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Bayesian Model on Fatigue Crack Growth Rate of Type 304 Stainless Steel

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1. Introduction

One of the most important issues with structural materials of nuclear power plants is the aging management against fatigue, corrosion as well as radiation. The fatigue crack growth rate curve is typically estimated by deterministic methods in accordance with the ASME Boiler and Pressure Vessel Code Sec. XI. [1] The reliability of nuclear materials must also consider the environmental effect. This can be overcome by probabilistic methods that estimate the degradation of materials.

In this study, fatigue tests were carried out on Type 304 stainless steel (STS 304) to obtain a fatigue crack growth rate curve and Paris' law constants. Tests were conducted on a constant load and a constant delta K, respectively. The unknown constants of Paris' law were updated probabilistically by Bayesian inference and the method can be used for the probabilistic structural integrity assessment of other nuclear materials.

2. Methods and Results

2.1 Materials and Tensile Test Procedure

Type 304 stainless steel has a high resistance to fatigue and corrosion, so it is widely used for the pipe materials of NPPs. The chemical composition of Type 304 stainless steel is summarized in Table I. To determine the mechanical properties of Type 304 stainless steel, tensile testing was performed in accordance with ASTM E8/E8M-15a. [2] The rectangular tension test specimens are shown in Fig. 1. The test was controlled by a servo-hydraulic testing control machine named Instron® Model 8516 with a load capacity of 100kN. The specimens were tested at room temperature (25℃) in air at a strain rate of 0.75mm/min. The strain gauge length is measured by Reliant extensometers as in Fig. 2.

Table I: Chemical composition of Type 304 stainless steel (STS 304)

Chemical Composition (%)	STS 304
C	0.044
Si	0.47
Mn	1.15
P	0.038
S	0.002
Ni	8.00
Cr	18.14
Mo	0.22
N	0.023
Cu	0.34
Fe	Bal.

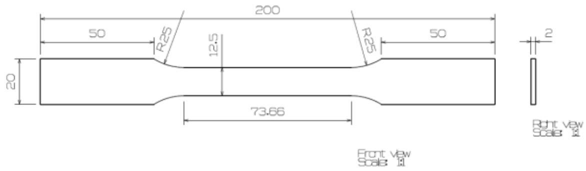

Fig. 1. Rectangular tension test specimens for tensile test

Fig. 2. Tensile testing of Type 304 stainless steel

2.2 Fatigue Test Procedure

SEN (Single Edge Notch) specimens for fatigue testing were made in accordance with ASTM E647-13ae1, papers from Gary S. Was et al., a dissertation from Il Soon Hwang, and a thesis from Jae Young Yoon. [3-6] The specimen drawing is shown in Fig. 3. The machined notch for specimens was made by electrical-discharge machining (EDM). The geometrical factor in the correlation between applied load and stress intensity factor is given in equation (1). A comparison of this equation with theoretical data shows a maximum difference of 6% at $a/W = 0.621$.

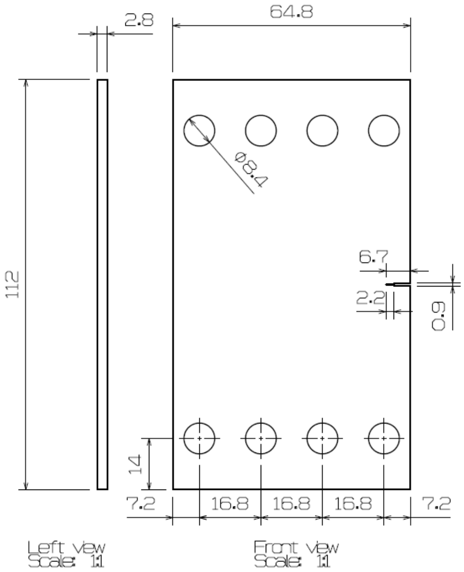

Fig. 3. SEN (Single Edge Notched) specimens for fatigue testing

$$K = \frac{P\sqrt{a}}{BW}\left[1.986 + 1.782\left(\frac{a}{W}\right) + 6.998\left(\frac{a}{W}\right)^2 - 21.505\left(\frac{a}{W}\right)^3 + 45.351\left(\frac{a}{W}\right)^4\right] \quad (1)$$

K is the stress intensity factor ($MPa\sqrt{m}$), P is the applied load (N), a is the crack length (mm), B is the thickness of specimens (mm) and W is the width of specimens (mm).

