Application of the Modified Power Method to 2D Core Simulation

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1. Introduction

The modified power method (MPM) was proposed to get the first two eigenmodes simultaneously [1-6]. It has been extended to get even higher eigenmodes with a general solution strategy recently [7-10]. In order to get the first *N* eigenmodes, *N* coarse meshes should be used. This method works well for 1D problems since all the eigenvalues are discrete. However, the application of this method to multi-dimensional problems may be problematic due to the degeneracy issues. In these cases, in order to get the first *N* eigenmodes, more than *N* coarse meshes should be used. In this paper, the corresponding modification to the MPM will be discussed, and the 2D C5G7 benchmark results will be presented to show the performance.

2. Methods and Results

2.1 The MPM Adopting More Coarse Meshes

The key of the MPM is to solve the following linear equation system:

$$\mathbf{WX} = \mathbf{VX\Lambda}, \tag{1}$$

where $\mathbf{W},\mathbf{V} \in R^{M\times N}$ and $\mathbf{X},\mathbf{\Lambda} \in R^{N\times N}$. M is the number of meshes, while N is the number of modes. $V_{i,j}$ is the integration of j^{th} fission source over i^{th} mesh before the power operation, $V_{ij} = \int_{Ri} \psi_j dr$, while $W_{i,j}$ is the corresponding integration after the power operation, $W_{ij} = \int_{Ri} A\psi_j dr$. The $\mathbf{X}$ matrix contains the linear coefficients to combine the source distributions to get the better eigenfunction estimations. $\mathbf{\Lambda}$ is a diagonal matrix and its diagonal elements are the eigenvalues of the system.

In the case of M>N, a matrix $\mathbf{G} \in R^{N\times M}$ is used to multiply the both sides of Eq. (1):

$$\begin{aligned} &\mathbf{GWX} = \mathbf{GVX\Lambda}, \\ &(\mathbf{GW}) = \mathbf{GVX\Lambda X}^{-1} = (\mathbf{GVX})\mathbf{\Lambda}(\mathbf{GVX})^{-1}(\mathbf{GV}) \\ &= \mathbf{P}^{TM}(\mathbf{GV}). \end{aligned} \tag{2}$$

The $\mathbf{G}$ matrix is chosen at the beginning cycles satisfying $\mathbf{GV} = \mathbf{I}^{N\times N}$. Actually, $\mathbf{G}$ can be an arbitrary matrix if only $(\mathbf{GV})$ is a full rank matrix. $\mathbf{P}^{TM}$ is the transfer matrix that can be calculated at the end of every cycle according to Eq. (2).

From Eq. (2) it can be seen that the eigenvalues of the transfer matrix are the eigenvalues of the system. The eigen-decomposition of the transfer matrix, $\mathbf{P}^{TM} = \mathbf{Q\Lambda Q}^{-1}$, provides the eigenvectors that can be used to solve the linear combination factors:

$$\begin{aligned} &\mathbf{GVX} = \mathbf{Q}, \\ &\mathbf{X} = (\mathbf{GV})^{-1}\mathbf{Q}. \end{aligned} \tag{3}$$

The fission sources can be updated as:

$$(\psi'_1,...,\psi'_N) = (\psi_1,...,\psi_N)\mathbf{X}. \tag{4}$$

2.2 The 2D C5G7 Benchmark Results

The 2D C5G7 benchmark was modeled using multigroup Monte Carlo with the MPM. The 2D one quarter core was shown in Fig. 1. The detailed geometry and cross section data can be found in the benchmark specification.

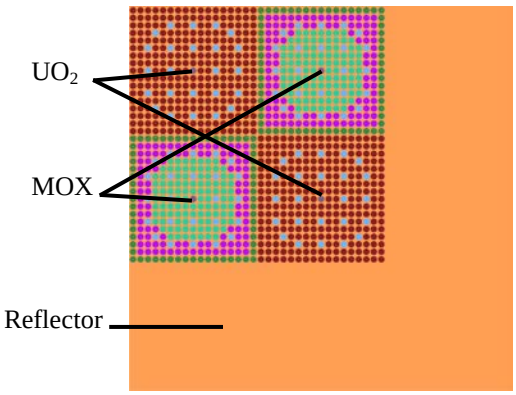

Fig. 1. The 2D C5G7 core model.

All the simulations were done with 50 inactive cycles, 150 active cycles and 200,000 histories per cycle. The uniform 4-by-4 meshes and 4-by-5 meshes across the core region were used to get the first 16 eigenmodes.

The Shannon entropy results are shown in Fig. 2. The simulation of MPM with 4-by-4 meshes to get the first 16 eigenmodes is stable at the first 60 cycles, but after that it becomes unstable. The behavior is similar even if the histories per cycle increased dramatically. However,

the simulation of MPM with 4-by-5 meshes to get the first 16 eigenmodes is stable all the time.

