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Model Heater Design for the Experimental Simulation of Nuclear Reactor Core

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1. Introduction

For the replication of the prototypic behaviors into the small scaled model test facility, similarity analysis is usually used. In the nuclear system, there have been developed several scaling laws for the design of small test facility, and they provides approximate or relatively good design methods. Among them the approach of Ishii et al. [1] is known to give much flexibility in the design of test facility. Just like the other scaling laws, this approach also provides several global dimensionless groups that should be conserved in model. Based on these global similarities, the bottom approach or local scaling is additionally conducted.

The major phenomena in the transient of nuclear system are concerned not only with fluid but also with solid structures. In particular, the core, as a principal heat source of the system, is of important solid structure. In the prototypic nuclear system, the fuel of the core is mainly metal uranium or oxide uranium, whereas the model heating part is usually composed of electrical heater. Moreover, the heating rod both in the prototype and the model are not homogeneous material, such that the nuclear fuel is composed of uranium (or oxide uranium), cladding, and gap material, and commercial electric heater is composed of sheath material, heating element, and insulating material. And the core materials of solid in both systems are different each other. All of these make the scaling analysis abstruse, because the past scaling laws discussed only the homogeneous material. Even more, it looks more difficult to derive a new similarity relation for heat structure of the heterogeneous materials. In spite of such difficulties in the core similarity analysis, the surface temperature of fuel or heater is of great importance as a figure of merit in nuclear safety.

This study deals with the scaling analysis of heating part and design of electric heater for the conservation of surface temperature. In order to overcome the problem of heterogeneous material composition, equivalent properties formula was firstly derived. And based on this formula, the commercial heater was assessed in order to conserve the surface temperature. In the past works this subject seems to be rarely investigated. And the similarity analysis related with these equivalent thermal properties has not been intensively tried.

2. Equivalent Property Formula

2.1 Concept of Equivalent Thermal Conductivity in Slab System in the Absence of Heat Generation

Fig. 1 shows the schematics of the concept of the equivalent thermal conductivity in slab system where there is no source. The thermal diffusions, which is expressed as the temperature difference between left side and right side, are expected same each other, even though the local temperature profiles inside are different.

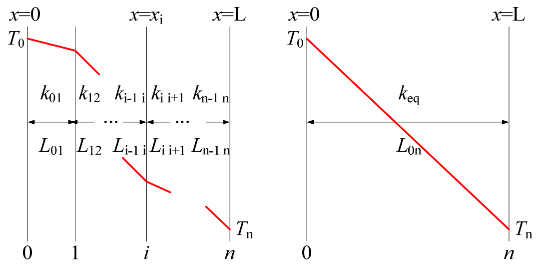

Fig. 1. Schematics of equivalent thermal conductivity in slab system in the absence of heat generation.

One-dimensional heat conduction equation for fuel region, under the assumption of the dominant heat diffusion along the x direction, is given by

$$k\frac{d^2T}{dx^2}=0 \tag{1}$$

, where k, T, and x is thermal conductivity (W/m-K), temperature (K), and coordinate (m). The boundary conditions are imposed by

$$\begin{aligned} &T=T_0 \ \ at \ \ x=x_0 \\ &T=T_1 \ \ at \ \ x=x_1 \\ &\ldots \\ &T=T_i \ \ at \ \ x=x_i \\ &\ldots \\ &T=T_n \ \ at \ \ x=x_n \end{aligned} \tag{2}$$

The solution for i-th section of the width $L_{i\,i+1}$ is

$$T(x)=(T_i-T_{i-1})\frac{x}{L_{i-1\,i}}+T_{i-1} \tag{3}$$

Thus, the heat flux at x=x$_i$, $\dot{q}_i''$, is given by

$$\dot{q}_i'' = -k_{i-1\,i}\frac{dT}{dx} = \frac{k_{i-1\,i}}{L_{i-1\,i}}\left(T_{i-1}-T_i\right) \quad (4)$$

Or

$$\left(T_{i-1}-T_i\right) = \frac{L_{i-1\,i}}{k_{i-1\,i}}\dot{q}_i'' \quad (5)$$

For all sections, temperature differences between left and right surfaces can be similarly obtained. The heat flux at any location is same each other, i.e.,

$$\dot{q}_1'' = \dot{q}_2'' = \cdots = \dot{q}_i'' = \cdots = \dot{q}_n'' \equiv \dot{q}'' \quad (6)$$

Summing up all the temperature differences yields

$$\left(T_0-T_n\right) = \left(\frac{L_{01}}{k_{01}} + \frac{L_{12}}{k_{12}} + \cdots + \frac{L_{i-1\,i}}{k_{i-1\,i}} + \cdots + \frac{L_{n-1\,n}}{k_{n-1\,n}}\right)\dot{q}'' \quad (7)$$

On the other hand, for the equivalent slab shown in Fig. 1 (b), following relation can be obtained

$$\left(T_0-T_n\right) = \frac{L_{0n}}{k_{eq}}\dot{q}'' \quad (8)$$

Thus, the equivalent thermal conductivity is

$$\frac{1}{k_{eq}} = \sum_{i=1}^{n}\frac{L_{i-1i}}{L_{0n}}\frac{1}{k_{i-1i}} \quad (9)$$

As shown in above equation, the equivalent thermal conductivity is a harmonic average of all thermal conductivities with the weighting of the width.

2.2 Concept of Equivalent Thermal Conductivity in Slab System in the Presence of Heat Source

If heat is generated in an section of the slab, the temperature at somewhere of the section is highest, and the temperature decreases linearly according to the distance from the heat generation section. So, in this case the temperature difference between the left and right of the slab cannot mean the appropriate temperature gradient depending on the thermal conductivities. However, considering the symmetric condition as Fig. 2, the temperature difference between centerline and surface has its own meanings.

For the first section the heat conduction equation of Eq. (1) is applied. The general solution for this equation is

$$T\left(x\right) = C_1 x + C_2 \quad (12)$$

, where C$_1$ and C$_2$ are integral constants that should be determined by boundary conditions. Following boundary conditions are applied for this section.

$$\frac{\partial T}{\partial x} = 0 \;\; at \;\; x = x_0$$
$$T = T_1 \;\; at \;\; x = x_1 \quad (13)$$

Thus, the final solution is

$$T\left(x\right) = T_1 \qquad for \;\; x_0 \le x \le x_1 \quad (14)$$

For the second section the applied boundary conditions are as follows.