The aim in fatigue crack growth rate testing is to maintain a predominantly elastic condition in the test specimen, thereby allowing results to be interpreted in terms of the crack-tip stress intensity that is defined by the linear-elastic theory. Thus, there are two constraints: First, stress applied on uncracked ligament do not yield more stress. Second, the specimen size requirement for SEN specimen amounts to restricting the monotonic plastic zone size ($2r_y$) to 25% of the specimen's uncracked ligament for elastic condition. Two requirements are as following equations (2) and (3). [3, 4]

$$\sigma_{\max} = \frac{P_{\max}}{B(W-a)} \le \sigma_y \quad (2)$$

σ is stress (MPa), P is the applied load (N), a is the crack length (mm), B is the thickness of specimens (mm) and W is the width of specimens (mm).

$$W - a \ge \frac{4}{\pi}\left(\frac{K_{\max}}{\sigma_y}\right)^2 \quad (3)$$

σ is stress (MPa), P is the applied load (N), a is the crack length (mm), B is the thickness of specimens (mm) and W is the width of specimens (mm).

If $\Delta P = 18kN$, the crack length should be below 37.2mm for the first constraints that apply stress below the yield strength. Additionally, the crack length is below 20.7mm and $K_{\max}$ is below $48.22MPa\sqrt{m}$ for elastic condition. Therefore fatigue test results should be considered below $\Delta K = 43.4MPa\sqrt{m}$.

The fatigue test is conducted as shown in Fig. 4 and Fig. 5. Grip and specimens are insulated with Teflon to prevent direct current from flowing in the wrong direction.

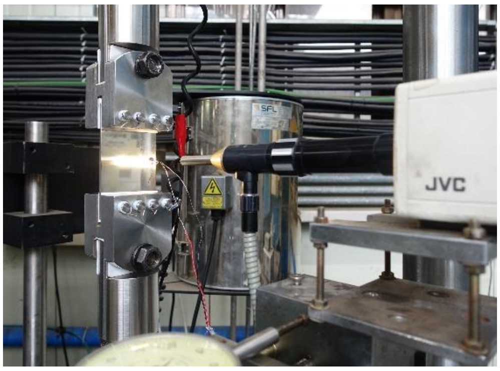

Fig. 4. Fatigue testing of Type 304 stainless steel (front view)

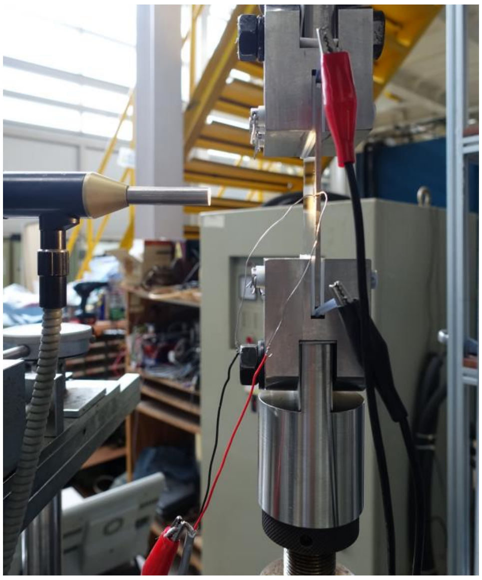

Fig. 5. Fatigue testing of Type 304 stainless steel (side view)

The direct current and voltage measurement wire is attached to the specimens as shown in Fig. 6. Pt diameter 1mm wire is spot welded on the top and bottom of the specimens. Type 304 stainless steel has wire with diameter 0.02mm spot welded near the notch, as shown in Fig. 6. The Pt wire is covered with heat-shrink tubing to avoid current leakage and the voltage measure wire shielding is power grounded to reduce noise.

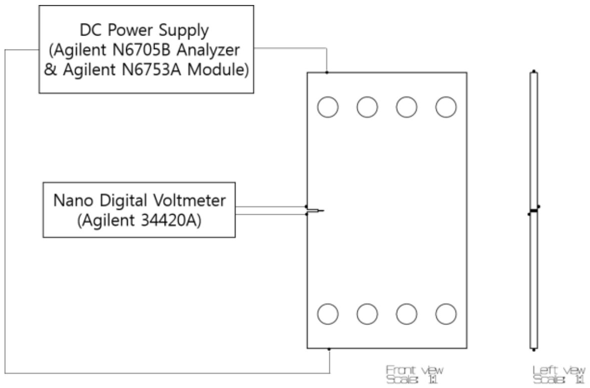

Fig. 6. Wire attachment positions on specimens

To make 1mm pre-crack on the specimens, the fatigue condition is shown in Table Ⅱ. After pre-cracking, test condition is shown in Table Ⅲ.

