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An analytical method for calculating stresses and strains of ATF cladding based on thick-walled theory

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1. Introduction

Since the Fukushima accident, a decrease in the oxidation rate of Zr-based alloys at high temperatures is a key issue involved in improving the accident tolerance of the fuel assembly. Thus, the development of accident tolerant fuel (ATF) is a major concern in nuclear research fields at the present time. One of promising ATF concepts is the coated cladding, which take advantages such as high melting point, a high neutron economy, and low tritium permeation rate [1].

To evaluate the mechanical behavior and performance of the coated cladding, we need to develop the specified model to simulate the ATF behaviors in the reactor. In particular, the model for simulation of stress and strain for the coated cladding should be developed because the previous model, which is 'FRACAS', is for one body model. The FRACAS module employs the analytical method based on thin-walled theory [2]. According to thin-walled theory, radial stress is defined as zero but this assumption is not suitable for ATF cladding because value of the radial stress is not negligible in the case of ATF cladding.

Recently, a structural model for multi-layered ceramic cylinders based on thick-walled theory was developed [3]. Also, FE-based numerical simulation such as BISON has been developed to evaluate fuel performance [4].

In this paper, an analytical method based on thick-walled theory has been studied to calculate stress and strain of ATF cladding. In order to prescribe boundary conditions of the analytical method, two algorithms were employed which are called subroutine 'Cladf' and 'Couple' of FRACAS, respectively. To evaluate the developed method, equivalent model using finite element method was established and stress components of the method were compared with those of equivalent FE model.

2. Theoretical background

Some of the theoretical backgrounds used to analytical method are described in this section. FRACAS is a mechanical module in FRAPCON/ FRAPTRAN code system and consists of a set of independent subroutines according to gap status. A gap means a space between pellet and cladding and this gap status is divided broadly into two parts that one is open and the other is closed gap. Subroutine 'Cladf' is for open gap status and 'Couple' is for closed gap status. Two subroutines have different boundary conditions at each other and these conditions were equally applied to the analytical method that was developed in this study. However, FRACAS is based on thin-walled theory and radial stress is treated as zero accordingly. In the case of ATF cladding, radial stress should be considered and the equations for calculating stress were newly derived by thick-walled theory shown in Eqs. (1)-(3). In conclusion, the assumptions applied to the analytical method are defined as follows; the cladding is modeled as a thick-walled cylinder with uniform temperature, pressures, and radial displacement of the inside surface; the pellet is not deformable (Rigid pellet); When the contact occurs, slip of pellet against cladding is not allowed (no slip).

2.1 Stress components based on thick-walled theory

In the case of thick-walled pressure vessels, such as a nuclear containment vessel, the external force may not vary significantly in the axial direction. In such case, a plane strain model can be used [5].

Assuming that strain occurs in the elastic region, thermal expansion is induced by uniform temperature gradient, and material properties are different at each layer, stress components in cylindrical coordinate for general plane strain problem are obtained from Eqs. (1)-(3) respectively [3,5]. These stress components expressed by radial displacement *u* in Eq. (4) are function of radial coordinate *r*, where subscript *i* indicate interface between each layer. *R* is a radial position shown in Fig. 1. Hence, we have five unknowns, $C_{1,1}$, $C_{1,2}$, $C_{2,1}$, $C_{2,2}$, and ε_0 because other values are prescribed constants. To solve for theses unknowns, boundary conditions and loading conditions of the 'Cladf' and 'Couple' were applied to the analytical method.

$$\sigma_{r,i} = \frac{E_i}{1+v_i}\left(\frac{C_{1,i}}{1-2v_i} - \frac{C_{2,i}}{r^2} + \frac{v_i\varepsilon_0}{1-2v_i}\right) - \frac{\alpha_i E_i}{(1-v_i)r^2}\int \Delta T r dr, \qquad (1)$$

$$R_i \le r \le R_{i+1}, \text{ for } i = 1,2$$

$$\sigma_{\theta,i} = \frac{E_i}{1+v_i}\left(\frac{C_{1,i}}{1-2v_i} + \frac{C_{2,i}}{r^2} + \frac{v_i\varepsilon_0}{1-2v_i}\right) - \frac{\alpha_i E_i \Delta T}{1-v_i}$$

