Multiple Beta Spectrum Analysis Method Based on Spectrum Fitting

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1. Introduction

Beta spectrum has a wide range of energy distribution by beta-decay with neutrino. When the sample of several mixed radioactive nuclides is measured, it is difficult to divide each nuclide due to the overlapping of spectrums. For this reason, simple mathematical analysis method for spectrum analysis of the mixed beta ray source has been studied[1,2,3]. However, existing research was in need of more accurate spectral analysis method as it has a problem of accuracy. The study will describe the contents of the separation methods of the mixed beta ray source through the analysis of the beta spectrum slope based on the curve fitting to resolve the existing problem.

2. Methods and Results

Referring to ICRU[4], spectrum data of each radionuclide which is about energy per count ratio is obtained. Based on these data, a mixed beta ray source spectrum data was prepared. By using a commercially available program, MATLAB, the spectrum function due to each radionuclide was defined. The spectral analysis was carried out based on the derived function.

2.1 Data Preparation

Referring to ICRU, the information about the count ratio per the energy of ^{32}P, ^{90}Y and ^{106}Rh was used. The count ratios and energies of the each radionuclide were interpolated because the given energy value according to a type of radionuclides is different. So the count ratios and energies of the each radionuclide were interpolated based on the radionuclides of the lowest energy area. The result of data is shown in Fig. 1. The blue hexagram is ^{32}P, red pentagram is ^{90}Y and yellow diamond is ^{106}Rh.

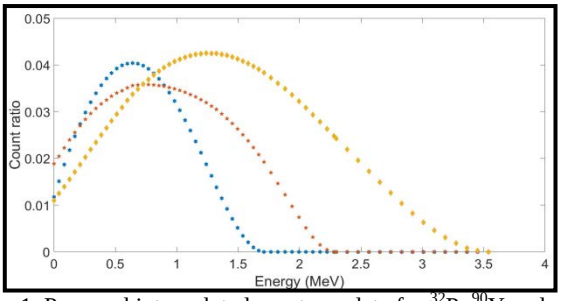

Fig. 1. Prepared interpolated spectrum data for ^{32}P, ^{90}Y and ^{106}Rh

On the basis of count ratio per specific energy for each radionuclide, which has been defined so, the count number per specific energy was assumed to be proportional in accordance with the intensity of radioactivity. In addition, in order to produce a mixed spectrum data, the count number of mixed nuclides is defined as sum of the count number per specific energy of each nuclide. To determine the applicability in cases of large variety, the data was prepared for three divided cases such as 1) the radioactivity of one of the radioactive nuclides is very strong, 2) the radioactivity of one of the radioactive nuclides is very small and 3) the values of the radioactivity of all the radioactive nuclide are similar. Example of each typical case of spectrum is shown at Fig. 2. The ratio of nuclides (^{32}P:^{90}Y:^{106}Rh) is 1:1:1: in blue hexagram, 1:1:2 in red pentagram and 2:2:1 in yellow diamond. The shape of spectrum varies with the mixed ratio of nuclides. So these data is used for application of proposed beta spectrum analysis method.

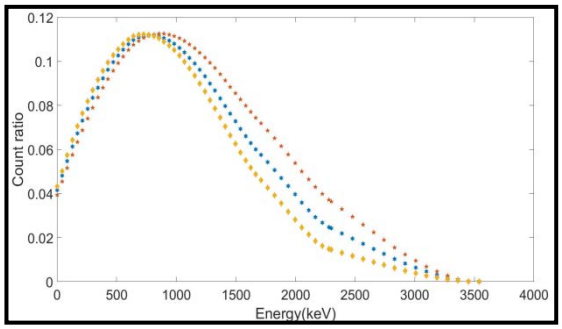

Fig. 2. Mixed spectrum for different mixed ratio

2.2 Curve Fitting Method

To simulate the spectrum of each nuclide, the functions for the spectral shape due to each radionuclide were derived by using the prepared data on count rate per specific energy. The curve fitting was performed using commercially available program MATLAB software. Fourier, polynomial, Gaussian and sum of sine methods are used for candidate functions to make beta spectrum. Also the highest order of fitting method is used for more precise application result. So the used fitting method and order is Fourier-8, polynomial-9, Gaussian-8 and sum of sine-8. However Gaussian fitting method shows poor application results in overall data like Fig. 3.

