# Preliminary Study of 1D Thermal-Hydraulic System Analysis Code Using the Higher-Order Numerical Scheme

Won Woong Lee [a], Jeong Ik Lee [a]
*[a]Dept. of Nuclear and Quantum engineering, Korea Advanced Institute of Science and Technology*
*291 Daehak-ro, (373-1, Guseong-dong), Yuseong-gu, Daejeon 305-701, Republic of KOREA*
*Tel: +82-42-350-5869, Fax: +82-42-350-3810, Email: woong.aa@kaist.ac.kr, jeongiklee@kaist.ac.kr*

## 1. Introduction

The existing nuclear system analysis codes such as RELAP5, TRAC, MARS and SPACE use the first-order numerical scheme in both space and time discretization. However, the first-order scheme is highly diffusive and less accurate due to the first order of truncation error. So, the numerical diffusion problem which makes the gradients to be smooth in the regions where the gradients should be steep can occur during the analysis, which often predicts less conservatively than the reality. Therefore, the first-order scheme is not always useful in many applications such as boron solute transport.

RELAP7 which is an advanced nuclear reactor system safety analysis code using the second-order numerical scheme in temporal and spatial discretization is being developed by INL (Idaho National Laboratory) since 2011. Therefore, for better predictive performance of the safety of nuclear reactor systems, more accurate nuclear reactor system analysis code is needed for Korea too to follow the global trend of nuclear safety analysis. Thus, this study will evaluate the feasibility of applying the higher-order numerical scheme to the next generation nuclear system analysis code to provide the basis for the better nuclear system analysis code development.

For this study, MARS code will be used as the reference code to identify the numerical diffusion problem which can arise in the first-order scheme. The higher-order scheme will be also tested for the numerical diffusion and dispersion problems as well. For this study, a separate single phase transient analysis code, namely NTS code, which is possible to calculate in the first-order and the second-order upwind scheme but mimics MARS solver is built in MATLAB environment.

## 2. Numerical Tests

This study will be conducted to identify the decreasing error depending on the increasing number of meshes and to evaluate effect of numerical diffusion and dispersion problems in a system analysis code with selected simple test cases. In addition, the sensitivity of the first-order and the second-order schemes for system transient analysis will be conducted.

To identify effects of the numerical diffusion and dispersion problems and the decreasing error depending on the increasing number of meshes, single phase pipe flow with a sine pulse of temperature is modeled by MARS and the NTS codes separately and the results are compared to each other. Fig. 1 shows the configuration of single phase pipe flow with sine pulse of temperature. In this test, the fluid flows at 1m/s through the pipe with cross sectional area of 0.5m$^2$ and 20m in length. The initial temperature and pressure of the fluid is 300K and 101,325Pa, respectively. The temperature of the injected fluid is changed with time as shown in Fig. 2. The pulse width is 5sec and the interval is 1.5 sec. This simulation is performed for several numbers of meshes to compare MARS with the NTS code. A sensitivity test depending on the first-order, the temporal second-order, the spatial second-order and the temporal and spatial second-order scheme is conducted.

