An analytical method for calculating stresses and strains of ATF cladding based on thick-walled theory

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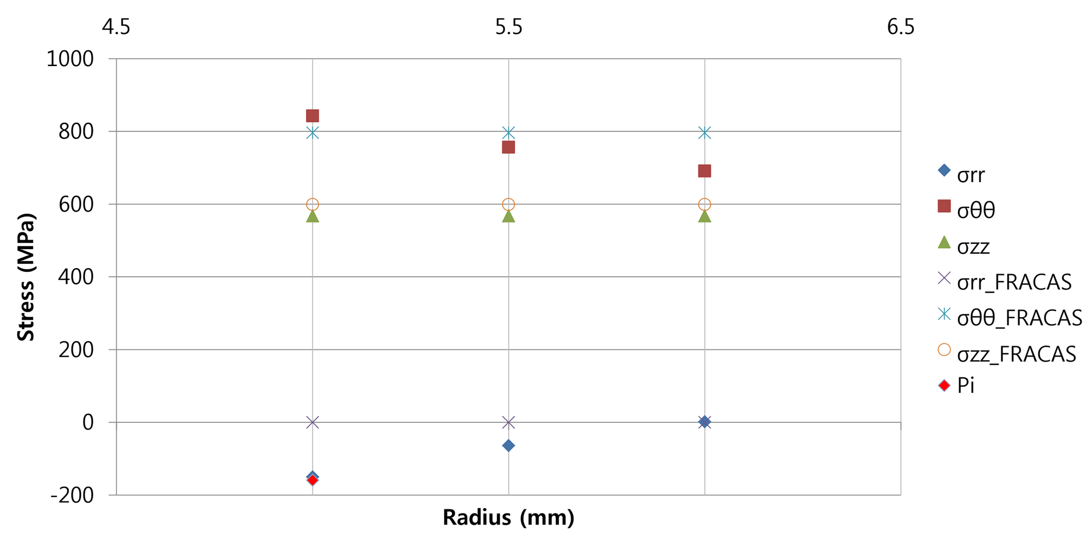
Introduction

❖ Mechanical module in FRAPCON/FRAPTRAN (FRACAS)

- FRACAS module calculates stress and strain of cladding with the prescribed conditions.
- According to gap status, FRACAS consists of subroutine 'cladf' for the open gap and subroutine 'couple' for the closed gap.
- Subroutine 'cladf' and 'couple' consider a thin cylindrical shell and therefore radial stress is treated as zero.
- Compared with results from equivalent FE model, radial stress should be considered because the value of radial stress is not negligibly.



< Comparison of stress components >

❖ Development of subroutines of FRACAS module based on thick-walled theory

- For ATF cladding composed of multi-layer, subroutine 'cladf' and 'couple' were newly modeled as a thick-walled cylinder to consider radial stress.
- To evaluate the developed method, equivalent model using finite element method was established and stress components of the method were compared with those of equivalent FE model.

Theoretical background

❖ Stress components based on thick-walled theory

- In the case of thick-walled pressure vessels, external force may not vary significantly in the axial direction and therefore plane stress model can be used.
- Based on the elasticity, stress components in cylindrical coordinate for general plane strain problem were obtained as below.

$$\sigma_{r,i} = \frac{E_i}{1+v_i}\left(\frac{C_{1,i}}{1-2v_i} - \frac{C_{2,i}}{r^2} + \frac{v_i\varepsilon_0}{1-2v_i}\right) - \frac{\alpha_i E_i}{(1-v_i)r^2}\int \Delta T r dr$$

$$\sigma_{\theta,i} = \frac{E_i}{1+v_i}\left(\frac{C_{1,i}}{1-2v_i} + \frac{C_{2,i}}{r^2} + \frac{v_i\varepsilon_0}{1-2v_i}\right) - \frac{a_i E_i \Delta T}{1-v_i} + \frac{\alpha_i E_i}{(1-v_i)r^2}\int \Delta T r dr$$

$$\sigma_{z,i} = E_i\varepsilon_0 + \frac{2v_i E_i C_{1,i}}{(1-2v_i)(1+v_i)} + \frac{2v_i^2 E_i \varepsilon_0}{(1-2v_i)(1+v_i)} - \frac{a_i E_i \Delta T}{1-v_i}$$