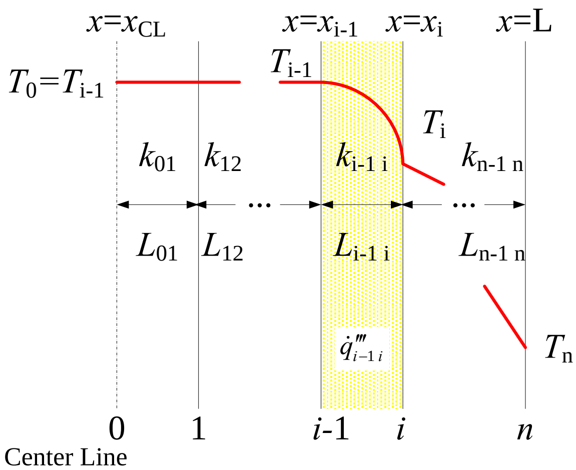

Fig. 2. Schematics of symmetric slab system in the presence of heat generation.

$$T = T_1 \;\; at \;\; x = x_1$$
$$T = T_2 \;\; at \;\; x = x_2 \quad (15)$$

And the final solution is

$$T\left(x\right) = \left(\frac{T_1 - T_2}{x_1 - x_2}\right)\left(x - x_1\right) + T_1 \quad for \;\; x_1 \le x \le x_2 \quad (16)$$

The heat flux at x=x$_1$ can be obtained from both Eq.(14) and Eq. (16), these two valued should be same.

$$\dot{q}''\left(x_1\right) = 0 = -k_{12}\frac{T_1 - T_2}{x_1 - x_2} \quad (17)$$

Thus, both the interface temperatures are same

$$T_1 = T_2 \quad (18)$$

And Eq. (16) becomes

$$T\left(x\right) = T_1 \qquad for \;\; x_1 \le x \le x_2 \quad (19)$$

For the other sections, the same process can be applied, and it is found that all the interface temperatures are same.

$$T_1 = T_2 = \cdots = T_{i-1} \quad (20)$$

For the sections outside the heating section, the same process can be applied. For the most outer section the temperature profile is given by

$$T\left(x\right) = \left(\frac{T_{j-1} - T_j}{x_{j-1} - x_j}\right)\left(x - x_j\right) + T_j \quad (21)$$
$$for \;\; x_{j-1} \le x \le x_j \;\; and \;\; i+1 \le j \le n$$

The heat flux at x=x$_j$ is given by

$$\dot{q}_j'' = -k_{j-1\,j}\left.\frac{dT}{dx}\right|_{x=x_j} = \frac{k_{j-1\,j}}{L_{j-1\,j}}\left(T_{j-1}-T_j\right) \quad (22)$$

Or

$$\left(T_{j-1}-T_j\right) = \frac{L_{j-1\,j}}{k_{j-1\,j}}\dot{q}_j'' \quad (23)$$

For the heat generation section, $x_{i-1} \le x \le x_i$, the heat conduction equation becomes a heat source added form from Eq. (1).

$$k\frac{d^2T}{dx^2}+\dot{q}'''_{i-1\,i}=0 \quad (24)$$

, where $\dot{q}'''_{i-1\,i}$ is volumetric hear source (W/m^3) of constant value. Boundary conditions are as follow.

$$T=T_{i-1}\ \ at\ \ x=x_{i-1}$$
$$T=T_i\ \ at\ \ x=x_i \quad (25)$$

The solution is

$$T(x)-T_i=-\frac{\dot{q}'''_{i-1\,i}}{2k_{i-1\,i}}\left\{x^2-\left(x_{i-1}+x_i\right)x+x_{i-1}x_i\right\}+\left(\frac{T_{i-1}-T_i}{x_{i-1}-x_i}\right)\left(x-x_i\right) \quad (26)$$

Thus, the heat flux at x=x$_i$, $\dot{q}''_i$, is given by

$$\dot{q}''_i=-k_{i-1\,i}\left.\frac{dT}{dx}\right|_{x=x_i}=\frac{\dot{q}'''_{i-1\,i}}{2}L_{i-1\,i}+k_{i-1\,i}\left(\frac{T_{i-1}-T_i}{L_{i-1\,i}}\right) \quad (27)$$

And this heat flux is equated with the heat generation as follow.

$$\dot{q}''_iA_i=\dot{q}'''_{i-1\,i}L_{i-1\,i}A_i \quad (28)$$

, where A_i is cross-sectional area at x=x$_i$. From Eq. (27) and Eq. (28), the temperature difference across the width $L_{i-1\ i}$ is

$$T_{i-1}-T_i=\frac{L_{i-1\,i}}{2k_{i-1\,i}}\dot{q}''_i \quad (29)$$

Here all the heat fluxes at the interface are same each other.

$$\dot{q}''_i=\dot{q}''_j \quad (30)$$

Summing all the temperature differences from i-th section to n-th section yields (see Eq. (23) and Eq. (29))

$$T_{i-1}-T_n=\left(\frac{L_{i-1\,i}}{2k_{i-1\,i}}+\sum_{j=i+1}^{n}\frac{L_{j-1\,j}}{k_{j-1\,j}}\right)\dot{q}''_i \quad (31)$$

Here, let's introduce an equivalent slab. The left and right surfaces of the slab are located at x_{i-1} and x_n, respectively, with the consideration that the inner parts of heating sections, which are the same temperature, have meaningless temperature gradient. Its equivalent thermal conductivity is assumed k_{eq}. The total generated heat is same to the heat generation of i-th section in Fig. 2, and the heat is generated uniformly over the entire equivalent slab.

$$\dot{q}'''_{i-1\,i}L_{i-1\,i}A_i=\dot{q}'''_{i-1\,n}L_{i-1\,n}A_n \quad (32)$$

Then, the temperature difference can be obtained with referring to Eq. (29).

$$T_{i-1}-T_n=\frac{L_{i-1\,n}}{2k_{eq}}\dot{q}''_n \quad (33)$$

Of course, the heat flux at x=x$_n$ is same to that at x=x$_j$. Thus, from Eq. (31) and Eq. (33), following equivalent thermal conductivity is derived.

$$\frac{1}{k_{eq}}=\left(\frac{L_{i-1\,i}}{L_{i-1\,n}}\frac{1}{k_{i-1\,i}}+\sum_{j=i+1}^{n}\frac{2L_{j-1\,j}}{L_{i-1\,n}}\frac{1}{k_{j-1\,j}}\right) \quad (34)$$

It should be noticed that the effective equivalent thermal conductivity is not related with the inner part of the heating section, as the inner parts have uniform temperature, so the temperature difference is present in the heating section and its outer parts. The contribution of thermal conductivity of non-heating section is a half of that of heating section.