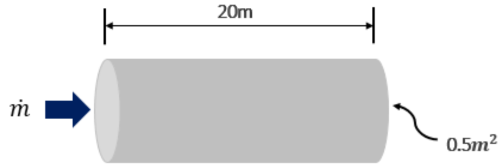

Fig. 1. Configuration of single phase pipe flow with sine pulse of temperature

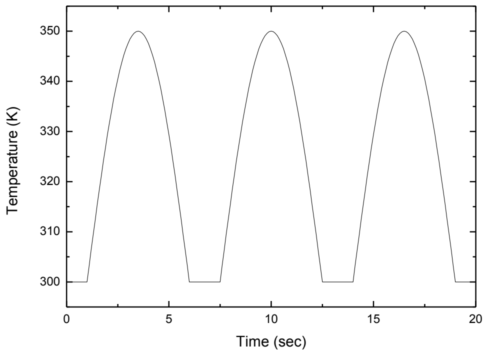

Fig. 2. Temperature profile of fluid injected at pipe inlet

## 3. Test Results

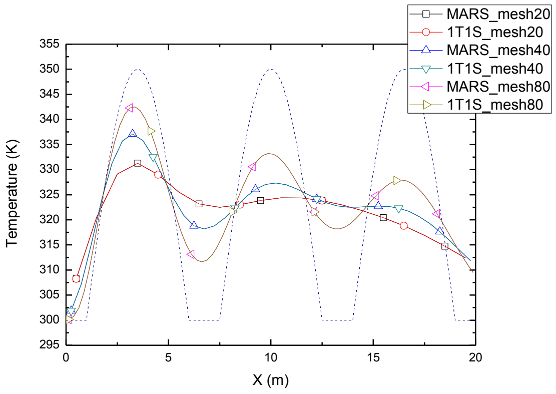

Fig. 5. Comparison of MARS and the 1st order NTS codes depending on the number of mesh in case of pulse width 5sec and interval 1.5sec of test 01

Fig. 5 shows the results of MARS code and the first-order NTS code depending on the number of meshes with a pulse like Fig. 2. In this graph, '1T1S' means the temporal first-order and the spatial first-order scheme of the NTS code. 'mesh10', 'mesh20', 'mesh30' means the number of meshes of 10, 20 and 30. As shown in this figure, a good agreement with MARS and the first-order NTS code results is observed. This is because the first-order NTS code is identical to MARS code in terms of the temporal and spatial numerical scheme and solver algorithms. Therefore, this figure validates that the first-order NTS code is identical to MARS code for single phase flow. However, the errors between the code results and the actual solution seem to originate from the numerical diffusion. The errors are decreased as the number of meshes increases. As shown in Fig. 5, the temperature peak is decreased while the injected water is passing through the pipe. The temperature distribution is severely distorted unlike the actual temperature distribution in the spatial and temporal first-order scheme. Therefore, it is identified that the first-order scheme is highly diffusive and less accurate in this test case.

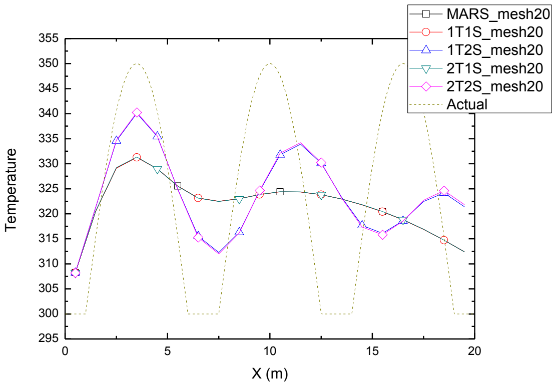

Fig. 6. Sensitivity results of MARS and NTS codes with mesh number 20 in case of pulse width 5sec and interval 1.5sec of test 01

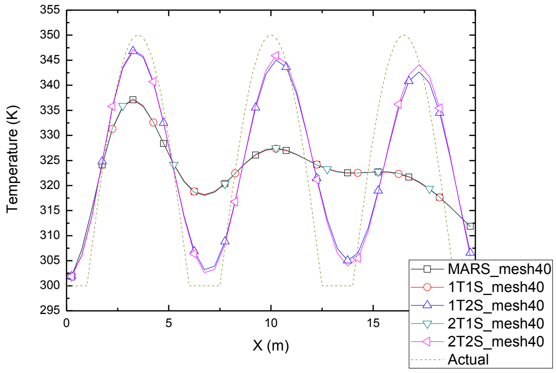

Fig. 7. Sensitivity results of MARS and NTS codes with mesh number 40 in case of pulse width 5sec and interval 1.5sec of test 01

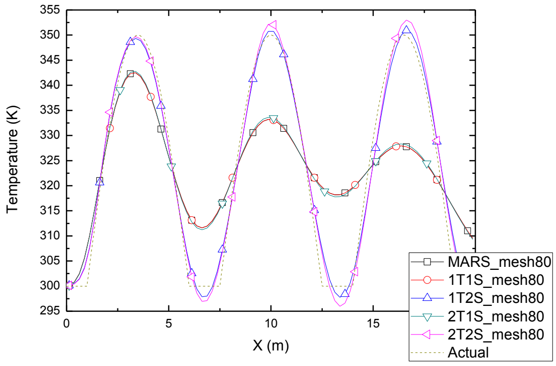

Fig. 8. Sensitivity results of MARS and NTS codes with mesh number 80 in case of pulse width 5sec and interval 1.5sec of test 01

