An Evaluation of Eigenvalue Uncertainty Caused by Monte Carlo Uncertainties of Multi-group Cross Section

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Outline

Introduction

- Motivation
- Goal

Method

- Monte Carlo Method to Calculate Multi-group Cross Section
- Error Propagation
- Correlated Sampling Method

Result

- Problem 1
- Problem 2

Conclusion

Introduction

Reactor Analysis

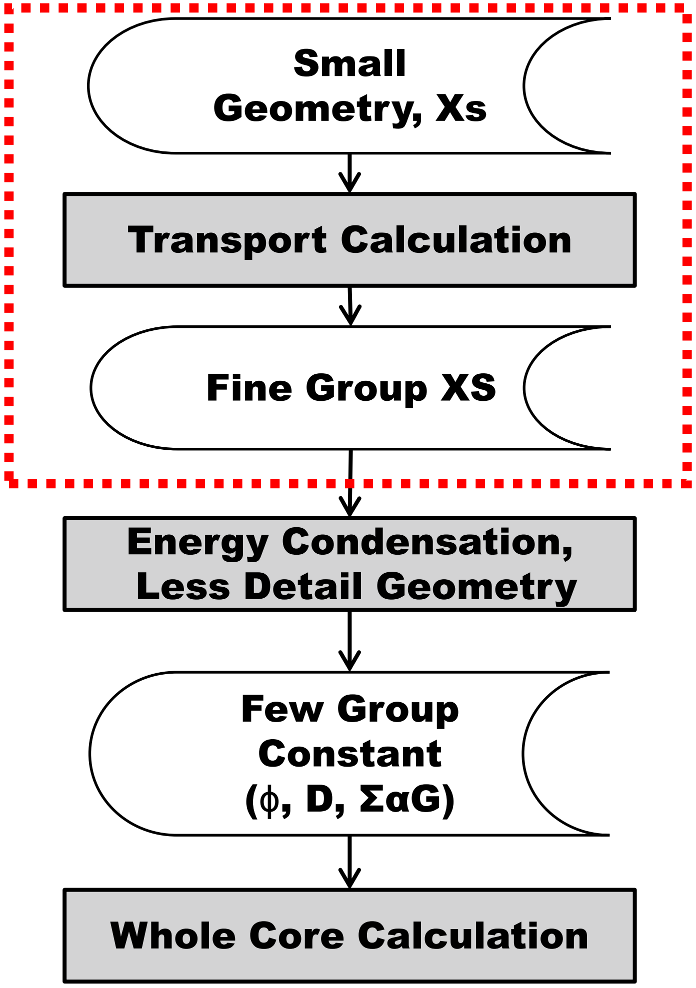
- **Whole Core Calculation**
- **FGC is Calculated from Fine Group Xs**
- **Fine Group Xs**

  Detailed in Geometry, Detailed Energy
- **Accurate Calculations Required**

  Uncertainty of fine group Xs may occur

  significant biases in next steps

Modern Reactor Analysis Scheme



Introduction

- **Consider Complicate Geometry Pin cell**

  - VHTR TRISO particle Fuel, Research Reactor

- **Group XS Generation for Complicate Geometry**

  - **Monte Carlo(MC) Method can be advantageous**

- **Monte Carlo Method Properties**

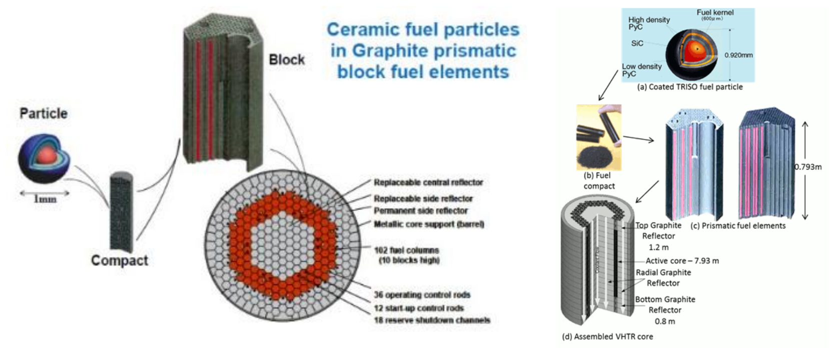
  - Geometry Description

  - Result with Uncertainty σ

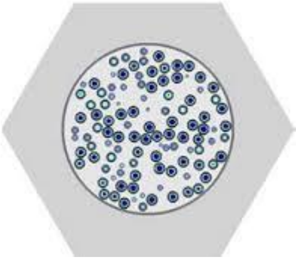
  - Computationally Intense: $\frac{1}{\sigma} \sim N^2$

