$$

Where μ is the mean of C and m, σ is the standard deviation of C and m

- Cumulative distribution function (CDF)

$$F(C,m) = \Phi\left(\frac{C,m-\mu_{C,m}}{\sigma_{C,m}}\right) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{C,m-\mu_{C,m}}{\sigma_{C,m}\sqrt{2}}\right)\right]$$

$$\text{Where } \Phi(x) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right], \mathrm{erf}(x) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x} e^{-t^2}\, dt$$

▷ Likelihood function [5]

- Assuming $x_i - \dot{a}$ normal distribution is $N(0, \sigma^2)$

$$L(C,m \mid \dot{a}) = \prod_{i=1}^{N} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[(x_i - \dot{a}(\mathrm{C,m})) - 0]^2}{2\sigma^2}}$$

Where σ is the standard deviation of $x_i - \dot{a}$

▷ Prior and posterior C and m distributions
- Random sampled using Monte Carlo simulation

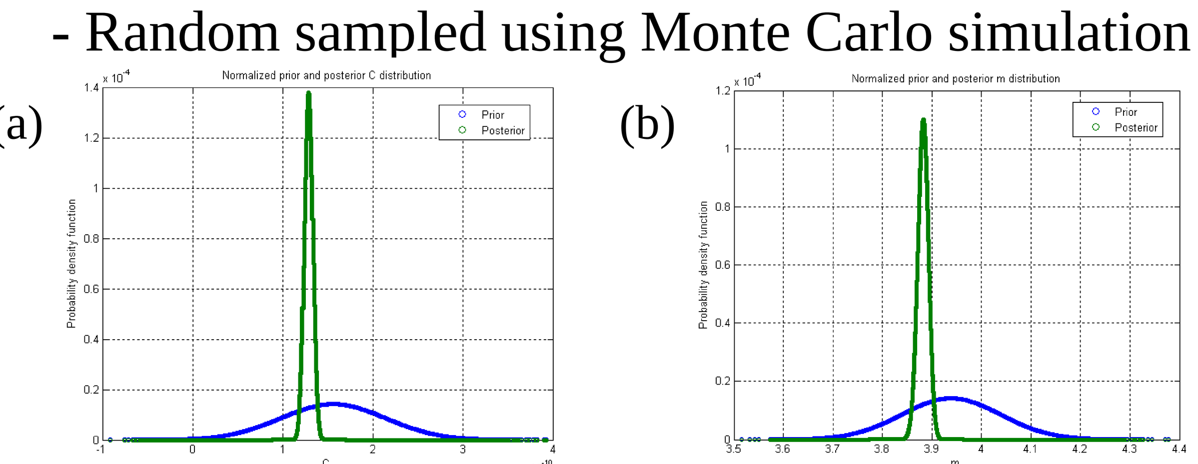

Fig 6. Prior and posterior constant (a) C and (b) m distribution

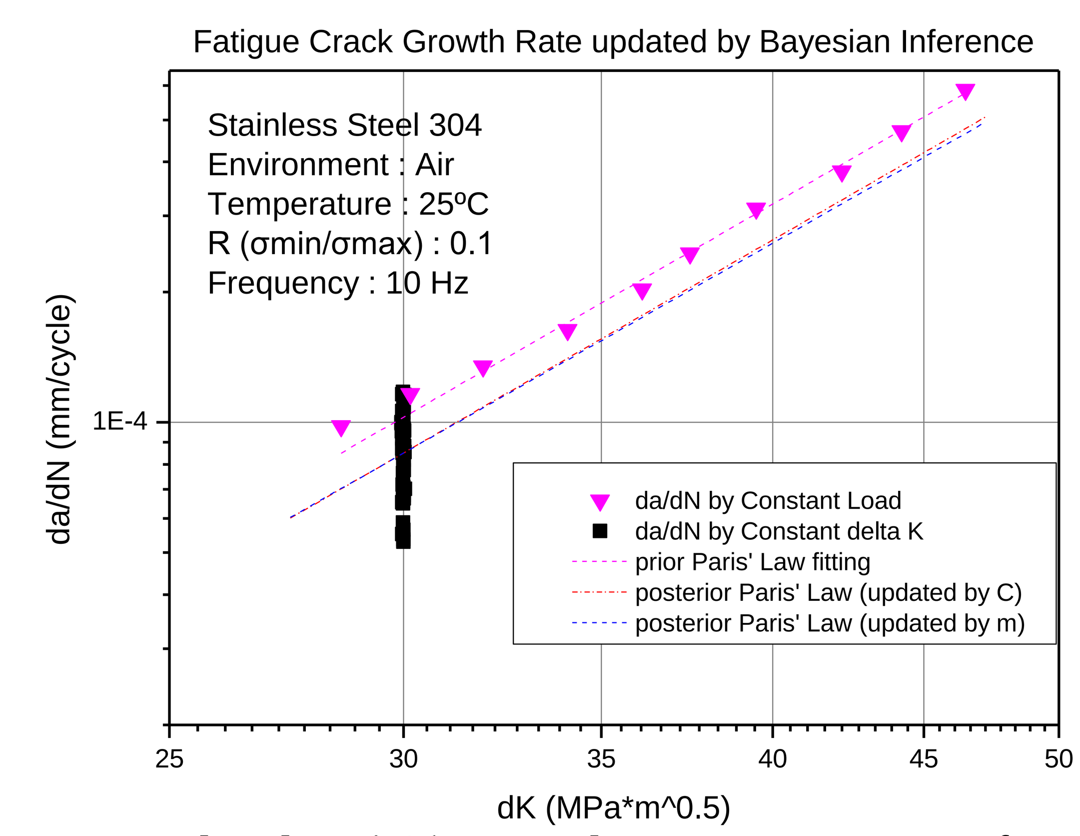

Fig 7. Updated Paris' law results using Bayesian inference

Table IV. Paris' law constant results of Type 304 stainless steel

Paris' law constant		Prior	Sampled prior	Posterior
C	Mean	1.5625×10^{-10}	1.5626×10^{-10}	1.2923×10^{-10}
	STD	5.8317×10^{-11}	5.8337×10^{-11}	2.1761×10^{-23}
Paris' law constant		Prior	Sampled prior	Posterior
m	Mean	3.9388	3.9388	3.8834
	STD	9.9560×10^{-2}	9.9562×10^{-2}	1.1213×10^{-4}

3. Conclusions

▷ Paris' law constants C and m for Type 304 stainless steel were determined by probabilistic method using Bayesian inference
→ Uncertainty of models' constant decreases dramatically

▷ Until now, remaining lives of NPPs are estimated by deterministic methods using a priori model to finally assess structural integrity.

▷ Bayesian approach can utilize in-service data derived from aged properties

→ A probabilistic method should be applied to consider the environment and material conditions

Reference

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