Figs. 6~8 show the sensitivity results of the NTS code depending on the first-order, the temporal second-order, the spatial second-order and the temporal and spatial second-order scheme compared to MARS code for number of meshes of 20, 40 and 80, respectively. In this figure, '1T2S' means the temporal first-order and the spatial second-order scheme of the NTS code. Based on this rule, legends '2T1S' and '2T2S' are designated to indicate different numerical schemes. As shown in this figure, the results of MARS and the 1T1S NTS codes are identical as expected. The results of the 2T1S NTS code are not much different from the results of MARS and the 1T1S NTS code. However, the 1T2S and the 2T2S NTS code results show different appearance with MARS, the 1T1S and the 2T1S NTS codes. The 1T2S and the 2T2S NTS code results are closer to the actual solution. In Fig. 7, the 1T2S and the 2T2S NTS code results indicate the decreased error for the mesh number 40. Also, the accuracy is better in Fig. 8. However, the numerical dispersion is identified in Fig. 8. In the 2T2S NTS code results, the numerical dispersion problem is more severe than in the 1T2S NTS code results. Therefore, when the second-order scheme in time and the second-order scheme in space are applied together, the numerical dispersion can occur. However, the only second-order scheme in time shows similar results to the first-order scheme results.

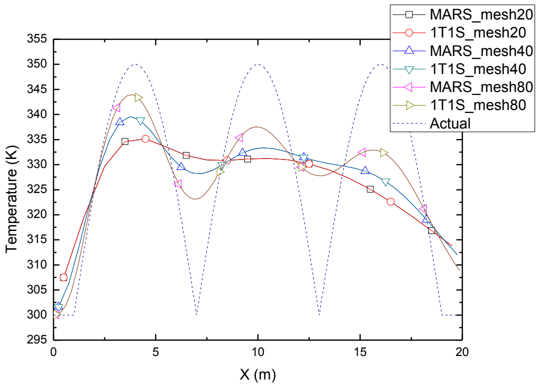

Fig. 9. Comparison of MARS and the 1st order NTS codes depending on the number of mesh in case of pulse width 6sec and interval 0sec of test 01

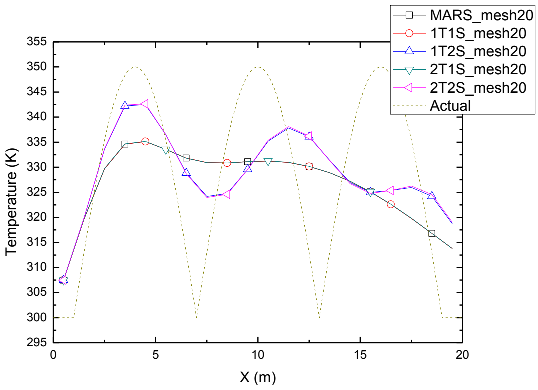

Fig. 10. Sensitivity results of MARS and NTS codes with mesh number 20 in case of pulse width 6sec and interval 0sec of test 01

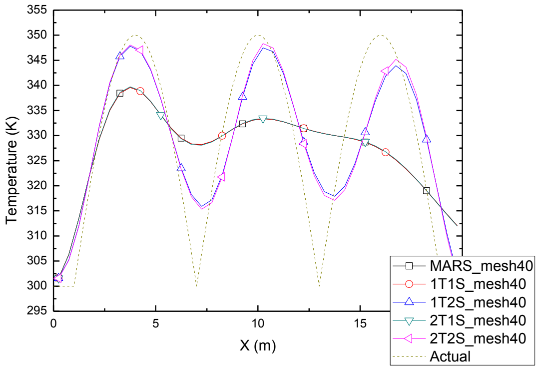

Fig. 11. Sensitivity results of MARS and NTS codes with mesh number 40 in case of pulse width 6sec and interval 0sec of test 01