2.3 Concept of Equivalent Volumetric Heat Capacity

Volumetric heat capacity is related with the rate of temperature rise. So the equivalent volumetric heat capacity can be obtained by this.

$$\left(\rho C_pV\right)_{eq}\Delta T=\sum_i\left(\rho_iC_{p,i}V_i\right)\Delta T \quad (35)$$

, where ρ, C_p, V, and ΔT are density, specific heat, volume, and temperature rise, respectively. Therefore, the equivalent volumetric heat capacity is

$$\left(\rho C_p\right)_{eq}=\sum_i\left(\rho_iC_{p,i}\frac{V_i}{V}\right) \quad (36)$$

2.4 Equivalent Thermal Conductivity in Cylindrical System in the Absence of Heat Source

The heat conduction along the radial direction is assumed to dominate in cylindrical system. Then the heat conduction equation is as follow.

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)=0 \quad (37)$$

, where r is radial coordinate. Boundary conditions are as follow.

$$\frac{\partial T}{\partial r}=0\ \ at\ \ r=0$$
$$T=T_0\ \ at\ \ r=r_0 \quad (38)$$

, where subscript 0 means outer surface. The solution is

$$T(r)=T_0$$

It means uniform temperature. As the inside temperature of the rod cannot be artificially controlled as another temperature, it is not practical to impose anther temperate inside the rod. For a rod of multi layers, the solution is same, uniform temperature profile. In such case the thermal conductivity does not affect the temperature profile, thus the equivalent thermal conductivity cannot be defined. Even for multi layers rode, the result is same.

2.5 Equivalent Thermal Conductivity in Cylindrical System in the Presence of Heat Source

2.5.1 In case that the heat source at core: Nuclear fuel

The nuclear fuel rod is composed of uranium metal or oxide uranium as a heating section, gap, and cladding, in the order from center to outside as shown in Fig. 3. Governing equation is

$$q''' + \frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0 \qquad (39)$$

And general solution is

$$T(r) = -\frac{q'''}{4k}r^2 + C_1 \ln r + C_2 \qquad (40)$$

, where C_1 and C_2 are integral constants to be determined by boundary conditions.

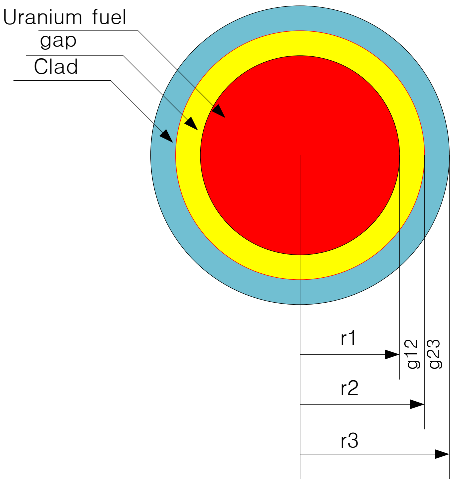

Fig. 3. Composition of nuclear fuel.

From center to r_1: Heat source is present

Boundary conditions are given by

$$\left.\frac{dT}{dr}\right|_{r=0} = 0$$

$$T(r_1) = T_1 \qquad (41)$$

And the solution is

$$T(r) = T_1 - \frac{\dot{q}'''_{01}}{4k_{01}}\left(r^2 - r_1^2\right) \qquad (42)$$

Thus, the temperature difference between center and r_1 is

$$T_0 - T_1 = \frac{\dot{q}'''_{01}}{4k_{01}} r_1^2 \qquad (43)$$

From r_1 to r_2: No heat source

Boundary conditions are

$$T(r)\big|_{r=r_2} = T_2$$

$$T(r)\big|_{r=r_1} = T_1 \qquad (44)$$

The solution is

$$T(r) = \frac{T_2 - T_1}{\ln(r_2/r_1)}\ln(r/r_2) + T_2 \qquad (44)$$

The total heat flow at r_2 is

$$\dot{q}\big|_{r=r_2} = -k_{12}A_2\left[\frac{dT}{dr}\right]_{r=r_2}$$

$$= -k_{12}(2\pi r_2 L)\left[\frac{T_2 - T_1}{\ln(r_2/r_1)}\frac{1}{r_2}\right] \qquad (45)$$

$$= \dot{q}'''_{01}\left(\pi r_1^2 L\right)$$

And temperature difference is

$$T_1 - T_2 = \frac{\dot{q}'''_{01} r_1^2 \ln(r_2/r_1)}{2k_{12}} \qquad (46)$$

From r_2 to r_3: No heat source

Boundary conditions are

$$T(r)\big|_{r=r_3} = T_3$$

$$T(r)\big|_{r=r_2} = T_2 \qquad (47)$$

And the solution is

$$T(r) = \frac{T_3 - T_2}{\ln(r_3/r_2)}\ln(r/r_3) + T_3 \qquad (48)$$

The total heat flow at r_3 is

$$\dot{q}\big|_{r=r_3} = -k_{23}A_3\left[\frac{dT}{dr}\right]_{r=r_3}$$

$$= -k_{23}(2\pi r_3 L)\left[\frac{T_3 - T_2}{\ln(r_3/r_2)}\frac{1}{r_3}\right] \qquad (49)$$

$$= \dot{q}'''_{01}\left(\pi r_1^2 L\right)$$

Thus, the temperature difference

$$T_2 - T_3 = \frac{\dot{q}'''_{01} r_1^2 \ln(r_3/r_2)}{2k_{23}} \qquad (50)$$

Summing all the temperature differences

$$T_0 - T_3 = \dot{q}'''_{01}\left[\frac{r_1^2}{4k_{01}} + \frac{r_1^2 \ln(r_2/r_1)}{2k_{12}} + \frac{r_1^2 \ln(r_3/r_2)}{2k_{23}}\right] \qquad (51)$$

Equivalent fuel rod: Uniform heat generation

The equivalent fuel rod is the rod whose diameter is same to the original one. And the heat is generated uniformly over the entire rod, and the total generated heat is same to original one.