Table II: Constant load pre-crack testing

Frequency (Hz)	10
Environment	Air
Temperature (℃)	25
ΔP (kN)	18
R ($\sigma_{\min} / \sigma_{\max}$)	0.1

Table III: Constant load fatigue test condition

Frequency (Hz)	5
Environment	Air
Temperature (℃)	25
ΔP (kN)	18
R ($\sigma_{\min} / \sigma_{\max}$)	0.1

The crack length is measured by both DCPD (Direct Current Potential Drop) methods and visual measurement with travelling microscope. DCPD and crack length relationship follow Johnson's equation. [7]

$$a = \frac{2W}{\pi} \cos^{-1} \left(\frac{\cosh\left(\frac{\pi y}{2W}\right)}{\cosh\left\{ \frac{U}{U_0} \cosh^{-1}\left(\frac{\cosh\left(\frac{\pi y}{2W}\right)}{\cos\left(\frac{\pi a_0}{2W}\right)} \right) \right\}} \right) \quad (4)$$

U is the potential drop (V), U_0 is the reference potential drop (V), a is the crack length (mm), a_0 is the reference crack length (mm), y is the length between notch centerline and the voltage measurement point shown in Fig. 7 (mm) and W is the width of specimens (mm).

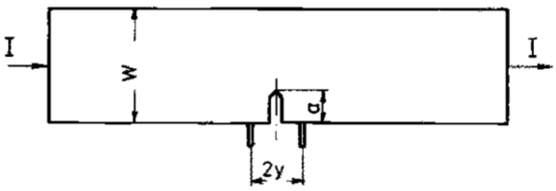

Fig. 7. Johnson's equation for SEN (Single Edge Notch) specimens [7]

Constant ΔK test is performed at $\Delta K = 30MPa\sqrt{m}$. The 1mm pre-cracking is made in the same environment with constant load test and frequency at $10Hz$. After pre-cracking, the test frequency is conducted at $5Hz$.

2.3 Bayesian Inference and Monte Carlo Simulation

Bayes' theorem is the conditional probability as shown in the following equation.

$$p(\theta \mid y) = \frac{p(y \mid \theta) p(\theta)}{\int p(y \mid \theta) p(\theta) d\theta} \propto p(y \mid \theta) p(\theta) \quad (5)$$

This equation shows that an unknown parameter, θ , can be solved with known y data. Therefore, $p(\theta)$ is the prior probability distribution and $p(\theta \mid y)$ is posterior probability distribution. $p(\mathrm{y} \mid \theta)$ is the likelihood distribution. [8]

To extract continuous random variables, Monte Carlo simulation is used for uniformly random sampling γ between 0<y<1. After that, corresponding x can be randomly sampled in the cumulative distribution function, as shown in Fig. 8. Therefore, θ can be obtained by random sampling. [9]

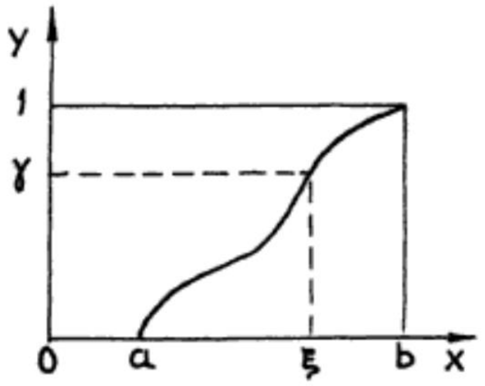

Fig. 8. The function for modeling of a continuous random variable [9]

2.4 Tensile Test Results

The stress-strain curve is shown in Fig. 9 and the mechanical properties of Type 304 stainless steel are shown in Table Ⅳ.