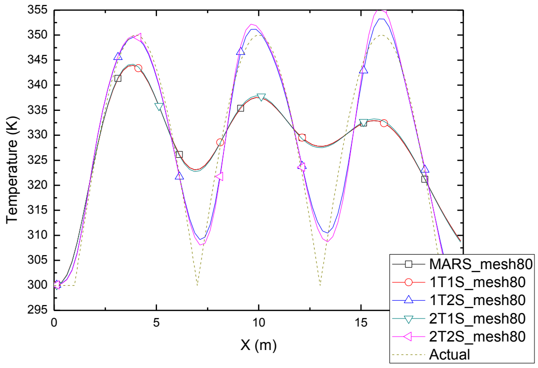

Fig. 12. Sensitivity results of MARS and NTS codes with mesh number 80 in case of pulse width 6sec and interval 0sec of test 01

Figs. 9~12 show the results of MARS and the NTS codes for mesh number 20, 40 and 80 when changing the temperature pulse to width of 6sec and interval of 0sec. In this case, similar results are obtained as well.

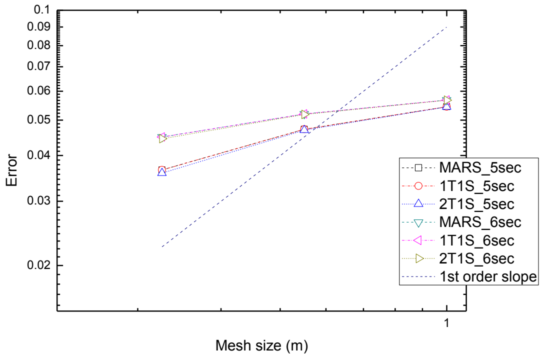

Fig. 13. Comparison of truncation error of the spatial 1st schemes depending on mesh size in test 01

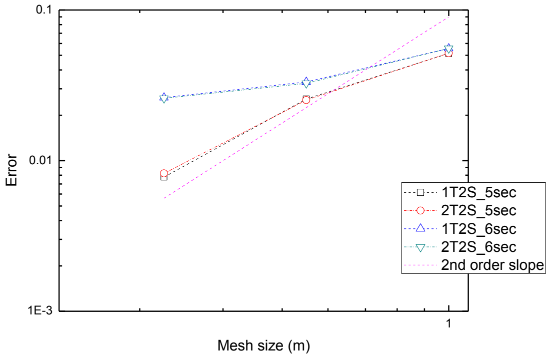

Fig. 14. Comparison of truncation error of the spatial 2nd schemes depending on mesh size in test 01

The truncation errors of the spatial first order schemes (MARS, 1T1S and 2T1S NTS codes) and the spatial second order schemes (1T2S and 2T2S NTS codes) are compared in Figs. 13 and 14, respectively. As shown in Figs. 13 and 14, it is identified that the truncation error is decreased as the number of meshes is increased and the temporal second-order scheme is similar to the temporal first-order scheme.

Table III. Maximum Courant number of NTS codes in case of pulse width 5sec and interval 1.5sec of test 01

| | 1T1S | 1T2S | 2T1S | 2T2S |
|---|---|---|---|---|
| Mesh 20 | 1.0 | 0.2628 | 0.7687 | 0.1349 |
| Mesh 40 | 1.0224 | 0.2499 | 0.6398 | 0.124 |
| Mesh 80 | 1.0147 | 0.26 | 0.6119 | 0.128 |

Table IV. Maximum Courant number of NTS codes in case of pulse width 6sec and interval 0sec of test 01

| | 1T1S | 1T2S | 2T1S | 2T2S |
|---|---|---|---|---|
| Mesh 20 | 1.0 | 0.27 | 0.75 | 0.14 |
| Mesh 40 | 1.0224 | 0.27 | 0.6498 | 0.136 |
| Mesh 80 | 1.0147 | 0.248 | 0.632 | 0.124 |

Tables III and IV show the maximum allowable Courant numbers of the NTS codes in case of pulse with width 5sec and interval 1.5sec and pulse of width 6sec and interval 0sec, respectively. Similar maximum Courant numbers are obtained in each scheme as shown in Tables III and IV. In 1T1S NTS code, the maximum Courant number is about 1.0 as expected. The maximum Courant number is about 0.26, 0.7 and 0.13 in 1T2S, 2T1S and 2T2S NTS codes, respectively. It is shown that the spatial second order schemes (1T2S and 2T2S) have significantly lower maximum Courant number.