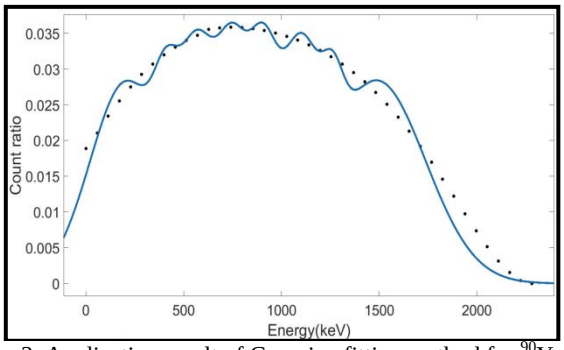

Fig. 3. Application result of Gaussian fitting method for ^{90}Y spectrum data

Also the fitting results of Fourier and polynomial methods have poor performance at end region of spectrum. So these methods are not used for fitting the data because the end point of spectrum is important for predicting the quantity of specific nuclide's activity. Therefore these three methods are excluded for fitting the beta spectrum. Finally accuracy test of sum of sine fitting method is processed to select suitable fitting method. E_{max} and E_{max-10} values are used for the test. E_{max} is the count rate ratio value at the maximum energy and E_{max-10} is the value at 10th before maximum energy. The meaning of these values is explained in the peak definition part. The comparison result of ICRU reference and application result of sum of sine fitting method is explained at Table. The sum of sine fitting method shows good performance. Follow the result, sum of sine fitting method is applied for spectrum analysis for mixed beta nuclide source.

Table I: The comparison result of ICRU reference and application result of sum of sine fitting method

	E_{max}		
	ICRU	Fitted	Difference
^{32}P	0	-0.0004358	0.0004358
^{90}Y	0	-0.0005993	0.0005993
^{106}Rh	0	-0.0004992	0.0004992
	E_{max-10}		
	ICRU	Fitted	Difference
^{32}P	0.01626	0.01625	1.0438E-05
^{90}Y	0.01920	0.01919	1.3114E-05
^{106}Rh	0.01488	0.01488	1.9279E-06

So the final fitted function by using sum of sine method is equation (1). Constant and R^2 for the equation of each nuclide are explained in Table II. R^2 is the coefficient of determination that indicates how well data fit a fitted function. These fitted equations of spectrum shape for each nuclide are used for separating each nuclides spectrum from mixed spectrum data.

$$Y = a_1\sin(b_1E+c_1) + a_2\sin(b_2E+c_2) + a_3\sin(b_3E+c_3) + a_4\sin(b_4E+c_4) + a_5\sin(b_5E+c_5) + a_6\sin(b_6E+c_6) + a_7\sin(b_7E+c_7) + a_8\sin(b_8E+c_8) \quad (1)$$

Table II: Constants and R^2 for the fitted function equation of each nuclide

	^{32}P	^{90}Y	^{106}Rh
a_1	0.03891	0.03955	0.03887
b_1	0.001807	0.001353	0.0008726
c_1	0.165	0.2609	0.1572
a_2	0.007272	0.005739	0.008922
b_2	0.003622	0.002707	0.001749
c_2	0.3526	1.321	0.2465
a_3	0.001474	0.001353	0.0008175
b_3	0.007243	0.005417	0.003493
c_3	0.9177	0.6792	1.413
a_4	0.0005915	0.0007281	0.0005408
b_4	0.01087	0.00813	0.005243
c_4	1.156	1.244	1.578
a_5	0.0003062	0.0004385	0.0004028
b_5	0.01451	0.01085	0.006995
c_5	1.316	1.326	1.803
a_6	0.0001813	0.0002396	0.0002467
b_6	0.01817	0.01359	0.008759
c_6	1.441	1.486	1.803
a_7	0.0001201	0.0001777	0.0001553
b_7	0.02187	0.01637	0.01055
c_7	1.519	1.731	1.572
a_8	9.37e-05	0.0001611	0.0001154
b_8	0.02568	0.01922	0.01241
c_8	1.515	1.537	1.369
R^2	0.9999	0.9997	0.9999