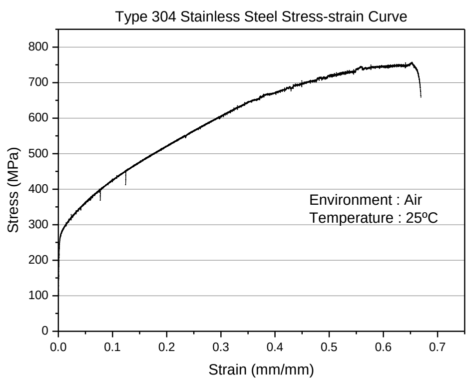

Fig. 9. Stress-strain curve of Type 304 stainless steel

Table IV: Mechanical properties of Type 304 stainless steel (STS 304)

Materials	STS 304
0.2 % Offset Yield Strength (MPa)	264.4
Ultimate Tensile Strength (MPa)	757.2
Elastic Modulus (GPa)	178.9
Elongation (%)	66.93

2.5 Fatigue Test Results

Fatigue test results are shown in Fig. 10. The test was carried out until the specimen was split into two pieces. ΔK and da/dN results are plotted on a log scale. Data results that satisfy the elastic condition and linear region are selected and fitted by Paris' law. Fitting results are shown in Fig. 11 and the following equation.

$$\frac{da}{dN} = C \times (\Delta K)^m = 1.56 \times (10)^{-10} \times (\Delta K)^{3.94} \qquad (6)$$

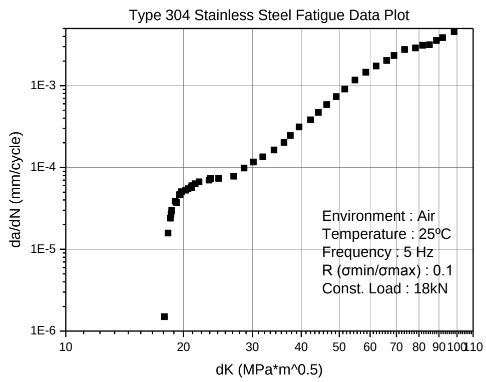

Fig. 10. Fatigue crack growth rate of Type 304 stainless steel at a constant load ΔP=18kN

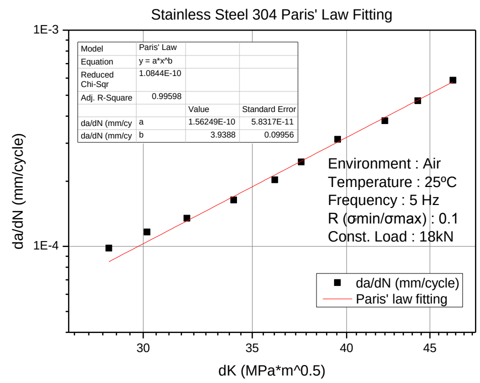

Fig. 11. Fatigue crack propagation data and Paris' law of Type 304 stainless steel at a constant load ΔP=18kN

The constant $\Delta K = 30 MPa\sqrt{m}$ test results are shown in Fig. 12. There are 36 data results and the mean crack growth rate is $8.40 \times 10^{-5}\, mm/cycle$ with standard deviation as 1.54×10^{-5}. In this test, DCPD crack length and visual measured crack length are compared with the results shown in Fig. 13. A linear fitting result shows that the slope is 1.00 and $R^2 = 0.99$, so the DCPD measured crack length very well. Fig. 14 and Fig. 15 show pictures of notch and crack with travelling microscope.

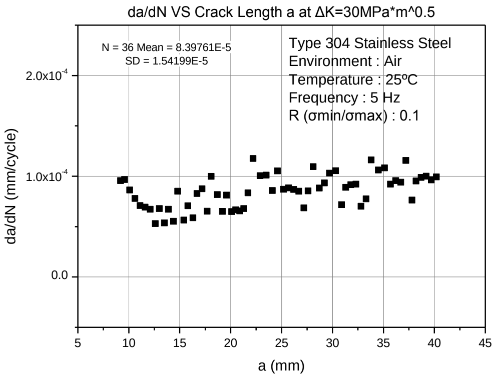

Fig. 12. Fatigue crack propagation data of Type 304 stainless steel at a constant $\Delta K = 30MPa\sqrt{m}$