  - Potentially Accurate

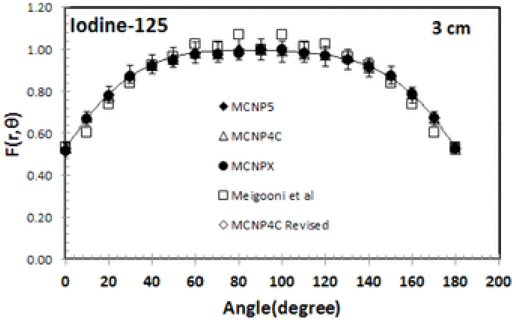
Complicate Geometry



Modeling



Uncertainty



Introduction

- **Group Xs Generated via MC**

  Each Group Xs has uncertainties.

- **MC Uncertainties causes biases in Results**

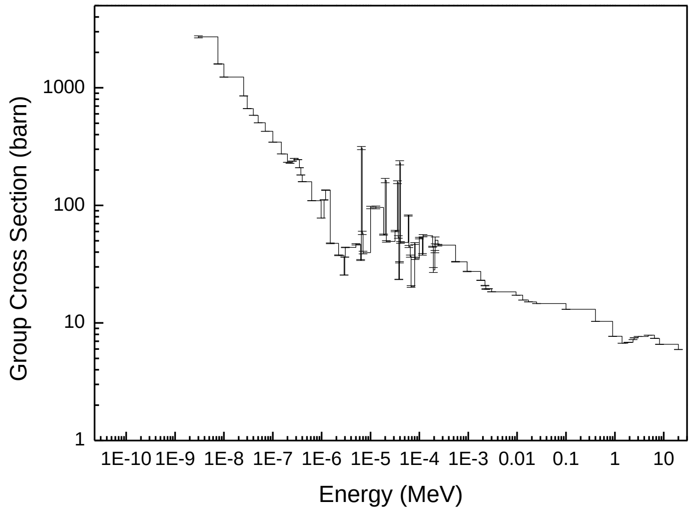
  Fine Group Xs is input of next calculation Steps, uncertainty of Group Xs may occur biases in results.

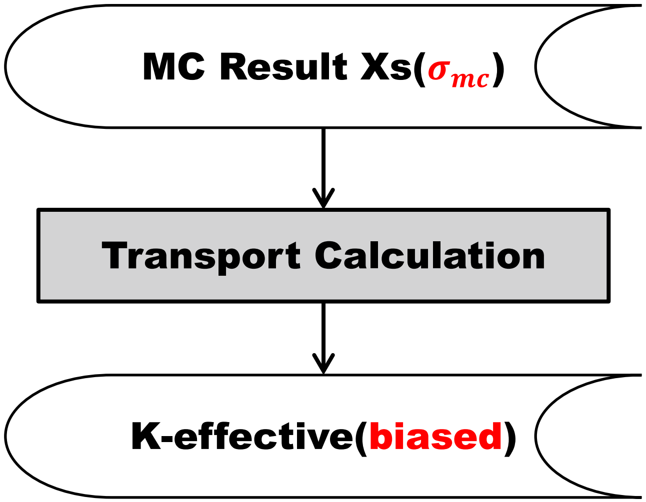
- **Potentially Accurate**

  More computational load, Better result

- **Goal of This Study**

  Evaluate Uncertainty of k-eff caused by MC uncertainty of group Xs.