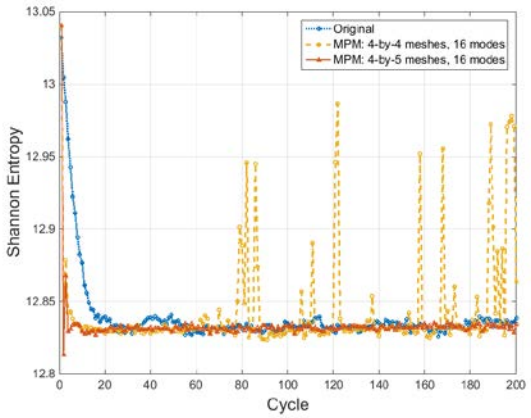

Fig. 2. The Shannon entropy results of different methods.

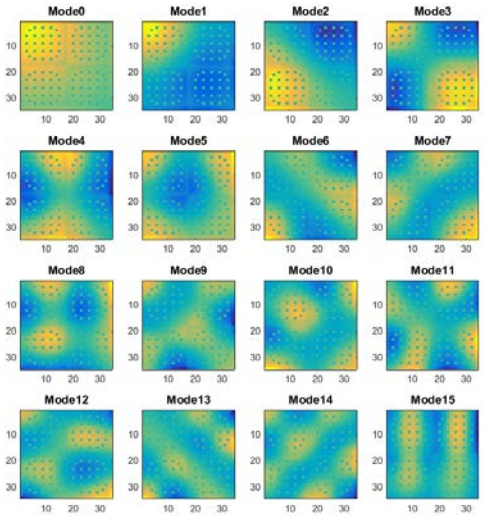

Fig. 3. The first 16 fission source eigenmodes of the 2D C5G7 model.

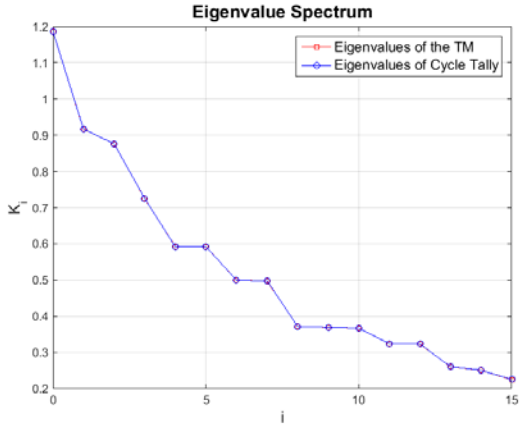

Fig. 4. The eigenvalue spectrum with 16 eigenmodes obtained using the 4-by-5 meshes.

Table I: The First 16 Eigenvalues of the 2D C5G7 Model

Method	Eigenvalue	Result
Original	k_0	1.18642 (12)
Modified Power Method	k_0	1.18672 (08)
	k_1	0.91676 (17)
	k_2	0.87620 (19)
	k_3	0.72526 (17)
	k_4	**0.59174 (20)**
	k_5	**0.59193 (18)**
	k_6	0.49945 (18)
	k_7	0.49757 (19)
	k_8	0.37095 (17)
	k_9	0.36950 (20)
	k_{10}	0.36657 (19)
	k_{11}	0.32432 (19)
	k_{12}	0.32296 (20)
	k_{13}	0.26014 (18)
	k_{14}	0.25012 (18)
	k_{15}	0.22492 (19)

The first 16 eigenmode fission source distributions are shown in Fig. 3. Fig. 4 shows the eigenvalue spectrum. The eigenvalues of the TM are the eigenvalues of the finally accumulated TM, and the eigenvalues of cycle tally are the averages of cycle tallied eigenvalues, which are calculated as:

$$k_i = \frac{\sum_j \left| w_j[i] \right|}{N_{HPC}}, \; i = 0,\ 1,\ \ldots,\ \mathrm{N}-1, \qquad (5)$$

where i is the eigenmode index, j is the neutron index, and N_{HPC} is the total number of histories per cycle. Table I lists the tallied eigenvalues, where k_0 is tallied with collision estimator, absorption estimator and track length estimator, while the higher mode eigenvalues are tallied according to Eq. (5), so their statistical errors are bigger than the fundamental mode. As there is no symmetry of the core model along the diagonal from upper right to lower left, the 1st and 2nd eigenmodes are not degenerated. The degeneracy may only happen for the 4th and 5th eigenmodes.

3. Conclusions

Due to the possible degeneracy issues for the eigenmodes of multi-dimensional problems, the application of modified power method requires more number of meshes than the number of eigenmodes. A preprocessing matrix can be used to multiply the fission source integrals, so that the transfer matrix can be solved as usual. The 2D C5G7 benchmark was modeled with the new method, and the results demonstrated the effectiveness of this method for practical problems. The application of this method to 3D problems is guaranteed

since there are no essential differences between 2D and 3D for the degeneracy issues.

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