Fig. 4 shows the application result of sum of sine fitting method in case of ^{90}Y. Black point is the count ratio per specific energy of ICRU data and blue line is the result of sum of fitting method. The fitting result is good agreement with reference data.

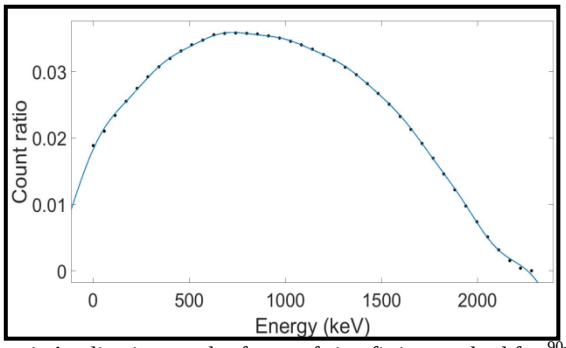

Fig. 4. Application result of sum of sine fitting method for ^{90}Y spectrum data

2.3 Nuclide Definition and Separation

The maximum energy of the mixed spectrum is used for defining the kind of radionuclide by using one of the typical characteristics of the beta nuclides that the maximum energy is different due to each kind of beta radionuclides. There are two assumptions are needed for separating spectrum of each radionuclide for mixed

spectrum data. First assumption is the count rate of E_{max} and E_{max-10} channel is the result generated by single radionuclide. The next is the number of count per specific energy (C_E) has proportional relationship with the value of multiplying count ratio and activity such as equation (2). A is radioactivity and CR_E is count ratio at energy E

$$C_E = A * CR_E \quad (2)$$

Also through the information of the nuclide, the spectral distribution function of the specific radionuclide can be identified and constant value which is multiplied by a function is defined by using the value of the E_{max-10}. The number of count per specific energy is to be derived out through the function of the derived radionuclides and separated by removing from the spectrum data of the mixed beta sample. This process is repeated until all the nuclides were defined. The result of separation process can be shown in Fig. 5.

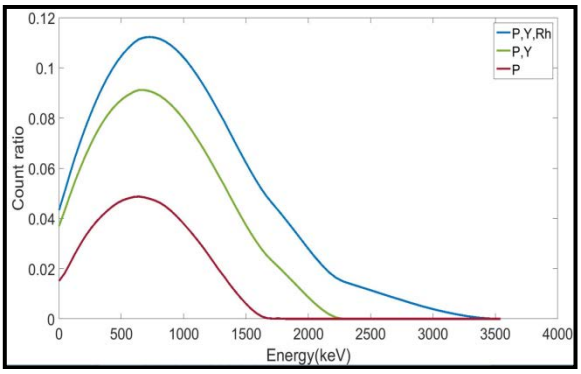

Fig. 5. The result of applying mixed beta spectrum separating method for mixed spectrum of ^{32}P, ^{90}Y and ^{106}Rh

The biggest spectrum which color is blue is mixed spectrum of ^{32}P, ^{90}Y and ^{106}Rh. First process is the definition of the ratio of ^{106}Rh by using the ratio of E_{max-10} value. Next step is multiplying the ratio of E_{max-10} value to the spectral function of ^{106}Rh and deducting the value of count per specific energy from mixed spectrum data. So the spectrum of ^{106}Rh is defined and separated from mixed spectrum data. So the Middle scale spectrum with green which means mixed spectrum of ^{32}P, ^{90}Y is obtained. Same process is repeated for definition of spectrum of ^{90}Y. Finally all the nuclides are defined and separated from multiple beta spectrums.