METHOD

Method

Group XS and Uncertainty

- **Group Averaged Xs**

  - $\sigma_g = \dfrac{\int_V \int_{E_g} \int_{4\pi} \sigma(r,E)\phi(r,E,\Omega)d\Omega dEdr}{\int_V \int_{E_g} \int_{4\pi} \phi(r,E,\Omega)d\Omega dEdr}$

- **Track Length Estimator**

  - $\sigma_g = \dfrac{\sum_i \sigma_i W_i L_{g,i}}{\sum_i W_i L_{g,i}}$

- **Uncertainty of group Xs**

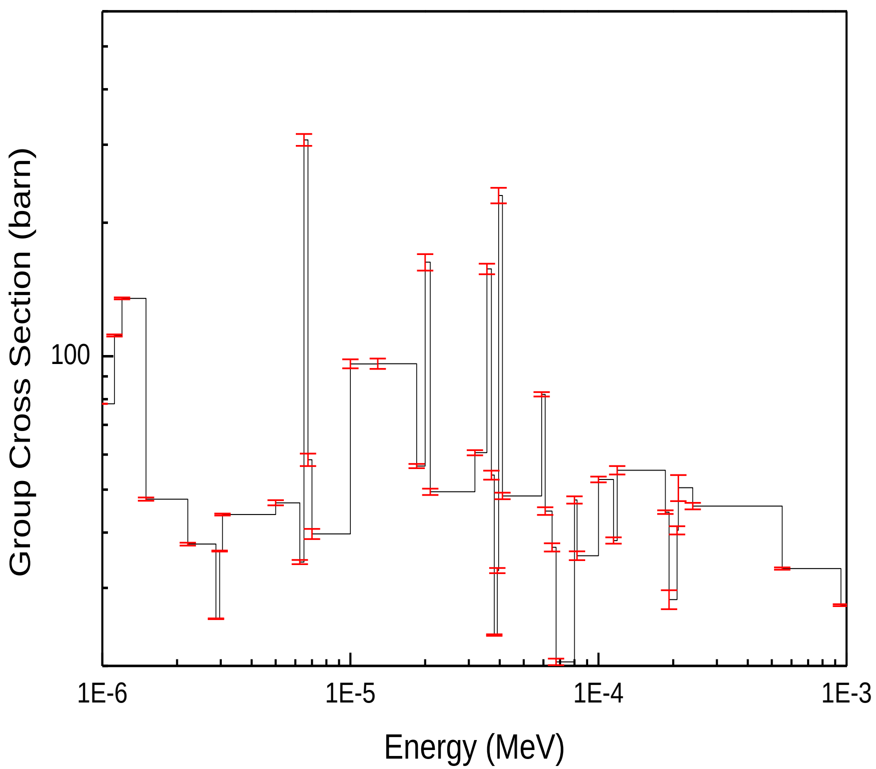
  - Correlation between to variables

  - $\bar{x} = \dfrac{\bar{\boldsymbol{u}}}{\bar{\boldsymbol{v}}}, r_{\bar{x}} = \sqrt{\dfrac{\sum u_i^2}{(\sum u_i)^2} + \dfrac{\sum v_i^2}{(\sum v_i)^2} - \dfrac{2\sum u_i v_i}{\sum u_i \sum v_i}}$

  - $\sigma(\bar{x}) = \bar{x} \times r_{\bar{x}}$

- **SUIT Code (HYU)**

Part of u-235 Total Xs (81g)



Method

Uncertainty Propagation

- Assume eigenvalue is function of each group cross section

$$k_{eff} = f\left(Xs_{g1}, Xs_{g2}, Xs_{g3}, \dots, Xs_{gn}\right)$$

- Uncertainty of eigenvalue can be written as follow:

$$\sigma\left(k_{eff}\right)^2 \approx \frac{\partial^2 k_{eff}}{\partial Xs_{g1}^2}\sigma\left(Xs_{g1}\right)^2 + \frac{\partial k_{eff}}{\partial Xs_{g1}}\frac{\partial k_{eff}}{\partial Xs_{g2}}cov\left(Xs_{g1}, Xs_{g2}\right) + \frac{\partial k_{eff}}{\partial Xs_{g1}}\frac{\partial k_{eff}}{\partial Xs_{g3}}cov\left(Xs_{g1}, Xs_{g3}\right) + \cdots$$

- It simply written as matrix form:

$$\sigma\left(k_{eff}\right)^2 \approx JVJ^T$$

where $J = (\frac{\partial k_{eff}}{\partial Xs_{g1}}, \frac{\partial k_{eff}}{\partial Xs_{g2}}, \dots, \frac{\partial k_{eff}}{\partial Xs_{gn}})$, $\quad V(i,j) = cov\left(Xs_{g,i}, Xs_{g,j}\right)$