$$+\frac{\alpha_i E_i}{(1-v_i)r^2}\int \Delta T r dr, R_i \le r \le R_{i+1}, \text{ for } i = 1, 2 \qquad (2)$$

$$\sigma_{z,i} = E_i\varepsilon_0 + \frac{2v_i E_i C_{1,i}}{(1-2v_i)(1+v_i)} + \frac{2v_i^2 E_i \varepsilon_0}{(1-2v_i)(1+v_i)} - \frac{\alpha_i E_i \Delta T}{1-v_i}$$

$$R_i \le r \le R_{i+1}, \text{ for } i = 1, 2 \qquad (3)$$

$$u_i = C_{1,i} r + \frac{C_{2,i}}{r} + \left(\frac{1+v_i}{1-v_i}\right)\frac{\alpha_i}{r}\int \Delta T r dr$$

$$R_i \le r \le R_{i+1}, \text{ for } i = 1, 2 \qquad (4)$$

2.2 Boundary conditions based on subroutine 'Cladf'

Subroutine 'Cladf' is for open gap status and this status means no contact between pellet and cladding. In the case of 'Cladf', boundary conditions are given by internal and external pressure shown in Fig. 1 [6].

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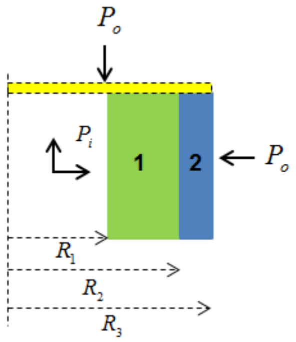

Fig. 1. Schematic drawing of subroutine 'Cladf'

To solve five unknowns in Eqs. (1)-(3), five independent boundary conditions can be imposed shown in Eqs. (5)-(9).

$$\sigma(R_1)_{r,1} = -P_i \qquad (5)$$

$$\sigma(R_2)_{r,1} = \sigma(R_2)_{r,2} \qquad (6)$$

$$\varepsilon(R_2)_{\theta,1} = \varepsilon(R_2)_{\theta,2} \qquad (7)$$

$$\sigma(R_3)_{r,2} = -P_o \qquad (8)$$

$$A_1\sigma_{z,1} + A_2\sigma_{z,2} = P_i\pi R_1^2 - P_0\pi R_3^2 \qquad (9)$$

Boundary conditions (5) and (8) prescribe radial stresses equal to internal pressure and outer pressure at the inner and outer surface of the cladding respectively. Boundary condition (6) states continuity of radial stresses and (7) means that each layer have equal hoop strain at the interface. Finally, axial force balance is set in Eq. (9) because cylinder has closed ends [7].

2.3 Boundary conditions based on subroutine 'Couple'

Subroutine 'Couple' is for closed gap where contact between pellet and cladding occurs in this case. Fundamental assumptions of 'Couple' are that pellet is rigid, no slip occurs between pellet and cladding, and radial displacement of outer surface of pellet is equal to that of inner surface of cladding. With this assumption, radial displacement and axial strain are given as boundary conditions shown in Fig. 2 [6].

Because axial strain is prescribed as a boundary condition in the case of 'Couple', we have four unknowns, $C_{1,1}, C_{1,2}, C_{2,1}, C_{2,2}$, except ε_0. To solve for these unknowns, four boundary conditions can be established shown in Eqs. (10)-(13).

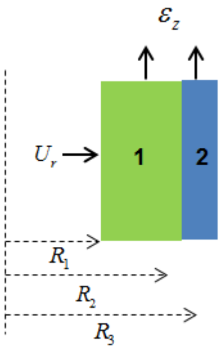

Fig. 2. Schematic drawing of subroutine 'Couple'

$$u(R_1)_1 = u_0 \qquad (10)$$

$$\sigma(R_2)_{r,1} = \sigma(R_2)_{r,2} \qquad (11)$$

$$\varepsilon(R_2)_{\theta,1} = \varepsilon(R_2)_{\theta,2} \qquad (12)$$

$$\sigma(R_3)_r = 0 \qquad (13)$$

Boundary condition (11) and (12) are equal to the one in Eq. (6) and (7). With fundamental assumptions of 'Couple', radial displacement of cladding at the inner surface is given as u_0 in Eq. (10) that is induced by deformation of pellet. At the steady state of fuel rod operation, inner pressure is so high that influence of outer pressure can be negligible in Eq. (13).

