On the Development of Multi-Dimensional RELAP5 with Conservative Convective Terms

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Contents of discussions

- Multi-dimensional calculations are needed to correctly evaluate the behavior of the downcomer and/or core.
- RELAP5m33 is still one-dimensional. But it is not so difficult to improve RELAP5m33 to the multi-dimensional code by implementing the 3-dimensional momentum fluxes because its programming structure is almost ready for the improvement.
- There are various ways of handling momentum flux terms in multi-fluid multi-dimensional codes.
  - Conservative momentum equations
  - Non-conservative momentum equations
  - Phase intensive non-conservative momentum equations
  - Semi-conservative momentum equations
  - How to select the proper equations ?
- Since RELAP5 uses the non-conservative form, it is natural to use the non-conservative form for implementing the 3-dimensional momentum fluxes.

Contents of discussions

- But it is pointed out that using the non-conservative momentum flux terms is not proper for the open body problem such as the downcomer analysis.
- RELAP5-3D and MARS-Multi-D are developed but they use the non-conservative form of the momentum equations. Therefore, they are not good for the open body problems. Another aspects to consider is that they are not open to the general RELAP5 users.
- One way to resolve these problems is to use the semi-conservative form of momentum equations. The approach is mathematically ok but it is not easy to interpret the semi-conservative momentum flux terms physically.
- The last way is to use the conservative momentum flux terms. COBRA-TF is the only code which explicitly uses the conservative momentum fluxes. But its discretization is inconsistent.
- In this presentation, it is shown that using the spatially conservative momentum equations are possible by adopting the temporally non-conservative form of the momentum equations with the retarded correction.

Treatment of momentum fluxes in codes

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SPACE	RELAP5	RELAP5-3D	TRAC/ TRACE	CATHARE	COBRA-TF	CUPID
3-d.	1-d	3-d	3-d	3-d	3-d	3-d
non-cons.	non-cons.	non-cons.	non-cons.	semi-cons.	cons.	semi-cons.
phase int.	phase int.	phase int.	phase int.	mass weight	regular	mass weight
rect cyl	network	rect cyl	rect cyl	rect cyl sph	rect	unst.
fvm	fdm	fdm	fdm	fvm	fvm	fvm

Comparisons between the semi-conservative and non-conservative momentum equations in the non-porous body

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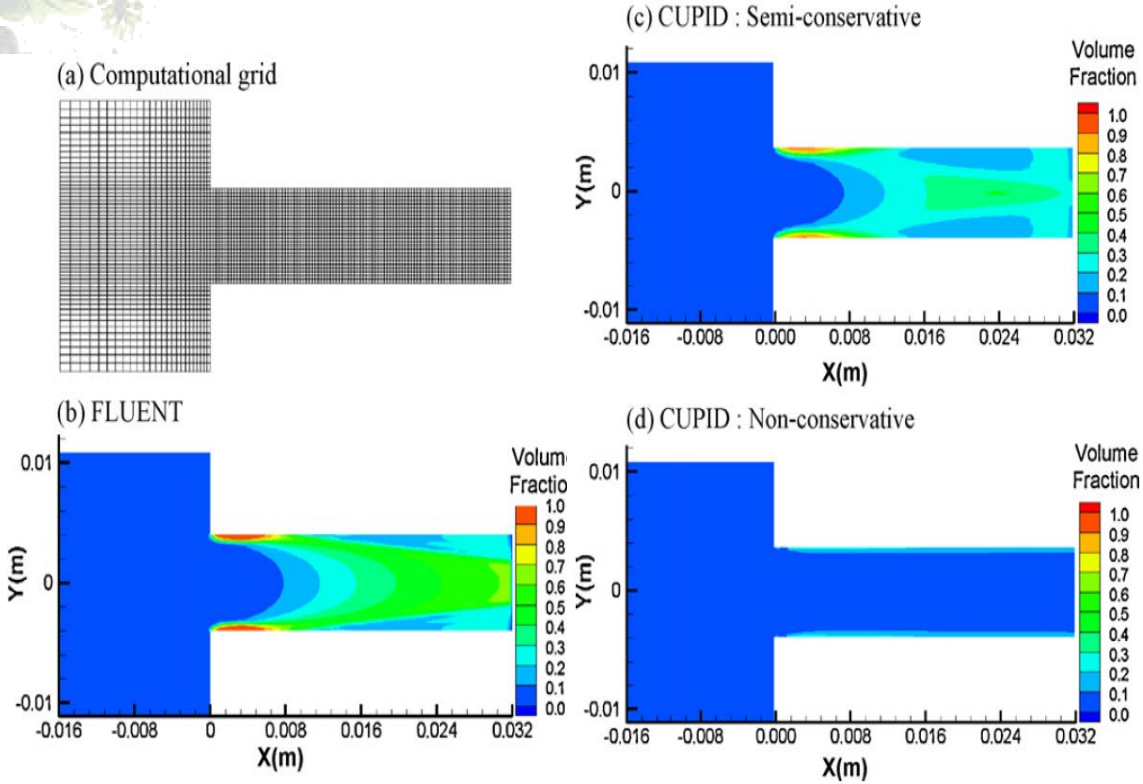

Non-conservative forms of momentum equations are not appropriate to model the open body problem. Instead semi-conservative forms can be used.