Method

Group XS and Uncertainty and Covariance

- $\sigma(k_{eff})^2 \approx JVJ^T$
- $V =$

$$\begin{pmatrix} \sigma^2_{xs_1} & \cdots & cov(xs_1, xs_n) \\ \vdots & \ddots & \vdots \\ cov(xs_n, xs_1) & \cdots & \sigma^2_{xs_n} \end{pmatrix}$$

- $\mathrm{Cov}(i, j) = \mathrm{Corr}(i, j) \times \sigma_i \times \sigma_j$

  - Union Tally

  - Assume $Corr_{flux} \approx Corr_{xs}$

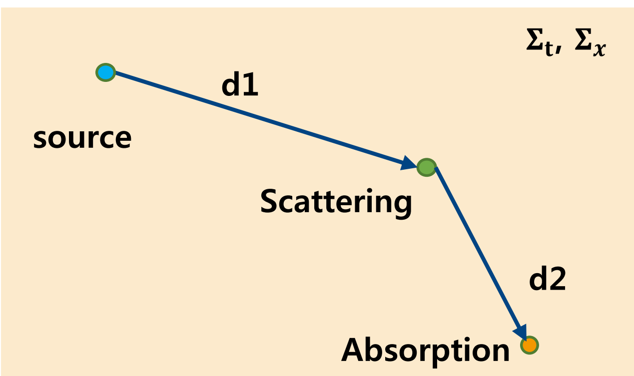
Method

- $\sigma(k_{eff})^2 \approx JVJ^T$

$$J = \left(\frac{\partial k_{eff}}{\partial Xs_{g1}}, \frac{\partial k_{eff}}{\partial Xs_{g2}}, \dots, \frac{\partial k_{eff}}{\partial Xs_{gn}}\right)$$

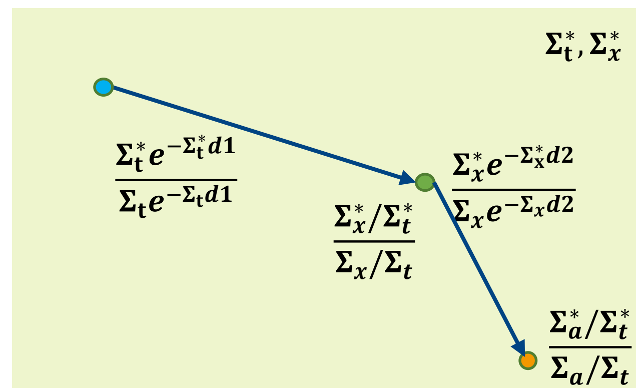
Correlated Sampling Method

- **Unperturbed System**



- *W0*

- **Perturbed System**



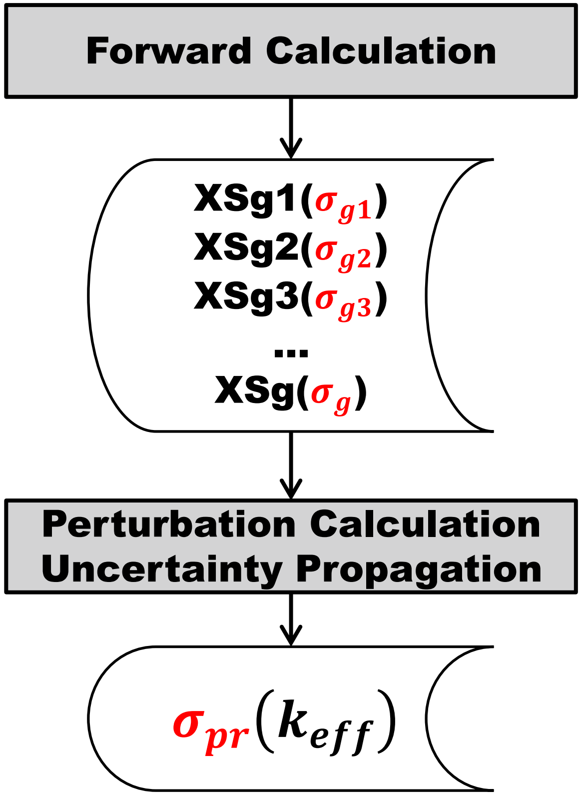
- $W^* = \frac{\Sigma_t^* e^{-\Sigma_t^* d1}}{\Sigma_t e^{-\Sigma_t d1}} \frac{\Sigma_x^*/\Sigma_t^*}{\Sigma_x/\Sigma_t} \frac{\Sigma_x^* e^{-\Sigma_x^* d2}}{\Sigma_x e^{-\Sigma_x d2}} \frac{\Sigma_a^*/\Sigma_t^*}{\Sigma_a/\Sigma_t} W_0$