3. Evaluation of the developed analytical method

3.1 Equivalent finite element model

Developed analytical methods based on 'Cladf' and 'Couple' were evaluated by comparison with equivalent finite element models shown in Fig.3. To establish these models, ABAQUS 6.12 was employed.

Material properties used for analysis are shown in table 1 and radial positions, R_1,R_2, and R_3, are set as 4.18mm, 4.67mm, and 4.75mm respectively. In the case of 'Cladf', external loading conditions are as follows; outer pressure is 15.5 Mpa, inner pressure is 1 Mpa. In the case of 'Couple', radial displacement at the inner surface and axial strain of cladding are a 0.004mm and 0.0004mm/mm respectively.

Table 1. Material properties of ATF cladding

	Elastic Modulus (Gpa)	Poisson Ratio (-)	Thermal expansion coefficient (μm/m℃)
Inner layer	80	0.3	1
Outer layer	279	0.21	0.5

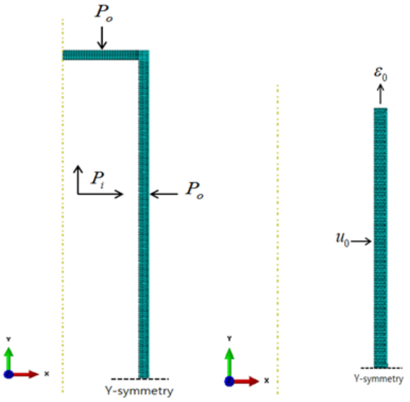

Fig. 3. The equivalent FE model for ‘Cladf’ (left) and ‘Couple’ (right)

3.2 Comparison of stress components between the analytical and equivalent FE models

Fig. 4 and 5 show stress distributions along the radius for the ‘Cladf’ and ‘Couple’. Full line represents stress components of analytical model and dotted line expresses those of FE model. Radial, axial, and hoop stresses are marked in different colors, black, red, and blue respectively. Compared with results from equivalent FE model, all stress components of analytical model were approximately identical in both cases.

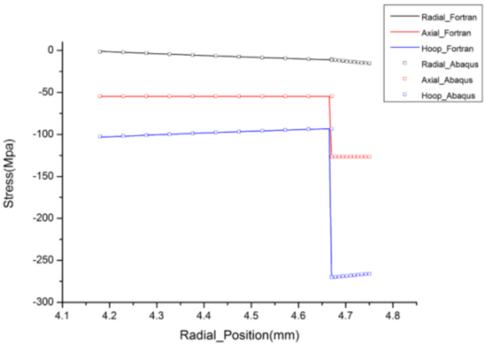

Fig. 4. Stress components of different two models for ‘Cladf’

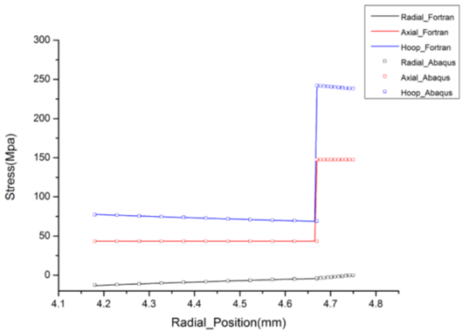

Fig. 5. Stress components of different two models for ‘Couple’

4. Conclusions

An analytical method that calculates stress components of ATF cladding was developed in this study. Thick-walled theory was used to derive equations for calculating stress and strain. To solve for these equations, boundary and loading conditions were obtained by subroutine ‘Cladf’ and “Couple’ and applied to the analytical method. To evaluate the developed method, equivalent FE model was established and its results were compared to those of analytical model. Based on the comparison of the analytical and FE model, two cases were approximately identical.

However, only thermo-elastic deformation was considered in this study. Further studies considering plastic behavior of ATF cladding such as creep and stress relaxation have to be performed.

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