Boundary conditions are

$$\left.\frac{dT}{dr}\right|_{r=0} = 0$$

$$T(r_3) = T_3 \qquad (52)$$

Thus, the temperature difference is

$$T_0 - T_3 = \frac{\dot{q}'''_{eq} r_3^2}{4k_{eq}} \quad (53)$$

As mentioned above, the heat generation rate is same to the original one, following relation is obtained.

$$\dot{q}'''_{eq}\left(\pi r_3^2 L\right) = \dot{q}'''_{01}\left(\pi r_1^2 L\right) \quad (54)$$

Thus, from Eq. (51), Eq. (53), and Eq. (54), the equivalent thermal conductivity is

$$\frac{1}{k_{eq}} = \left[\frac{1}{k_{01}} + \frac{2\ln\left(r_2/r_1\right)}{k_{12}} + \frac{2\ln\left(r_3/r_2\right)}{k_{23}}\right] \quad (55)$$

As in the case of slab, the contribution of non-heat generation section is a half of the heat generation section.

2.5.2 In case that the heat source at annulus: Electric heater

Electrical heat is somewhat different since the location of the heating section is not the center, as shown in Fig. 4.

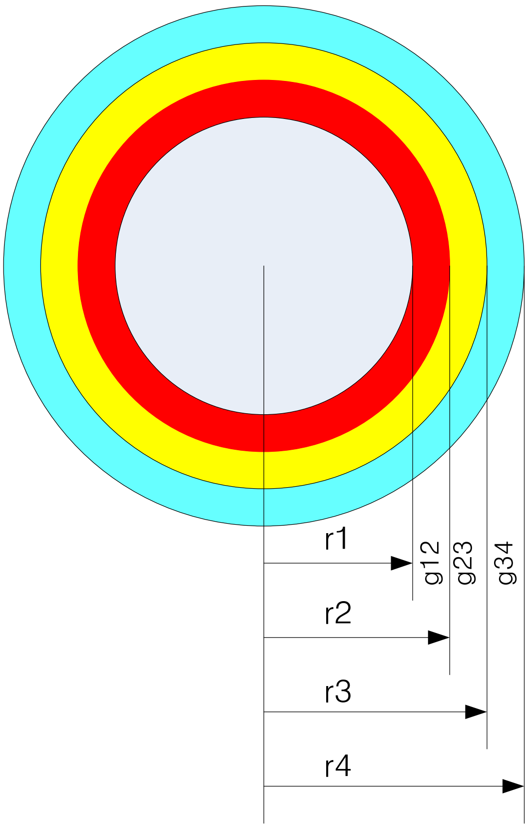

Fig. 4. Composition of electric heater.

The composition of electrical heater is core insulating material at the core, heating element, insulating material, and sheath in order for center to outside. The applied equation is same to that of nuclear fuel. Only the boundary conditions are slightly different.

<u>From center to r_1: No heat source</u>

Boundary conditions of this section are

$$\left.\frac{dT}{dr}\right|_{r=0} = 0$$

$$T(r_1) = T_1 \quad (56)$$

And the solution is

$$T(r) = T_1 \quad (57)$$

So, temperature difference is

$$T_0 - T_1 = 0 \quad (58)$$

<u>From r_1 to r_2: Heat source is present</u>

Boundary conditions are

$$T(r_1) = T_1$$

$$T(r_2) = T_2 \quad (59)$$

And the solution is

$$T(r) = -\frac{q'''_{12}}{4k_{12}}\left(r^2 - r_1^2\right) + \frac{\left(T_2 - T_1\right) + \frac{q'''_{12}}{4k_{12}}\left(r_2^2 - r_1^2\right)}{\ln\left(r_2/r_1\right)}\ln\left(r/r_1\right) + T_1 \quad (60)$$

The total heat flow at r_2 is

$$q|_{r=r2} = -k_{12}A_2\left[\frac{dT}{dr}\right]_{r=r_2}$$

$$= -k_{12}\left(2\pi r_2 L\right)\left[-\frac{q'''_{12}}{4k_{12}}\left(2r_2\right) + \frac{\left(T_2 - T_1\right) + \frac{q'''_{12}}{4k_{12}}\left(r_2^2 - r_1^2\right)}{\ln\left(r_2/r_1\right)}\frac{1}{r_2}\right] \quad (61)$$

$$= q'''_{12}\left\{\pi\left(r_2^2 - r_1^2\right)L\right\}$$

Thus, the temperature difference

$$T_1 - T_2 = \frac{q'''_{12}}{4k_{12}}\left[\left(r_2^2 - r_1^2\right) - 2r_1^2\ln\left(r_2/r_1\right)\right] \quad (62)$$

<u>From r_2 to r_3 and from r_3 to r_4: No heat source</u>

These two sections have the same form of solution and they are treated at a time. Boundary conditions are

$$T(r)\big|_{r=r_2} = T_2$$

$$T(r)\big|_{r=r_3} = T_3 \quad (63)$$

$$T(r)\big|_{r=r_3} = T_3$$

$$T(r)\big|_{r=r_4} = T_4 \quad (64)$$

Solutions are

$$T(r) = \frac{T_3 - T_2}{\ln(r_3/r_2)} \ln(r/r_3) + T_3 \quad (65)$$

$$T(r) = \frac{T_4 - T_3}{\ln(r_4/r_3)} \ln(r/r_4) + T_4 \quad (66)$$

Temperature difference by heat flow also can be obtained by the same method above.