- Storing perturbed weights, Several results can be calculated by one input

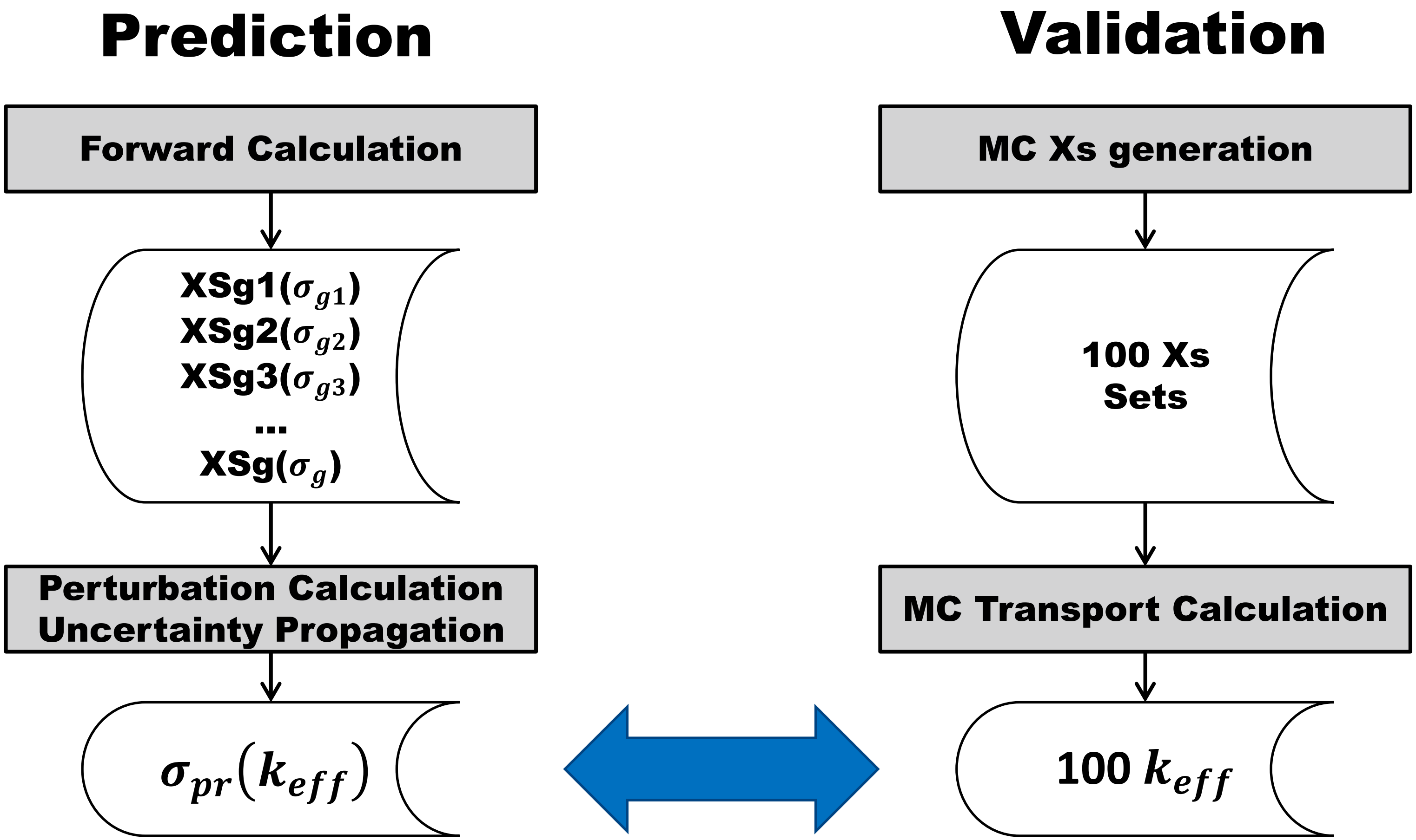
Method

$$\sigma\left(k_{eff}\right)^2 \approx JVJ^T$$

- V: MC Calculation to Generate XS
- J : Correlated Sampling Method



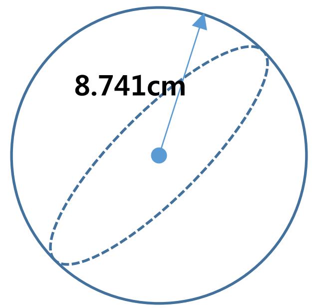
Method


Prediction
Validation
Forward Calculation
XSg1(σ_{g1})
XSg2(σ_{g2})
XSg3(σ_{g3})
...
XSg(σ_g)
Perturbation Calculation
Uncertainty Propagation
$\sigma_{pr}(k_{eff})$