Why not using the conservative forms ???

Semi-implicitly linearized mass equations

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$$\frac{\alpha_g^{n+1}\rho_g^{n+1}-\alpha_g^n\rho_g^n}{\Delta t}V+\sum_f \alpha_{g,f}^n\rho_{g,f}^n v_{g,f}^{n+1}A_f=\Gamma_g{}^{n+1}V \qquad (18)$$

$$\frac{\alpha_l^{n+1}\rho_l^{n+1}-\alpha_l^n\rho_l^n}{\Delta t}V+\sum_f \alpha_{l,f}^n\rho_{l,f}^n v_{l,f}^{n+1}A_f=-\Gamma_g{}^{n+1}V \qquad (19)$$

Semi-implicitly linearized momentum equations in conservative forms

$$\frac{\alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}v_{g,f}^{n+1} - \alpha_{g,f}^{n}\rho_{g,f}^{n}v_{g,f}^{n}}{\Delta t}V_f + \sum_{f'} \alpha_g^n \rho_g^n v_g^n v_{g,f'}^n A_{f'}$$

$$= -\alpha_{g,f}^n\left(p_{f''}^{n+1} - p_{f'''}^{n+1}\right)A_f - \alpha_{g,f}^n\rho_{g,f}^n F_g^w v_{g,f}^{n+1} V_f$$

$$-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^n\left(v_{g,f}^{n+1} - v_{l,f}^{n+1}\right)V_f$$

$$+\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1} + \theta v_{l,f}^{n+1}\right)V_f + \alpha_{g,f}^n\rho_{g,f}^n g V_f \qquad (20)$$

$$\frac{\alpha_{l,f}^{n+1}\rho_{l,f}^{n+1}v_{l,f}^{n+1} - \alpha_{l,f}^{n}\rho_{l,f}^{n}v_{l,f}^{n}}{\Delta t}V_f + \sum_{f'} \alpha_l^n \rho_l^n v_l^n v_{l,f'}^n A_{f'}$$

$$= -\alpha_{l,f}^n\left(p_{f''}^{n+1} - p_{f'''}^{n+1}\right)A_f - \alpha_{l,f}^n\rho_{l,f}^n F_l^w v_{l,f}^{n+1} V_f$$

$$-\alpha_{g,f}^n\rho_{g,f}^n\alpha_{l,f}^n\rho_{l,f}^n F_i^{n+1}\left(v_{l,f}^{n+1} - v_{g,f}^{n+1}\right)V_f$$

$$-\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1} + \theta v_{l,f}^{n+1}\right)V_f + \alpha_{l,f}^n\rho_{l,f}^n g V_f \qquad (21)$$

Semi-implicitly linearized momentum equations in non-conservative forms

$$
\begin{aligned}
&\alpha_{g,f}^{n}\rho_{g,f}^{n}\frac{v_{g,f}^{n+1}-v_{g,f}^{n}}{\Delta t}V_f+\sum_{f'}\alpha_{g}^{n}\rho_{g}^{n}v_{g}^{n}v_{g,f'}^{n}A_{f'}\\
&\quad -v_{g,f}^{n}\sum_{f'}\alpha_{g}^{n}\rho_{g}^{n}v_{g,f'}^{n}A_{f'}=-\alpha_{g,f}^{n}\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f\\
&\quad -\alpha_{g,f}^{n}\rho_{g,f}^{n}F_{g}^{w}v_{g,f}^{n+1}V_f-\alpha_{g,f}^{n}\rho_{g,f}^{n}\alpha_{l,f}^{n}\rho_{l,f}^{n}F_{i}^{n}\left(v_{g,f}^{n+1}-v_{l,f}^{n+1}\right)V_f\\
&\quad +\Gamma_{g}^{n}\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f-\Gamma_{g}^{n}v_{g,f}^{n+1}V_f+\alpha_{g,f}^{n}\rho_{g,f}^{n}gV_f \quad (22)
\end{aligned}
$$

$$
\begin{aligned}
&\alpha_{l,f}^{n}\rho_{l,f}^{n}\frac{v_{l,f}^{n+1}-v_{l,f}^{n}}{\Delta t}V_f+\sum_{f'}\alpha_{l}^{n}\rho_{l}^{n}v_{l}^{n}v_{l,f'}^{n}A_{f'}\\
&\quad -v_{l,f}^{n}\sum_{f'}\alpha_{l}^{n}\rho_{l}^{n}v_{l,f'}^{n}A_{f'}=-\alpha_{l,f}^{n}\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f\\
&\quad -\alpha_{l,f}^{n}\rho_{l,f}^{n}F_{l}^{w}v_{l,f}^{n+1}V_f-\alpha_{g,f}^{n}\rho_{g,f}^{n}\alpha_{l,f}^{n}\rho_{l,f}^{n}F_{i}^{n+1}\left(v_{l,f}^{n+1}-v_{g,f}^{n+1}\right)V_f\\
&\quad -\Gamma_{g}^{n}\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f-\Gamma_{g}^{n}v_{l,f}^{n+1}V_f+\alpha_{l,f}^{n}\rho_{l,f}^{n}gV_f \quad (23)
\end{aligned}
$$