$$T_2 - T_3 = \frac{q'''_{12}\left(r_2^2 - r_1^2\right)\ln(r_3/r_2)}{2k_{23}} \quad (67)$$

$$T_3 - T_4 = \frac{q'''_{12}\left(r_2^2 - r_1^2\right)\ln(r_4/r_3)}{2k_{34}} \quad (68)$$

Summing all the temperature differences yields

$$T_0 - T_4 = q'''_{12}\left[\frac{1}{4k_{12}}\left\{\left(r_2^2 - r_1^2\right) - 2r_1^2\ln(r_2/r_1)\right\} + \frac{\left(r_2^2 - r_1^2\right)\ln(r_3/r_2)}{2k_{23}} + \frac{\left(r_2^2 - r_1^2\right)\ln(r_4/r_3)}{2k_{34}}\right] \quad (69)$$

Equivalent electrical heater: Uniform heat generation

Through the same processes above the temperature difference for equivalent electrical heater rode can be obtained.

$$T_0 - T_4 = \frac{q'''r_4^2}{4k_{eq}} \quad (70)$$

The same heat generation rate is same each other and

$$q'''_{eq}\left(\pi r_4^2 L\right) = q'''_{12}\left\{\pi\left(r_2^2 - r_1^2\right)L\right\} \quad (71)$$

Therefore, the equivalent thermal conductivity is correlated as follow.

$$\frac{1}{k_{eq}} = \left[\frac{1}{k_{12}}\left\{1 - \frac{2r_1^2\ln(r_2/r_1)}{\left(r_2^2 - r_1^2\right)}\right\} + \frac{2\ln(r_3/r_2)}{k_{23}} + \frac{2\ln(r_4/r_3)}{k_{34}}\right] \quad (72)$$

This equation indicates that the thermal conductivity of the core insulation material does not influence the equivalent thermal conductivity, but that the size of core insulation material contributes to it.

3. Similarity Analysis in Heat Conduction if Fuel Rod

The heat conduction equation in 1 fuel rod can be written as follow.

$$\rho_s c_{p,s}\frac{\partial T_s}{\partial t} + k_s\nabla^2 T_s = \frac{\dot{q}_t}{N_f\left(\frac{\pi}{4}\delta_f^2 l_f\right)} \quad (73)$$

, where $\dot{q}_t$, N_f , δ_f , and l_f , are total core heat generation rate, total number of fuel rod, fuel rod diameter, and fuel rod length, respectively, and subscript s means structure. Let's introduce following non-dimensionalizing variables

$$\tau = \frac{tu_0}{l_0},\ \theta = \frac{\Delta T}{\Delta T_0}\left(= \frac{T_{out} - T}{T_{out} - T_{in}}\right),\ \nabla^{*2} = \delta^2\nabla^2 \quad (74)$$

, where u_0 and l_0 are reference velocity and length that have been used in fluid part non-dimensionalization. Thus, following dimensionless equation is obtained.

$$\frac{\partial\theta_s}{\partial\tau} + T^*\nabla^{*2}\theta_s - Q_s = 0 \quad (75)$$

In above equation following dimensionless numbers are used.

Time ratio number $T^* \equiv \left(\frac{l_0/u_0}{\delta^2/\alpha_s}\right)$ (76)

Heat source number $Q_s = \frac{\dot{q}_t}{\rho_s c_{p,s} u_0 \Delta T_0 N_f\left(\frac{\pi}{4}\delta_f^2\frac{l_f}{l_0}\right)}$

(77)

The conservation of time ratio number leads to the constraint for the fuel rod diameter.

$$\delta_R = \alpha_{s,R}^{1/2} l_{0,R}^{1/4} \quad (78)$$

, where α_s is thermal diffusivity ($k_s/\rho_s c_{p,s}$), and the subscript R means the ratio of model to prototype. As shown in this equation, the equivalent thermal properties are used to determined the fuel diameter.

Heat generation rate scale should be same to the flowrate scale according to Ishii et al..

$$\dot{q}_R = \dot{m}_R = \rho_R u_{0,R} a_{0,R} = a_{0,R} l_{0,R}^{1/2} \quad (79)$$

Based on this relation, the conservation of heat source number in Eq. (77) result in following rod number relation.

$$N_{f,R} = \frac{l_{0,R}^{3/2}}{k_{s,R}} \quad (80)$$

4. Equivalent Property in Prototypic Nuclear Fuel

Based on the concepts in section 2.2 and 2.3, the equations for the equivalent properties such as equivalent thermal conductivity and equivalent volumetric heat capacity can be derived as shown in section 2.5. These equations are applied to calculate the equivalent properties of the prototypic nuclear fuel, uranium metal fuel and oxide fuel as following sections.

4.1 Uranium Metal Fuel

As a fuel in the prototypic nuclear system, the equivalent properties of uranium metal fuel were calculated. The uranium fuel is composed with U for fuel, sodium in the gap, and specification of uranium metal fuel is in Table I.

Table I: Specification and Properties of Uranium Metal Fuel

	U-fuel	Sodium-Gap	Cladding
Diameter [m]	6.98E-3	0	7.406E-3
Radius [m]	3.49E-3	0	3.703E-4
Thickness [m]	3.49E-3	0	2.13E-4
Length [m]	0.976		
Heat conductivity *k* [W/m-K]	38.8	65.88	20.5
Specific heat capacity c_p [J/kg-K]	176	1260	546
Density ρ [kg/m^3]	19070	825.6	7363.887
Volume [m^3]	3.735E-5	0	4.680E-6

In the table, the gap thickness is assumed to 0 m as fuel-cladding contact due to the swelling and thermal expansion of fuel. For that reason, the gap conductance was not considered. Moreover, the properties are in the case when the fuel temperature is at 800 K.

Applying the equations for the equivalent properties, the equivalent thermal conductivity k_{eq} and equivalent volumetric heat capacity $(\rho c_p)_{eq}$ were obtained and shown in the following Table II.