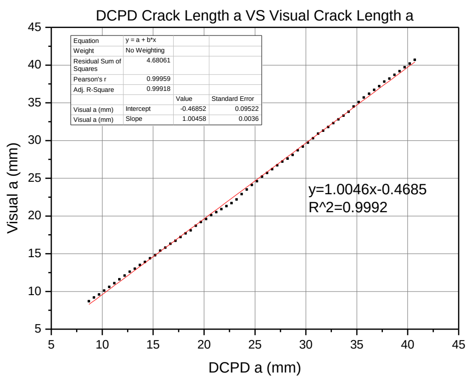

Fig. 13. DCPD crack length a versus visual crack length a

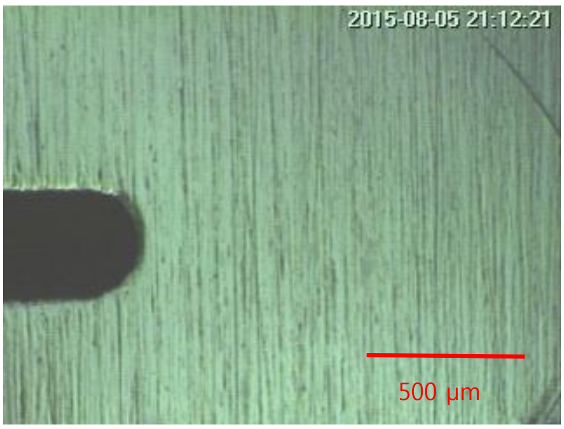

Fig. 14. Picture of notch with travelling microscope

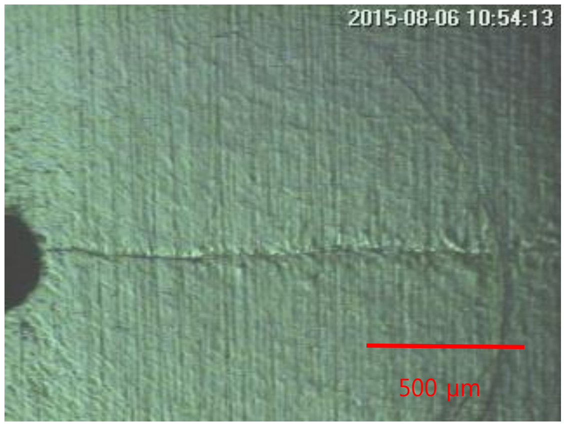

Fig. 15. Picture of crack with travelling microscope

2.6 Bayesian Updating

The following equations are deduced from Bayes' theorem.

$$f(C,m\,|\,\dot{a}) = kL(\mathrm{C,m}\,|\,\dot{a})\,f(\mathrm{C,m}) \qquad (7)$$

The unknown parameters are Paris' law constant C and m. These unknown parameters will be updated with fatigue crack growth rate results at a constant $\Delta K = 30MPa\sqrt{m}$ where C and m are assumed normal distributions. Therefore, the probability distribution function of C and m is like equation (8) while the cumulative distribution function is like equation (9).

$$f(C,m) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(C,m-\mu_{C,m})^2}{2\sigma_{C,m}{}^2}} \qquad (8)$$

$$F(C,m) = \Phi\left(\frac{C,m-\mu_{C,m}}{\sigma_{C,m}}\right) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{C,m-\mu_{C,m}}{\sigma_{C,m}\sqrt{2}}\right)\right] \quad (9)$$

$$\textit{Where } \Phi(x) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{x}{\sqrt{2}}\right)\right], \mathrm{erf}(x) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x} e^{-t^2}dt$$

Where σ is the standard deviation of C and m, μ is the mean of C and m.

When m is fixed, 100,000 unknown parameter C were sampled with equation (9) using MATLAB. This probability distribution function is shown in Fig. 16. Constant m can sample in the same way when C is fixed.

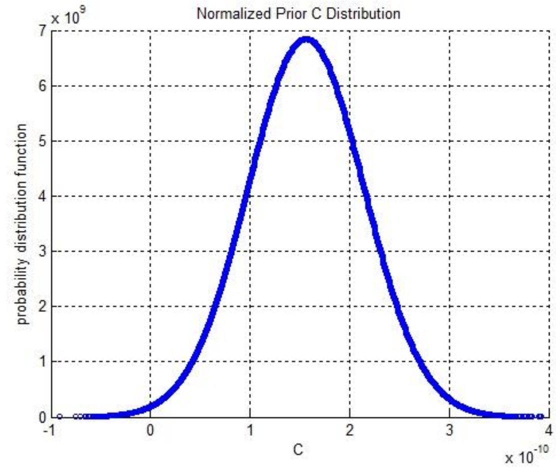

Fig. 16. Normalized prior probability distribution of constant C (when m is fixed)

$L(C,m\,|\,\dot{a}) = f(\dot{a}\,|\,C,m)$ is defined as a normal distribution of $x_i - \dot{a}$ with $N(0,\sigma^2)$. x_i is the i-th crack growth rate data obtained from a constant ΔK test. The normalized likelihood distribution is shown as Fig. 17. [10]

$$L(C,m\,|\,\dot{a}) = \prod_{i=1}^{N} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[(x_i - \dot{a}(\mathrm{C,m})) - 0]^2}{2\sigma^2}} \qquad (10)$$