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Solving for the intermediate velocities: RELAP5

Non-conservative form and phase-intensive form are same

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} v_{g,f}^{n+1} \\ v_{l,f}^{n+1} \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix} \left(\delta p_{f''}^{n+1} - \delta p_{f'''}^{n+1} \right) + \begin{pmatrix} t \\ u \end{pmatrix} \qquad (24)$$

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Solving for the intermediate velocities: COBRA-TF

Because of solving for mass flow it needs more steps!!!

Change velocity to momentum flux before solving

$$\begin{pmatrix} v_{g,f}^{n+1} \rightarrow \dfrac{1}{\alpha_{g,f}^{n}\rho_{g,f}^{n}}\alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}v_{g,f}^{n+1} \\ v_{l,f}^{n+1} \rightarrow \dfrac{1}{\alpha_{l,f}^{n}\rho_{l,f}^{n}}\alpha_{l,f}^{n+1}\rho_{l,f}^{n+1}v_{l,f}^{n+1} \end{pmatrix} \tag{25}$$

Solving for momentum flux

$$\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}\begin{pmatrix} \alpha_{g,f}^{n+1}\rho_{g,f}^{n+1}v_{g,f}^{n+1} \\ \alpha_{l,f}^{n+1}\rho_{l,f}^{n+1}v_{l,f}^{n+1} \end{pmatrix} = \begin{pmatrix} r' \\ s' \end{pmatrix}\left(\delta p_{f''}^{n+1} - \delta p_{f'''}^{n+1}\right) + \begin{pmatrix} t' \\ u' \end{pmatrix} \tag{26}$$

Factoring velocities

$$\begin{pmatrix} \dfrac{1}{\alpha_{g,f}^{n}\rho_{g,f}^{n}}\alpha_{g,f}^{*}\rho_{g,f}^{*}v_{g,f}^{*} \rightarrow v_{g,f}^{*} \\ \dfrac{1}{\alpha_{l,f}^{n}\rho_{l,f}^{n}}\alpha_{l,f}^{*}\rho_{l,f}^{*}v_{l,f}^{*} \rightarrow v_{l,f}^{*} \end{pmatrix} \tag{27}$$

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COBRA-TF solution scheme

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Equivalent to solve the following equations

$$
\begin{aligned}
&\alpha_{g,f}^{n}\rho_{g,f}^{n}\frac{v_{g,f}^{n+1}-v_{g,f}^{n}}{\Delta t}V_f+\sum_{f'}\alpha_g^n\rho_g^n v_g^n v_{g,f'}^n A_{f'}\\
&\quad=-\alpha_{g,f}^{n}\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f-\alpha_{g,f}^{n}\rho_{g,f}^{n}F_g^{w}v_{g,f}^{n+1}V_f\\
&\qquad-\alpha_{g,f}^{n}\rho_{g,f}^{n}\alpha_{l,f}^{n}\rho_{l,f}^{n}F_i^{n}\left(v_{g,f}^{n+1}-v_{l,f}^{n+1}\right)V_f\\
&\qquad+\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f+\alpha_{g,f}^{n}\rho_{g,f}^{n}gV_f
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\alpha_{l,f}^{n}\rho_{l,f}^{n}\frac{v_{l,f}^{n+1}-v_{l,f}^{n}}{\Delta t}V_f+\sum_{f'}\alpha_l^n\rho_l^n v_l^n v_{l,f'}^n A_{f'}\\
&\quad=-\alpha_{l,f}^{n}\left(p_{f''}^{n+1}-p_{f'''}^{n+1}\right)A_f-\alpha_{l,f}^{n}\rho_{l,f}^{n}F_l^{w}v_{l,f}^{n+1}V_f\\
&\qquad-\alpha_{g,f}^{n}\rho_{g,f}^{n}\alpha_{l,f}^{n}\rho_{l,f}^{n}F_i^{n+1}\left(v_{l,f}^{n+1}-v_{g,f}^{n+1}\right)V_f\\
&\qquad-\Gamma_g^n\left((1-\theta)v_{g,f}^{n+1}+\theta v_{l,f}^{n+1}\right)V_f+\alpha_{l,f}^{n}\rho_{l,f}^{n}gV_f
\end{aligned}
\tag{29}
$$

with the assumptions

$$
\begin{pmatrix}
\dfrac{\partial\alpha_g\rho_g}{\partial t}=0\\
\dfrac{\partial\alpha_l\rho_l}{\partial t}=0
\end{pmatrix}
\tag{30}
$$

Inconsistent discretization !!!!!!!!