Table II: Equivalent Properties

	Uranium Metal Fuel
k_{eq} [W/m-K]	31.693
$(\rho c_p)_{eq}$ [J/m^3-K]	3164570

4.2 Oxide Fuel

As well as the uranium metal fuel, the equivalent properties of the oxide fuel are obtained and shown in the following Table III. Particularly, the gap conductance was considered in the calculation for the equivalent thermal conductivity. In the gap, the He gas was filled with the pressure at 3 atm. Thus, the following equation was applied to obtain the equivalent thermal conductivity.

$$\frac{1}{k_{eq}} = \left[\frac{1}{k_{01}} + \frac{2}{(r_2 h_{12})} + \frac{2\ln(r_3/r_2)}{k_{23}} \right] \quad (81)$$

In order to get the gap heat transfer coefficient h_{12} into above equation, the equations in the Westinghouse procedure was accepted to use whichever of the following two equations yields the larger gap conductance.

$$h = \left[\frac{k_g}{\delta/2 + 14.4\times10^{-6}} \right] \quad [\text{Btu/hr - ft}^2\text{-}^\circ\text{F}] \quad (82)$$

and

$$h = 1500\,k_g + \frac{4.0}{0.006 + 12\delta} \quad [\text{Btu/hr - ft}^2\text{-}^\circ\text{F}] \quad (83)$$

where k_g is thermal conductivity of the gas in the gap and δ is the gap width. This heat transfer coefficient is also shown in the Table III. Likewise with the uranium metal fuel, the properties at 800 K were used to calculate the equivalent properties.

Table III. Specification and Properties of Oxide Fuel

	UO_2-fuel	He-Gap	Cladding
Diameter [m]	8.268E-3	8.424E-3	9.694E-3
Radius [m]	4.134E-3	4.212E-3	4.847E-3
Thickness [m]	4.134E-3	7.800E-5	6.350E-4
Length [m]	0.976		
Heat conductivity *k* [W/m-K]	3.469	0.360	21.5l
Specific heat capacity c_p [J/kg-K]	311.54	5193	360
Density ρ [kg/m^3]	10729.2	0.144	6382
Volume [m^3]	5.240E-5	1.996E-6	1.764E-5
Heat transfer coefficient *h* [W/m^2-K]	-	21788.48	-

The equivalent properties of the oxide fuel were calculated in the same way as the uranium metal fuel. The properties are shown in the following Table IV.

Table IV: Equivalent Properties

	Oxide Fuel
k_{eq} [W/m-K]	3.095
$(\rho c_p)_{eq}$ [J/m^3-K]	4610286

5. Review of Thermal Properties for Electric Heater

The exact specification of commercial electrical heater is not exactly known. The assumed specifications are given in Table V. Even though this is assumed one, it is believed to be very similar to the exact one.

For the metal fuel in section 4.1 the required thermal diffusivity ratio according to model electrical heater

Table V: Commercial electric heater specification

diameter[mm]	6	8	10	12	14	16
sheath thickness[mm]	1	1	1	1	1	1
insulator thickness[mm]	1	1	1	1	1	1
nichrome thickness[mm]	0.1	0.1	0.1	0.1	0.2	0.2
core thickness[mm]	0.9	1.9	2.9	3.9	4.8	5.8

diameter is given in Fig. 5.

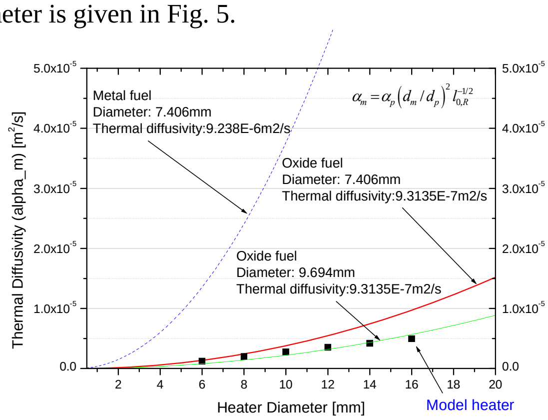

Fig. 5. Required thermal diffusivity ratio according to model electrical heater diameter

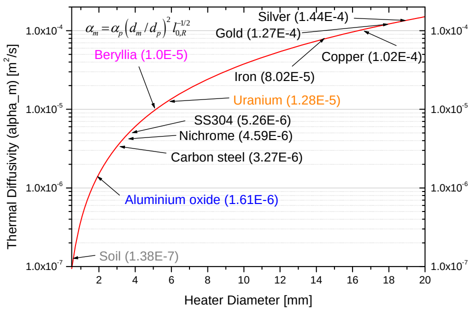

Fig. 6. Required thermal diffusivity and candidate heater material diffusivity

In Fig. 6 the candidate heater material diffusivities are expressed in contrast to the required diffusivity for metal fuel. It seems that the candidate materials are short for the similarity.

6. Design of Model Electric Heater

6.1 Model Electric Heater for Uranium Metal Fuel

The model electric heater with respect to uranium metal fuel is designed with the inhomogeneous heater material as Fig. 4 in the section 2.5.2. In this design, insulating materials are placed in the core and annulus at $g23$. Heating element is located at $g12$ between core and $g23$. Then, sheath is placed at the $g34$. With designing in such geometrical arrangement of the heater materials, the following materials were selected to compose the model heater.

- Insulating material: Al_2O_3
- Heating element: Nichrome wire
- Sheath material: Stainless Steel

The properties of above materials for the model design calculation are in Table VI.

Table VI: Properties of the Model Electric Heater Materials

	Al_2O_3 (insulator)	Nichrome (heat source)	SS (sheath)
Heat conductivity k [W/m-K]	9.989	21	22.8
Specific heat capacity c_p [J/kg-K]	1190	545	585
Density ρ [kg/m^3]	3811	8258.713	8055

Since the equivalent properties depend on sizes of the arranged materials, modeling with scaling methods are to modify the thickness of each material in the calculations of heater diameter, δ_{model}, by matching the calculated and required heat diffusivities, α_{model} to conserve the surface temperature similarity. To design with the surface temperature similarity, the minimum thickness of Nichrome wire was assumed to 0.01 mm. In addition, the insulator thickness at $g23$ was assumed to be always thicker than the Nichrome wire to prevent the electricity conduction to the sheath. The sheath thickness was based on the commercial pipe thickness.

The similarity of the model electric heater in respect to uranium metal fuel with SS sheath was simulated, and the results are in the following Table VII and VIII.