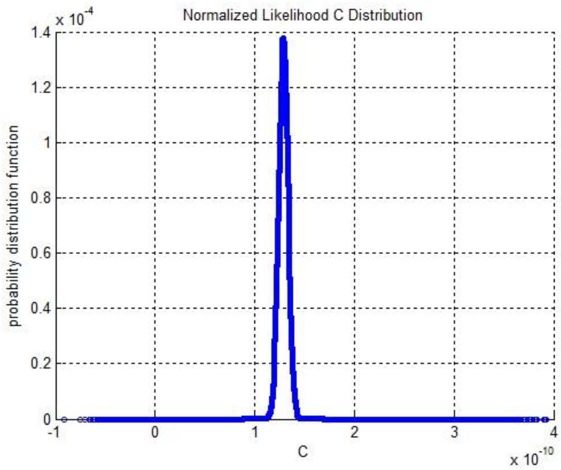

Fig. 17. Normalized likelihood distribution of constant C (when m is fixed)

Therefore, the posterior distribution is shown in Fig. 18 and the updated Paris' law graph is in Fig. 19. These results show that an unknown parameter's standard deviation is reduced considerably. The updated constant C and m is summarized in Table Ⅴ.

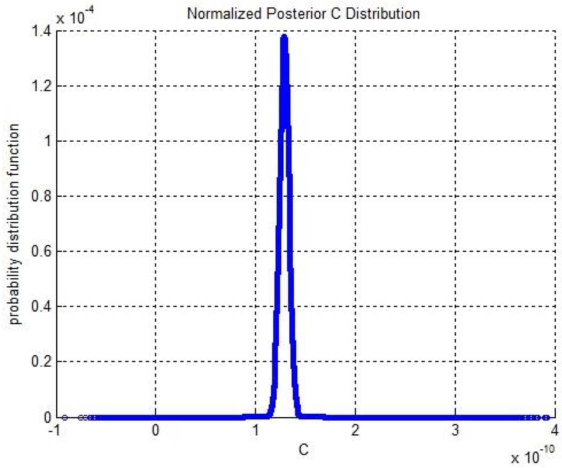

Fig. 18. Normalized posterior probability distribution of constant C (when m is fixed)

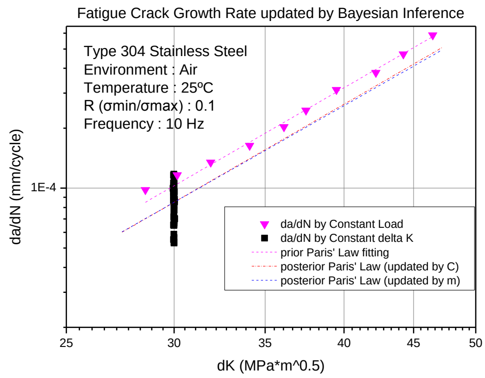

Fig. 19. Paris' law updated by Bayesian inference

Table V. Paris' law constant C and m

	C	
	Mean	STD
Prior	1.5625×10^{-10}	5.8317×10^{-11}
Sampled Prior	1.5626×10^{-10}	5.8337×10^{-11}
Posterior (m fixed)	1.2923×10^{-10}	2.1761×10^{-23}
Posterior (C fixed)	1.5626×10^{-10}	5.8337×10^{-11}
	m	
	Mean	STD
Prior	3.9388	9.9560×10^{-2}
Sampled Prior	3.9388	9.9562×10^{-2}
Posterior (m fixed)	3.9388	9.9562×10^{-2}
Posterior (C fixed)	3.8834	1.1213×10^{-4}

3. Conclusions

In this paper, Paris' law constants including C and m for Type 304 stainless steel were determined by probabilistic approach with Bayesian Inference. The Bayesian update process is limited in accuracy, because this method should assume initial data distribution. If we select an appropriate distribution, this updating method is powerful enough to get data results considering the environment and materials.

Until now, remaining lives of NPPs are estimated by deterministic methods using a priori model to finally assess structural integrity. Bayesian approach can utilize in-service inspection data derived from aged properties. Therefore, a probabilistic method should be applied to consider the environment and material conditions.

This method should be especially useful in estimating the remaining life of weld region of the NPPs because it is affected considerably by the environment. In future experiments, a reliability assessment for the weld region would be conducted.

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