Correct Approach - 1

Solve the temporally non-conservative momentum equations

$$\alpha_g\rho_g\frac{\partial \boldsymbol{v}_g}{\partial t}+\boldsymbol{v}_g\frac{\partial \alpha_g\rho_g}{\partial t}+\nabla\cdot\left(\alpha_g\rho_g\boldsymbol{v}_g\boldsymbol{v}_g\right)$$
$$=-\alpha_g\nabla p-\alpha_g\rho_g F_g^{w}\boldsymbol{v}_g-\alpha_g\rho_g\alpha_l\rho_l F_i\left(\boldsymbol{v}_g-\boldsymbol{v}_l\right)$$
$$+\Gamma_g\left((1-\theta)\boldsymbol{v}_g+\theta\boldsymbol{v}_l\right)+\alpha_g\rho_g\boldsymbol{g} \qquad (31)$$

$$\alpha_l\rho_l\frac{\partial \boldsymbol{v}_l}{\partial t}+\boldsymbol{v}_l\frac{\partial \alpha_l\rho_l}{\partial t}+\nabla\cdot\left(\alpha_l\rho_l\boldsymbol{v}_l\boldsymbol{v}_l\right)=$$
$$-\alpha_l\nabla p-\alpha_l\rho_l F_l^{w}\boldsymbol{v}_l-\alpha_g\rho_g\alpha_l\rho_l F_i\left(\boldsymbol{v}_l-\boldsymbol{v}_g\right)$$
$$-\Gamma_g\left((1-\theta)\boldsymbol{v}_g+\theta\boldsymbol{v}_l\right)+\alpha_l\rho_l\boldsymbol{g} \qquad (32)$$

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Discretized temporally non-conservative momentum equations

retarded correction

$$
\begin{aligned}
&\alpha^n_{g,f}\rho^n_{g,f}\frac{v^{n+1}_{g,f}-v^n_{g,f}}{\Delta t}V_f + v^{n+1}_{g,f}\frac{\alpha^n_{g,f}\rho^n_{g,f}-\alpha^{n-1}_{g,f}\rho^{n-1}_{g,f}}{\Delta t}V_f \\
&\quad+\sum_{f'}\alpha^n_g\rho^n_g v^n_g v^n_{g,f'}A_{f'} = -\alpha^n_{g,f}\left(p^{n+1}_{f''}-p^{n+1}_{f'''}\right)A_f \\
&\quad-\alpha^n_{g,f}\rho^n_{g,f}F^w_g v^{n+1}_{g,f}V_f - \alpha^n_{g,f}\rho^n_{g,f}\alpha^n_{l,f}\rho^n_{l,f}F^n_i\left(v^{n+1}_{g,f}-v^{n+1}_{l,f}\right)V_f \\
&\quad+\Gamma^n_g\left((1-\theta)v^{n+1}_{g,f}+\theta v^{n+1}_{l,f}\right)V_f + \alpha^n_{g,f}\rho^n_{g,f}gV_f \qquad (33)
\end{aligned}
$$

$$
\begin{aligned}
&\alpha^n_{l,f}\rho^n_{l,f}\frac{v^{n+1}_{l,f}-v^n_{l,f}}{\Delta t}V_f + v^{n+1}_{l,f}\frac{\alpha^n_{l,f}\rho^n_{l,f}-\alpha^{n-1}_{l,f}\rho^{n-1}_{l,f}}{\Delta t}V_f \\
&\quad+\sum_{f'}\alpha^n_l\rho^n_l v^n_l v^n_{l,f'}A_{f'} = -\alpha^n_{l,f}\left(p^{n+1}_{f''}-p^{n+1}_{f'''}\right)A_f \\
&\quad-\alpha^n_{l,f}\rho^n_{l,f}F^w_l v^{n+1}_{l,f}V_f - \alpha^n_{g,f}\rho^n_{g,f}\alpha^n_{l,f}\rho^n_{l,f}F^{n+1}_i\left(v^{n+1}_{l,f}-v^{n+1}_{g,f}\right)V_f \\
&\quad-\Gamma^n_g\left((1-\theta)v^{n+1}_{g,f}+\theta v^{n+1}_{l,f}\right)V_f + \alpha^n_{l,f}\rho^n_{l,f}gV_f \qquad (34)
\end{aligned}
$$