## 3. Conclusions

This study evaluated the feasibility of the higher-order numerical scheme for the next generation nuclear system analysis code.

First, a good agreement with MARS and the first-order NTS code results was observed in both test cases. Therefore, it can be concluded that the developed first-order NTS code is identical to MARS code under the single phase flow condition. Also, the error between the code results and the actual solution was decreased as the number of meshes was increased for the pipe flow case. Numerical diffusion issues were re-confirmed in the first-order scheme. Furthermore, it was shown that the temperature distribution is severely distorted due to the numerical diffusion in the pipe flow problem. The accuracy is enhanced in the spatial second-order scheme and the numerical diffusion problem is alleviated while indicates significantly lower maximum Courant limit and the numerical dispersion issue which produces spurious oscillation and non-physical results in the higher-order scheme. If the spatial scheme is the first-order scheme then the temporal second-order scheme provides almost the same result with the temporal first-order scheme. However, when the temporal second-order scheme and the spatial second-order scheme are applied together, the numerical dispersion can occur more severely.

For the more in-depth study, the verification and validation of the NTS code built in MATLAB will be conducted further and expanded to handle two phase flow conditions as well. In this study, all test cases were limited to the single phase flow to observe the effect of the numerical scheme only. However, it was identified that the numerical diffusion problem is alleviated and the accuracy is improved in the second-order spatial scheme for the simple pipe flow simulation. Therefore, these effects should be also checked for the two phase flow conditions further, since it is expected that these effects can be amplified in the two phase flow conditions where the variables change dramatically. Additionally, to stabilize the numerical solutions of the higher-order scheme, a slope limiter should be applied or more stable higher-order scheme should be implemented and tested in the developed code.

## REFERENCES

[1] J.H. Mahaffy, “Numerics of codes: stability, diffusion, and convergence”, Nuclear Engineering and Design, 145(131-145), 1993

[2] D.R. Liles, Wm. H. Reed, “A Semi-implicit Method for Two-Phase Fluid Dynamics”, Journal of Compuational Physics, 26(390~407), 1978

[3] Jae-Jun Jeong, Kwi Seok Ha, Bub Dong Chung, and Won Jae Lee, “A Multi-Dimensional Thermal-Hydraulic System Analysis Code, MARS 1.3.1”, Journal of the Korean Nuclear Society, Volume 31, Number 3, pp. 344~363, 1999

[4] H.K.Cho, H.D. Lee, I.K. Park, J.J. Jeong, “Implementation of a second-order upwind method in a semi-implicit two-phase flow code on unstructured meshes”, 37(606~614), 2010

[5] H.K. Versteeg, W. Malalasekera, An introduction to computational fluid dynamics, Longman Scientific & Technical, pp. 103~133, 1995

[6] Ray Berry, Ling Zou, Haihua Zhao, David Andrs, John Peterson, Hongbin Zhang, Richard Martineau, RELAP7: Demonstrating Seven-Equation, Two-Phase Flow Simulation in a Single-pipe, Two-Phase Reactor Core and Steam Seperator/Dryer, INL, 2013

[7] R.A. Berry, J.W. Peterson, H. Zhang, R.C. Martineau, H. Zhao, L. Zou, D. Andrs, RELAP-7 Theory Manual, INL, 2014

[8] David Andrs , Ray Berry, Derek Gaston, Richard Martineau, John Peterson, Hongbin Zhang, Haihua Zhao, Ling Zou, RELAP-7 Level 2 Milestone Report: Demonstration of a Steady State Single Phase PWR Simulation with RELAP-7, INL, 2012

[9] MARS CODE MANUAL VOLUME I: Code Structure, System models, and Solution Methods, KAERI, 2009

[10] MARS CODE MANUAL VOLUME II: Input Requirements, KAERI, 2009

[11] MARS CODE MANUAL VOLUME V: Models and Correlations, KAERI, 2009