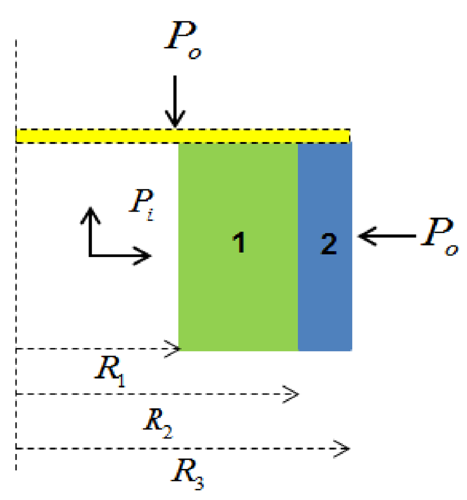
$$u = C_{1,i} r + \frac{C_{2,i}}{r} + \left(\frac{1+v_i}{1-v_i}\right)\frac{\alpha_i}{r}\int \Delta T r dr$$

$$R_i \le r \le R_{i+1},\ for\ i = 1, 2$$

- These stress components were expressed by radial displacement *u* where subscript *i* indicates interface between each layer.

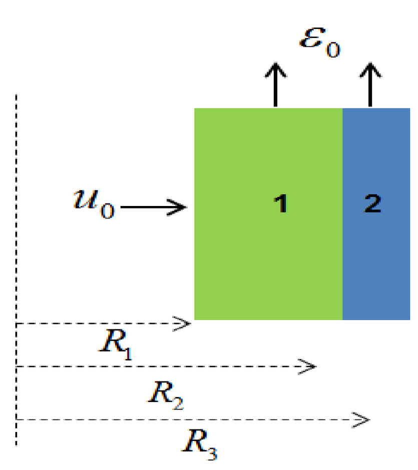
❖ Boundary conditions of subroutine 'cladf' and 'couple'

- For ATF cladding composed of two layers, 5 unknowns, $C_{1,1}, C_{1,2}, C_{2,1}, C_{2,2}, \varepsilon_0$ should be calculated to obtain stresses using prescribed boundary conditions.



$$\sigma(R_1)_{r,1} = -P_i$$
$$\sigma(R_2)_{r,1} = \sigma(R_2)_{r,2}$$
$$\sigma(R_3)_{r,2} = -P_o$$
$$\varepsilon(R_2)_{\theta,1} = \varepsilon(R_2)_{\theta,2}$$
$$A_1\sigma_{z,1} + A_2\sigma_{z,2} = P_i\pi R_1^2 - P_o\pi R_3^2$$

< Subroutine 'cladf' for open gap status>



$$\sigma(R_2)_{r,1} = \sigma(R_2)_{r,2}$$
$$\sigma(R_3)_r = 0$$
$$u(R_1)_1 = u_0$$
$$\varepsilon(R_2)_{\theta,1} = \varepsilon(R_2)_{\theta,2}$$
$$\varepsilon_0 = \varepsilon_z(given)$$

< Subroutine 'couple' for closed gap status>

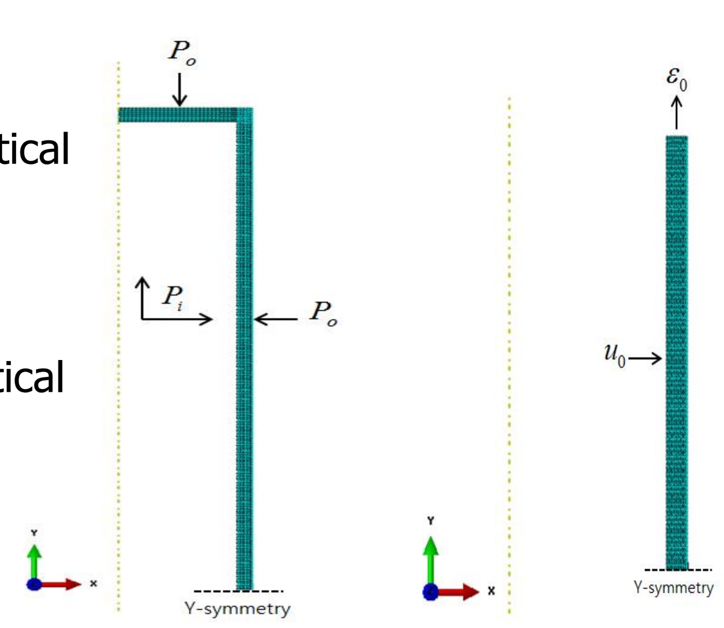
- Boundary conditions of subroutine 'cladf' are given by internal and external pressure.
- Boundary conditions of subroutine 'couple' are prescribed by radial displacement and axial strain.
- Fundamental assumptions of 'couple' that pellet is rigid, no slip occurs between pellet and cladding, and radial displacement of outer surface of pellet is equal to that of inner surface of cladding.