Correct approach - 2

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Use retarded corrected $\alpha\rho$

$$\begin{pmatrix} \dfrac{1}{\alpha_{g,f}^{n}\rho_{g,f}^{n} + \delta\alpha_{g,f}^{n}\rho_{g,f}^{n}}\alpha_{g,f}^{*}\rho_{g,f}^{*}v_{g,f}^{*} \rightarrow v_{g,f}^{*} \\ \dfrac{1}{\alpha_{l,f}^{n}\rho_{l,f}^{n} + \delta\alpha_{l,f}^{n}\rho_{l,f}^{n}}\alpha_{l,f}^{*}\rho_{l,f}^{*}v_{l,f}^{*} \rightarrow v_{l,f}^{*} \end{pmatrix} \tag{35}$$

where

$$\begin{pmatrix} \delta\alpha_{g,f}^{n}\rho_{g,f}^{n} = \dfrac{\alpha_{g,f}^{n}\rho_{g,f}^{n} - \alpha_{g,f}^{n-1}\rho_{g,f}^{n-1}}{\Delta t^{n-1}}\Delta t^{n} \\ \delta\alpha_{l,f}^{n}\rho_{l,f}^{n} = \dfrac{\alpha_{l,f}^{n}\rho_{l,f}^{n} - \alpha_{l,f}^{n-1}\rho_{l,f}^{n-1}}{\Delta t^{n-1}}\Delta t^{n} \end{pmatrix} \tag{36}$$

It is interesting to see that the retarded correction is already used in solving the non-conservative equations

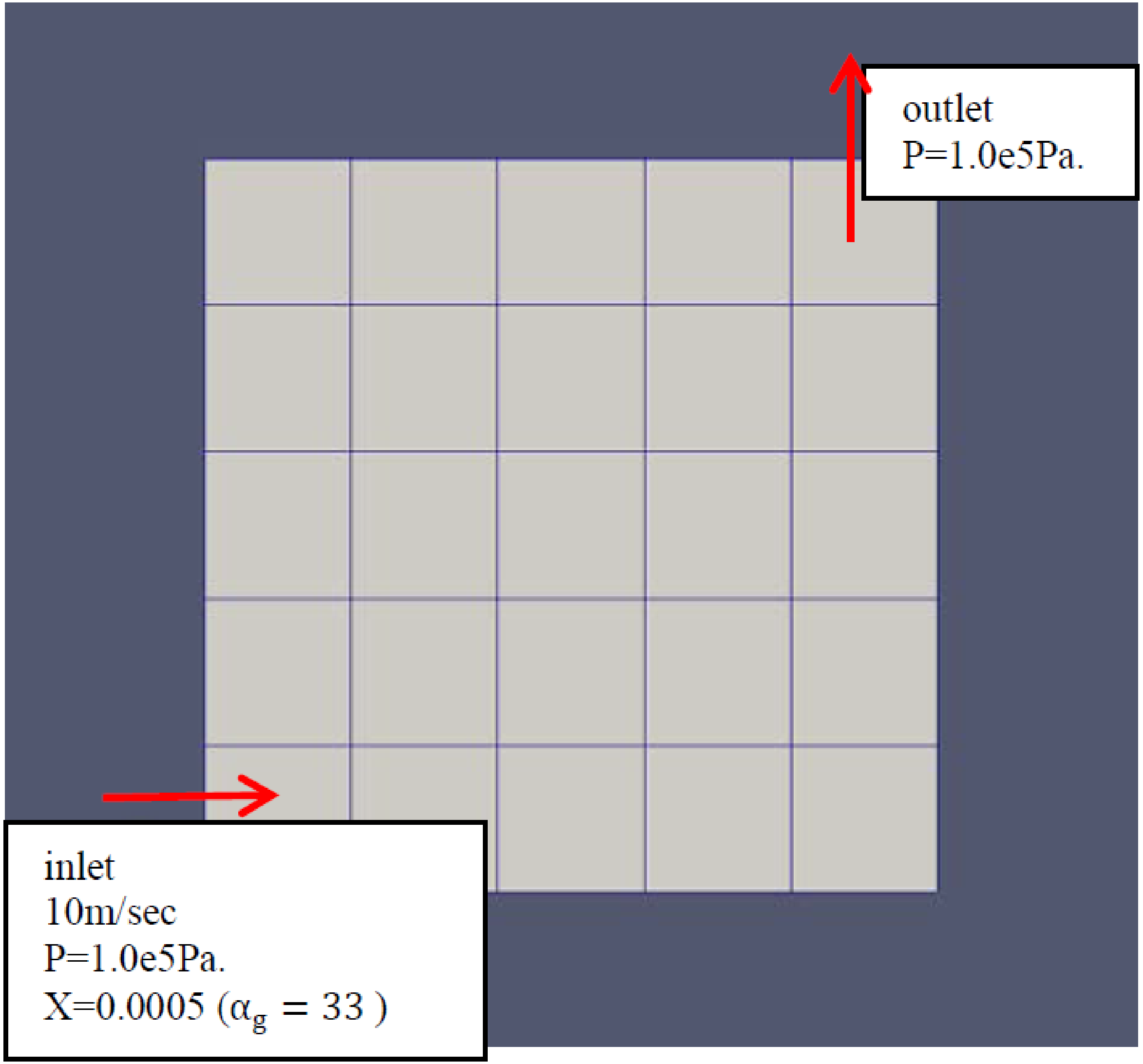
$$\Gamma_g V_f = \frac{\alpha_{g,f}^{n}\rho_{g,f}^{n} - \alpha_{g,f}^{n-1}\rho_{g,f}^{n-1}}{\Delta t}V_f + \sum_{f'} \alpha_{g,f'}^{n}\rho_{g,f'}^{n}v_{g,f'}^{n}A_{f'} \tag{37}$$

$$\Gamma_g{}^{n} V_f = \frac{\alpha_{l,f}^{n}\rho_{l,f}^{n} - \alpha_{l,f}^{n-1}\rho_{l,f}^{n-1}}{\Delta t}V_f + \sum_{f'} \alpha_{l,f'}^{n}\rho_{l,f'}^{n}v_{l,f'}^{n}A_{f'} \tag{38}$$