Table VII: Thicknesses of Heater Materials for the Commercial SS Sheath Thickness

	case	Core	Heat Source	Insula-tor	Sheath
		Al_2O_3	NiCr	Al_2O_3	SS
Thick-ness [mm]	1	1.504	0.01	0.01	1.651
	2	2.634	0.01	0.01	2.108
	3	4.222	0.01	0.01	2.108
	4	5.505	0.01	0.01	2.413
	5	6.736	0.01	0.01	2.769

Table VIII. Heat Diffusivity and Diameter of Model Electric Heater

Case	Model Required		Model Calculated	
	α_{model} [m^2/s]	δ_{model} [mm]	α_{model} [m^2/s]	δ_{model} [mm]
1	1.519E-5	6.350	3.243E-6	2.934
2	3.417E-5	9.525	4.113E-6	3.305
3	6.075E-5	12.70	6.000E-6	3.991
4	9.492E-5	15.875	6.706E-6	4.220
5	1.367E-4	19.050	7.095E-6	4.340

From the Table VII and VIII, the similarity with the commercial sheath thickness was analyzed. The insulator thickness margin was not enough to prevent the electricity conduction since the thickness was even reduced to the minimum thickness which is the same thickness with Nichrome wire. Consequently, the calculated heat diffusivity and diameter of the model electric heater could not be matched with the required heat diffusivity and diameter. As a result, the surface temperature similarity was identified to be not conserved.

As a solution, the replacement of the sheath material from Stainless Steel to Copper was considered since it has higher heat conductivity which can give higher heat diffusivity to contribute toward matching the calculated results with the required results. The properties of Copper are in the following Table IX.

Table IX: Properties of Copper for the Model Electric Heater Sheath Material

	Heat conductivity k [W/m-K]	Specific heat capacity c_p [J/kg-K]	Density ρ [kg/m^3]
Copper, *Cu*	366	433	1358

Applying the same conditions with the previous SS sheath calculation, the surface temperature similarity of the model electric heater in respect to uranium metal fuel with Cu sheath was simulated, and the results are in the following Table X and XI.

Table X: Thicknesses of Heater Materials for the Commercial Cu Sheath Thickness

	case	Core	Heat Source	Insula-tor	Sheath
		Al_2O_3	NiCr	Al_2O_3	Cu
Thick-ness [mm]	1	1.241	0.01	0.2735	1.651
	2	2.944	0.01	0.1574	1.651
	3	4.134	0.01	0.0982	2.108
	4	5.456	0.01	0.0590	2.413
	5	6.716	0.01	0.0302	2.769

Table XI: Heat Diffusivity and Diameter of Model Electric Heater

Case	Model Required		Model Calculated	
	α_{model} [m^2/s]	δ_{model} [mm]	α_{model} [m^2/s]	δ_{model} [mm]
1	1.519E-5	6.350	1.519E-5	6.350
2	3.417E-5	9.525	3.417E-5	9.525
3	6.075E-5	12.70	6.075E-5	12.70
4	9.492E-5	15.875	9.492E-5	15.875
5	1.367E-4	19.050	1.367E-4	19.050

From the Table X and XI, unlike a case of the SS sheath material, the insulator thickness margin was fairly enough to prevent the electricity conduction that is the thicker thickness than Nichrome wire. Consequently, the calculated heat diffusivity and diameter of the model electric heater could be matched well with the required heat diffusivity and diameter. As a result, the surface temperature similarity was identified to be conserved.

This Cu sheath has a low chemical reactivity with water, but it has a strong chemical disadvantage with Na in the case of Sodium-cooled Fast Reactor (SFR). Therefore, the SS coating on the Cu sheath surface was considered as shown in Fig. 4 since it can prevent the chemical disadvantages. In the calculation, the coating thickness was assumed 0.01 mm, and the results are in the Table XII and XIII.

Table XII: Thicknesses of the Coated Heater Materials for the Commercial Cu and SS Sheath Thicknesses

	case	Core	Heat Source	Insulator	Sheath	Coat
		Al_2O_3	NiCr	Al_2O_3	Cu	SS
Thick-ness [mm]	1	1.248	0.01	0.2664	1.651	0.01
	2	2.948	0.01	0.1531	1.651	0.01
	3	4.137	0.01	0.0944	2.108	0.01
	4	5.459	0.01	0.0554	2.413	0.01
	5	6.720	0.01	0.0267	2.769	0.01

Table XIII. Heat Diffusivity and Diameter of the Coated Electric Heater

Case	Model Required		Model Calculated	
	α_{model} [m^2/s]	δ_{model} [mm]	α_{model} [m^2/s]	δ_{model} [mm]
1	1.528E-5	6.370	1.528E-5	6.370
2	3.431E-5	9.545	3.431E-5	9.545
3	6.094E-5	12.720	6.094E-5	12.720
4	9.516E-5	15.895	9.516E-5	15.895
5	1.370E-4	19.070	1.370E-4	19.070

From the Table XII and XIII, the insulator thickness margin was good enough to prevent the conduction of electricity as in the Table X and XI of the model electric heater with Cu sheath without coating. Consequently, the calculated heat diffusivity and diameter of the model electric heater were matched well with the required heat diffusivity and diameter. As a result, the surface temperature similarity even considering with SS coating was identified to be conserved.

Copper is not physically suitable for the sheath material since it can be deformed with heating up and being pushed by the inner components of heater because of thermal expansion. Although coating on the sheath surface showed the conservation of similarity, the SS coating thickness is not sufficient to maintain the heater shape. For that reason, the double sheath electric heater was considered since the second sheath is required to be thicker for the heater shape. However, the double sheath is not appropriate due to the gap filled with air between two sheaths. Moreover, the thicker thickness of the SS second sheath is another reason to give lower heat diffusivity caused by its lower equivalent heat conductivity. Intuitively, the double sheath electric heater is not good to consider.

6.2 Model Electric Heater for Oxide Fuel

The model electric heater with respect to oxide fuel was designed as well as the model electric heater for uranium metal fuel. Properly, the composing materials were arranged like Fig. 4 in the previous section 6.1. Likewise, the same materials were selected to compose the model heater such as Al_2O_3 insulators, Nichrome wire, and stainless steel sheath. Furthermore, the surface temperature similarity of heater diameter is considered to conserve in the same way as section 6.1, by matching heat diffusivity. Equally, all of the assumed conditions for the component thicknesses in the section 6.1 was applied in this case.