Evaluation of the developed analytical method

❖ Equivalent finite element model

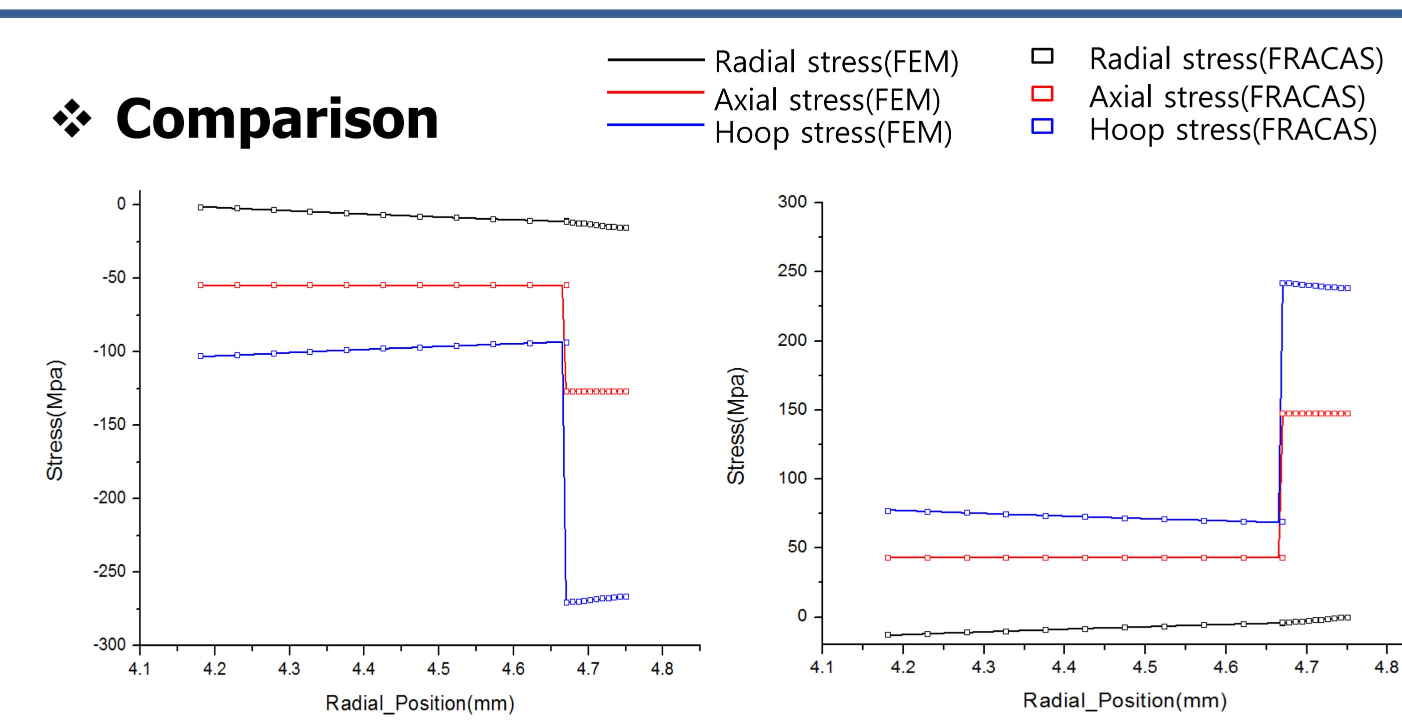
- Equivalent finite element model were developed to evaluate the developed analytical method.
- Material properties and boundary conditions were used as below.
- Compared with results from equivalent FE model, all stress components of analytical model were approximately identical in both cases.

E_1, E_2 = 80, 279 (Gpa)
α_1, α_2 = 1, 0.5 (μm/m℃)
P_i, P_o = 1, 15 (Mpa)
u_0 = 0.004 (mm)
ε_0 = 0.0004 (mm/mm)
ΔT = 100 (℃)
R_1, R_2, R_3 = 4.18, 4.67, 4.75 (mm)



< The equivalent FE model for 'cladf' and 'couple' >

❖ Comparison



< Comparison of stress components>

Conclusion

- An analytical method that calculates stress components of ATF cladding was developed in this study.
- Thick-walled theory was applied to develop subroutine 'cladf' and 'couple' of FRACAS based on thin-walled theory.
- The equivalent FE model was established to evaluate developed analytical method.
- In comparison with results of equivalent FEM model, all stress components of analytical model were approximately identical with those of FEM.