Implementation of the conservative momentum flux terms in RELAP5m33

1. Remove 1-dimensiobnal momentum flux terms.
2. Insert 3-dimensiobnal conservative momentum flux terms
3. Remove the term, $\Gamma_k^n v_f^{n+1}$.
4. Add the mass derivative term;

Validation problem definition



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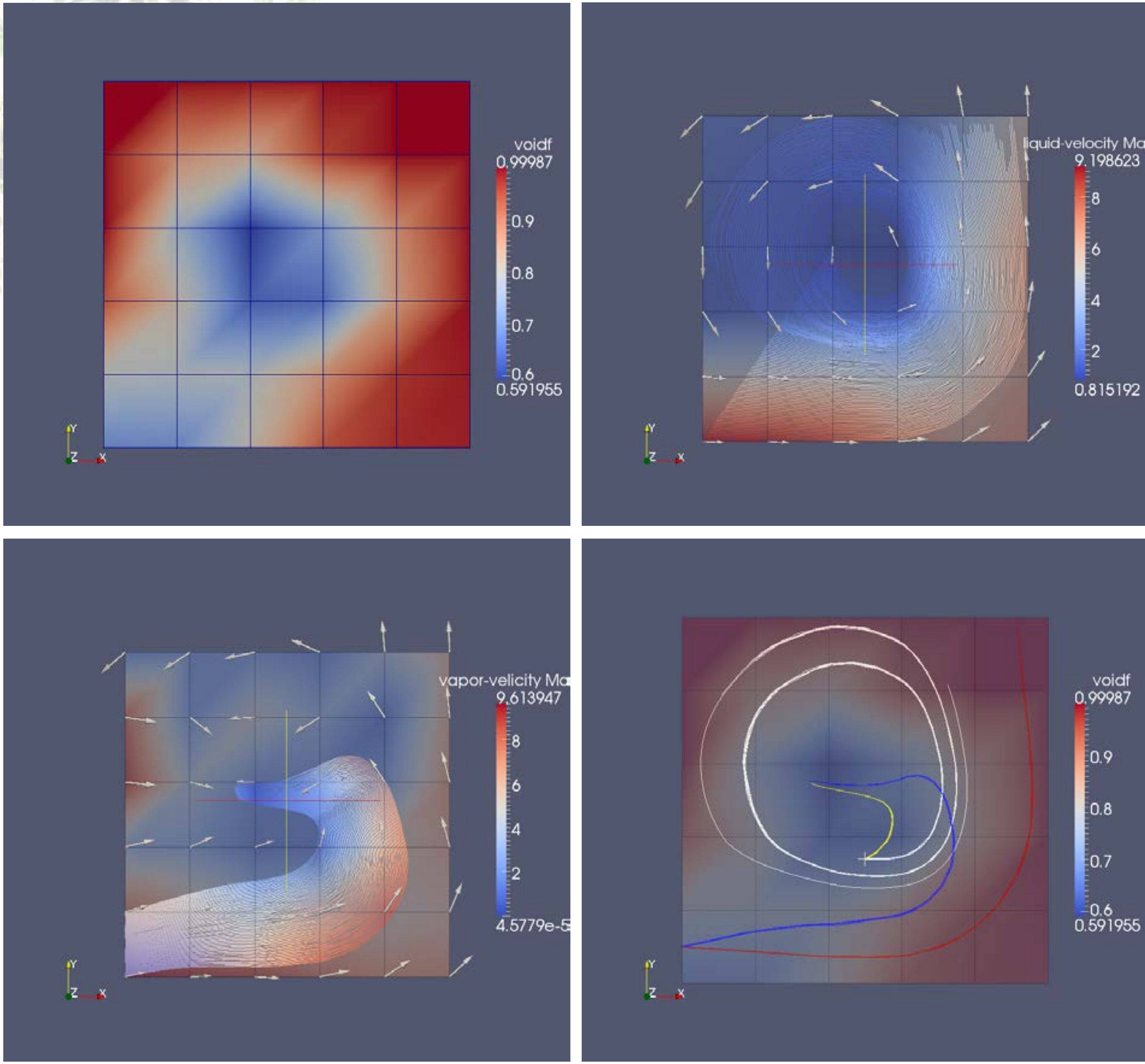
RELAP5 with conservative momentum fluxes