The Table III is the result of application of the proposed method to mixed spectrum data in various ratios of mixed nuclides. The data which is calculated by fitting and interpolation method is compared for the case of ^{32}P, ^{90}Y and ^{106}Rh mixed spectrum. The value of 'Standard' is the value of radioactivity calculated by interpolation. This value is calculated by summing the number of count per specific energy in prepared interpolated data. The value of 'Fitted' is the radioactivity calculated by fitting function. The number of count per specific energy is calculated by multiplying the value of ratio of E_{max-10} and count ratio per energy. Error is a relative error between Standard and Fitted data. The error of ^{106}Rh is always same because the definition of ratio of ^{106}Rh is the first step of separation.

Table III: The application result of the proposed method to mixed spectrum data in various ratios of mixed nuclides

	Mixed Ratio (^{32}P:^{90}Y:^{106}Rh = 1:1:1)		
	Standard	Fitted	Error (%)
^{32}P	1	1.0053	0.5333
^{90}Y	1.3227	1.3169	0.4353
^{106}Rh	1.8648	1.8667	0.1013
	Mixed Ratio (^{32}P:^{90}Y:^{106}Rh = 2:2:1)		
	Standard	Fitted	Error (%)
^{32}P	1.2	1.2051	0.4250
^{90}Y	1.5872	1.5843	0.3684
^{106}Rh	1.1189	1.1200	0.1013
	Mixed Ratio (^{32}P:^{90}Y:^{106}Rh = 1:2:2)		
	Standard	Fitted	Error (%)
^{32}P	0.6	0.6121	2.0138
^{90}Y	1.5872	1.5803	0.4353
^{106}Rh	2.2378	2.2401	0.1013
	Mixed Ratio (^{32}P:^{90}Y:^{106}Rh = 2:1:2)		
	Standard	Fitted	Error (%)
^{32}P	1.2	1.1988	0.0985
^{90}Y	0.7936	0.7891	0.5690
^{106}Rh	2.2378	2.2401	0.1013
	Mixed Ratio (^{32}P:^{90}Y:^{106}Rh = 2:1:1)		
	Standard	Fitted	Error (%)
^{32}P	1.5	1.4969	0.2069
^{90}Y	0.9920	0.9877	0.4353
^{106}Rh	1.3986	1.4000	0.1013
	Mixed Ratio (^{32}P:^{90}Y:^{106}Rh = 1:2:1)		
	Standard	Fitted	Error (%)
^{32}P	0.75	0.7635	1.7970
^{90}Y	1.9840	1.9767	0.3684
^{106}Rh	1.3986	1.4000	0.1013
	Mixed Ratio (^{32}P:^{90}Y:^{106}Rh = 1:1:2)		
	Standard	Fitted	Error (%)
^{32}P	0.75	0.7556	0.7501
^{90}Y	0.9920	0.9863	0.5690
^{106}Rh	2.7972	2.8001	0.1013

The relative error of various ratios of mixed nuclides has minimum value as 0.0985 and maximum value as 2.013. As a result, this method is considered that can separated mixed spectrum accurately. As seen in the Table III, error of ^{32}P has the biggest value in overall ratio of mixed nuclides because imperfection of fitting function causes error and this value is accumulated during the separation process. However it is very easy and fast way to distinguish each nuclide's peak spectrums. This method is applied to define the respective spectrum of each nuclide from the mixed beta spectrum due to each nuclide.

3. Conclusions

The method based on curve fitting for analyzing the multiple beta spectrums was proposed and evaluated by using the mixed spectrum of the three nuclides (^{32}P, ^{90}Y and ^{106}Rh) leading to improvement of the existing one. The fitting methods including It was understood that sum of sine fitting method was the best one of such proposed methods as Fourier, polynomial, Gaussian and sum of sine to obtain equation for distribution of mixed beta spectrum. It was shown to be the most appropriate for the analysis of the spectrum with various ratios of mixed nuclides. It was thought that this method could be applied to rapid spectrum analysis of the mixed beta ray source.

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