The surface temperature similarity of the model electric heater in respect to uranium metal fuel with SS sheath was simulated, and the results are in the following Tables XIV and XV.

Table XIV: Thicknesses of Heater Materials for the Commercial SS Sheath Thickness

	case	Core	Heat Source	Insula-tor	Sheath
		Al_2O_3	NiCr	Al_2O_3	SS
Thick-ness [mm]	1	0.708	0.01	0.807	1.651
	2	2.113	0.01	0.532	2.108
	3	3.850	0.01	0.382	2.108
	4	5.432	0.01	0.082	2.413
	5	6.736	0.01	0.01	2.769

Table XV: Heat Diffusivity and Diameter of Model Electric Heater

Case	Model Required		Model Calculated	
	α_{model} [m^2/s]	δ_{model} [mm]	α_{model} [m^2/s]	δ_{model} [mm]
1	9.917E-7	6.350	9.917E-7	6.350
2	2.231E-6	9.525	2.231E-6	9.525
3	3.967E-6	12.70	3.967E-6	12.70
4	6.198E-6	15.875	6.198E-6	15.875
5	8.926E-6	19.050	7.095E-6	16.984

From the Table XIV, the insulator thickness margins for cases 1~4 were enough to prevent the electricity conduction, whereas the insulator thickness margin for case 5 was not good enough since the thickness was reduced to the minimum thickness which is the same thickness with Nichrome wire. Moreover for cases 1~4 from the Table XV, the calculated heat diffusivity and diameter of the model electric heater were matched well with the required heat diffusivity and diameter. However, the case 5 showed that the calculated and required heat diffusivity and diameter of the model electric heater were not matched. Consequently, the surface temperature similarities of cases 1~4 were identified to be conserved, while case 5 was not. From this scaling analyses results, the model electric heater for oxide fuel can be the other solution to the model electric heater for uranium metal fuel.

Due to the case 5 from Table XIV and XV, the replacement of the sheath material from Stainless Steel to Copper was considered. Applying the same conditions with the previous SS sheath calculation, the surface temperature similarity of the model electric heater in respect to uranium metal fuel with Cu sheath was simulated, and the results are in the following Table XVI and XVII.

Table XVI: Thicknesses of Heater Materials for the Commercial Cu Sheath Thickness

	case	Core	Heat Source	Insula-tor	Sheath
		Al_2O_3	NiCr	Al_2O_3	Cu
Thick-ness [mm]	1	0.046	0.01	1.4678	1.651
	2	1.168	0.01	1.9338	1.651
	3	2.501	0.01	1.7304	2.108
	4	4.034	0.01	1.4805	2.413
	5	5.479	0.01	1.2670	2.769

Table XVII: Heat Diffusivity and Diameter of Model Electric Heater

Case	Model Required		Model Calculated	
	α_{model} [m^2/s]	δ_{model} [mm]	α_{model} [m^2/s]	δ_{model} [mm]
1	9.917E-7	6.350	9.917E-7	6.350
2	2.231E-6	9.525	2.231E-6	9.525
3	3.967E-6	12.700	3.967E-6	12.700
4	6.198E-6	15.875	6.198E-6	15.875
5	8.926E-6	19.050	8.926E-6	19.050

From the Tables XV and XVI, by comparison with the Tables X and XI, the oxide fuel gives certainly enough margins to design the insulator thickness. The calculated and required heat diffusivity and diameter of the model electric heater were matched well. Therefore, the surface temperature similarity was identified to be conserved.

As the same reasons for the Cu sheath in the previous section like the strong chemical disadvantage with Na, a SS coating on the Cu sheath surface was considered again with the same assumptions and conditions. The results are in the Table XVIII and XIX.

Table XVIII: Thicknesses of the Coated Heater Materials for the Commercial Sheath Thickness

	case	Core	Heat Source	Insulator	Sheath	Coat
		Al_2O_3	NiCr	Al_2O_3	Cu	SS
Thick-ness [mm]	1	0.046	0.01	1.4678	1.651	0.01
	2	1.168	0.01	1.9338	1.651	0.01
	3	2.501	0.01	1.7304	2.108	0.01
	4	4.034	0.01	1.4805	2.413	0.01
	5	5.479	0.01	1.2670	2.769	0.01

From the Table XVIII, the insulator thickness margins showed same as the insulator thickness margins of the model electric heater without coating as shown in Table XVI. In the Table XIX, the calculated and required heat diffusivity and diameter of the model electric heater were matched well. Consequently, the surface temperature similarity was identified to be conserved.

Table XIX Heat Diffusivity and Diameter of the Coated Electric Heater

Case	Model Required		Model Calculated	
	α_{model} [m^2/s]	δ_{model} [mm]	α_{model} [m^2/s]	δ_{model} [mm]
1	9.980E-7	6.370	9.980E-7	6.370
2	2.241E-6	9.545	2.241E-6	9.545
3	3.979E-6	12.720	3.979E-6	12.720
4	6.214E-6	15.895	6.214E-6	15.895
5	8.944E-6	19.070	8.944E-6	19.070

As shown above two sections, the model electric heater with the SS sheath is not suitable to design. For that reason, the replacement of the sheath materials from SS to Cu was considered and achieved the conservation of the surface temperature. However, the SS coating on the Cu sheath surface was considered as well due to the chemical disadvantages and also maintained the surface temperature similarity. Lastly, the oxide fuel was considered to achieve the surface temperature similarity alternatively. Since the equivalent thermal conductivity of the oxide fuel is relatively lower enough than the one of the uranium metal fuel, designing the model electric heater became easier due to better insulator thickness margin. Thus, the sheath material replacement, the sheath surface coating, and the oxide fuel are the measures to achieve the conservation of surface temperature.

7. Conclusions

For the design of model electrical heater for the simulation of nuclear reactor core thermal behavior, equivalent thermal properties was introduced. The equivalent thermal property formula was derived and it was applied to commercial electrical heater manufacturing in its size specification. By coating the copper sheath with stainless steel material the required thermal diffusivity was successfully obtained.

REFERENCES

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