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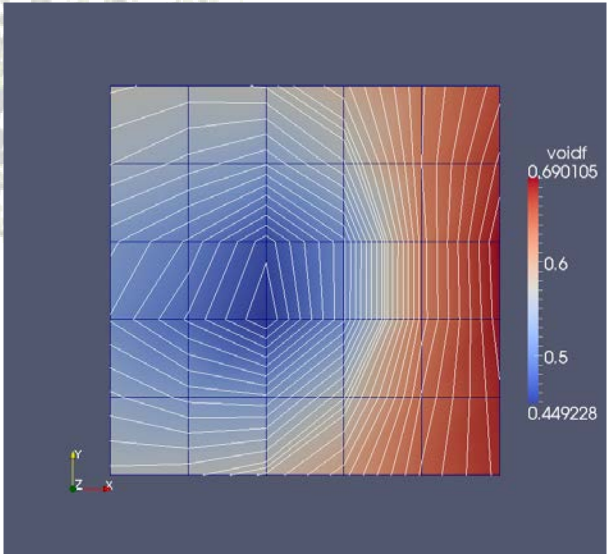
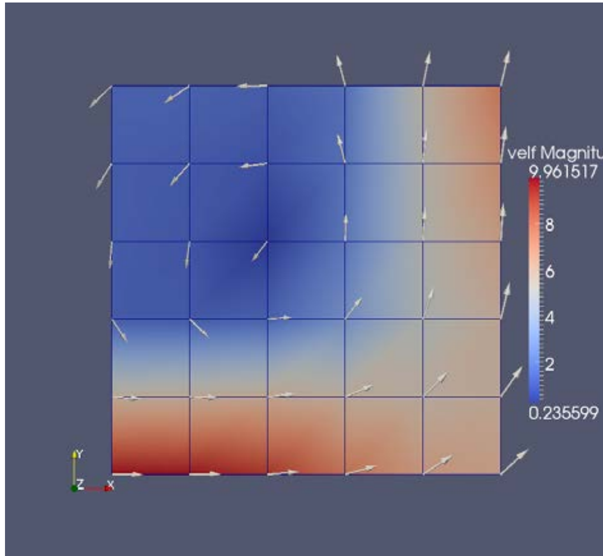
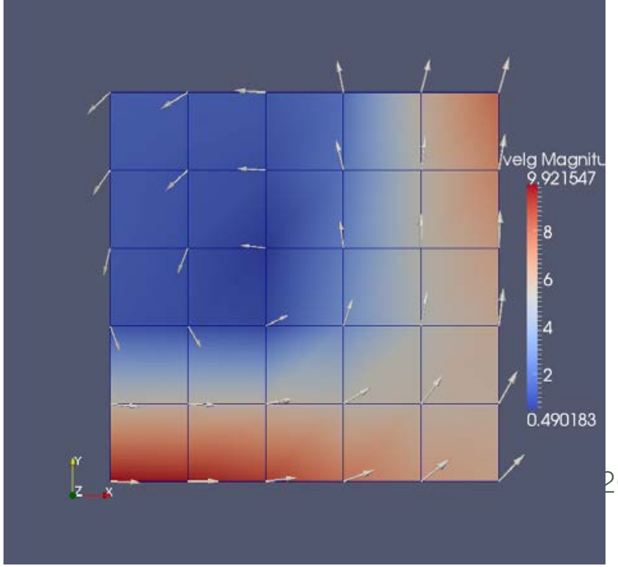
Stream lines

MARS-Multi-D







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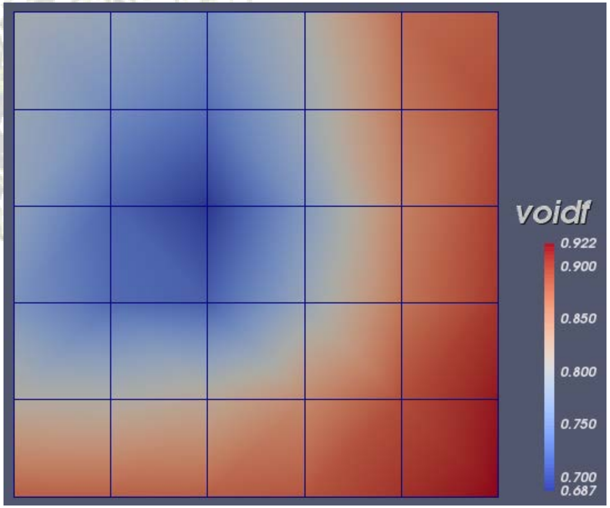
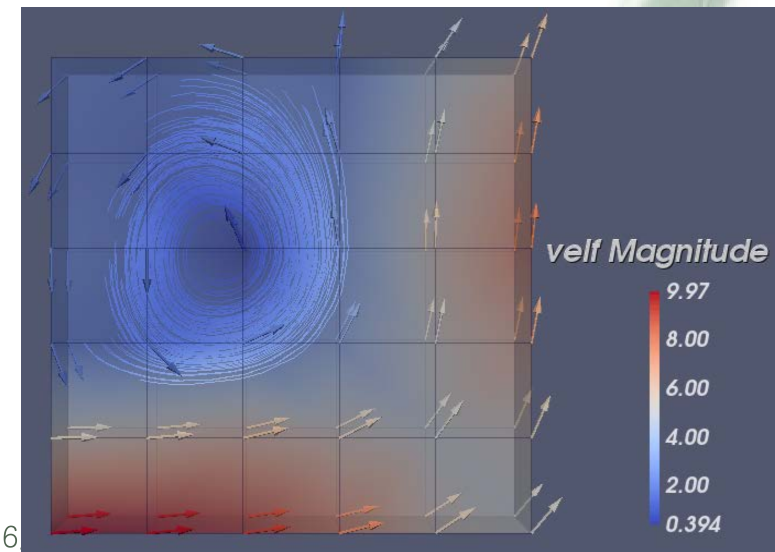
CUPID





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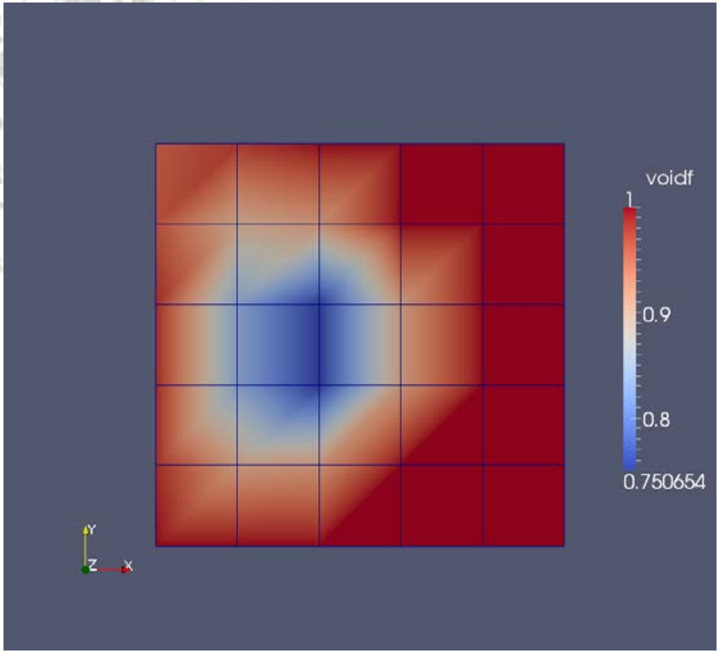
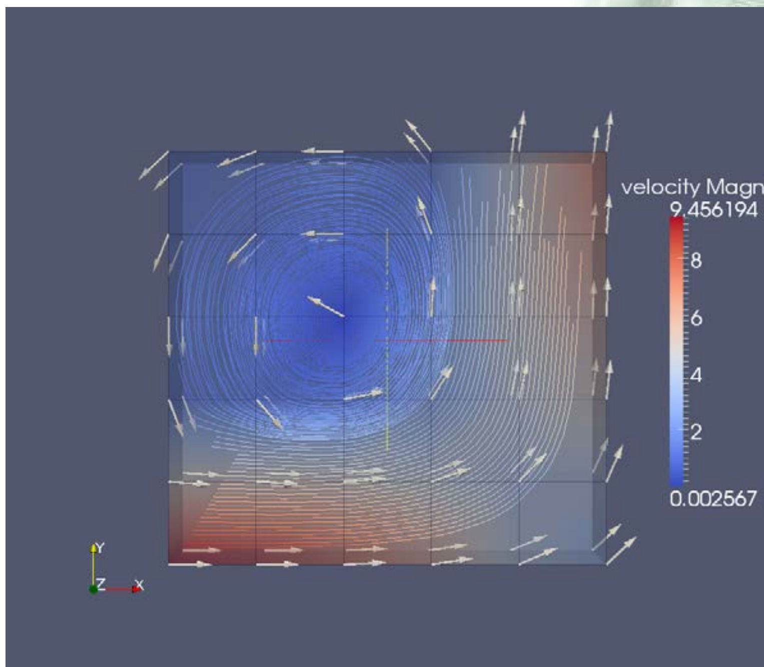
TRACE





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Conclusions

- From the present study, it can be concluded that the intuitive centrifugal phase separation is reproduced with the conservative, semi-conservative or non-conservative convective terms in the momentum equations.
- But the non-conservative form in finite difference approach may not be good for the problem that have strong gradient of the volume fraction in open body. MARS Multi-D and SPACE are such codes.
- The way handling the fully conservative momentum fluxes in COBRA-TF and its derivatives are not correct.
- Two suggestions are made
  - * Using the temporally non-conservative equations with retarded corrections
  - * Using the retarded mass correction when solving for intermediate velocities
- The implementation of the conservative convective terms in RELAP5 seems to be successful. Further elaboration of improvement activities such as input handling system may be necessary to develop the RELAP5 as a fully multi-dimensional code.

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