MC Xs generation
100 Xs
Sets
MC Transport Calculation
100 k_{eff}

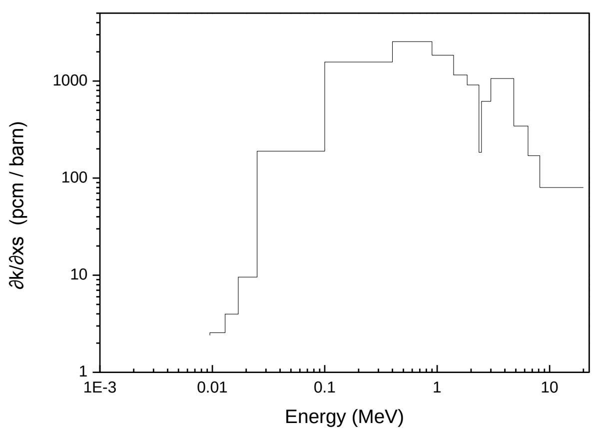
RESULT

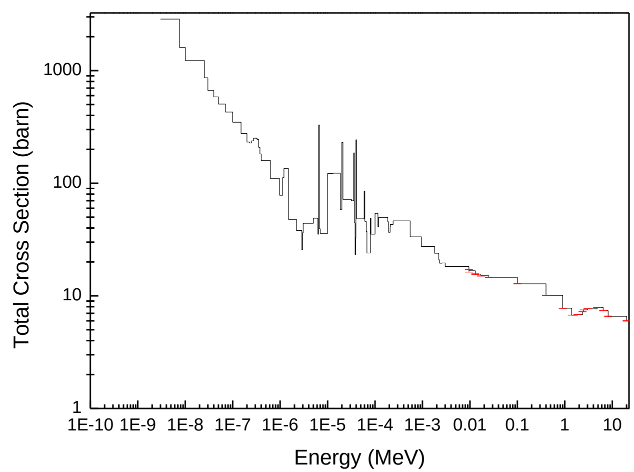
Result

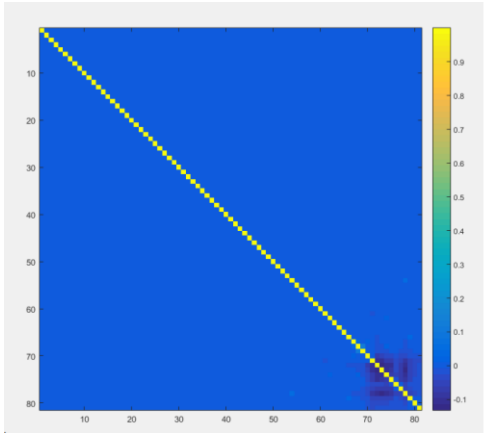
Godiva



Material	Uranium(5.27 u235 w/o)
Density	18.74g/cc
Number of Particle	3000



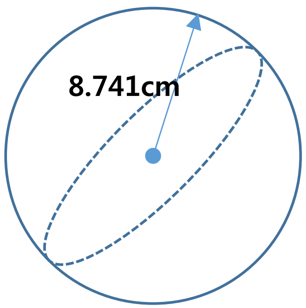




- $\sigma_{pr}(k_{eff}) \approx 42\ pcm$

Result

Godiva



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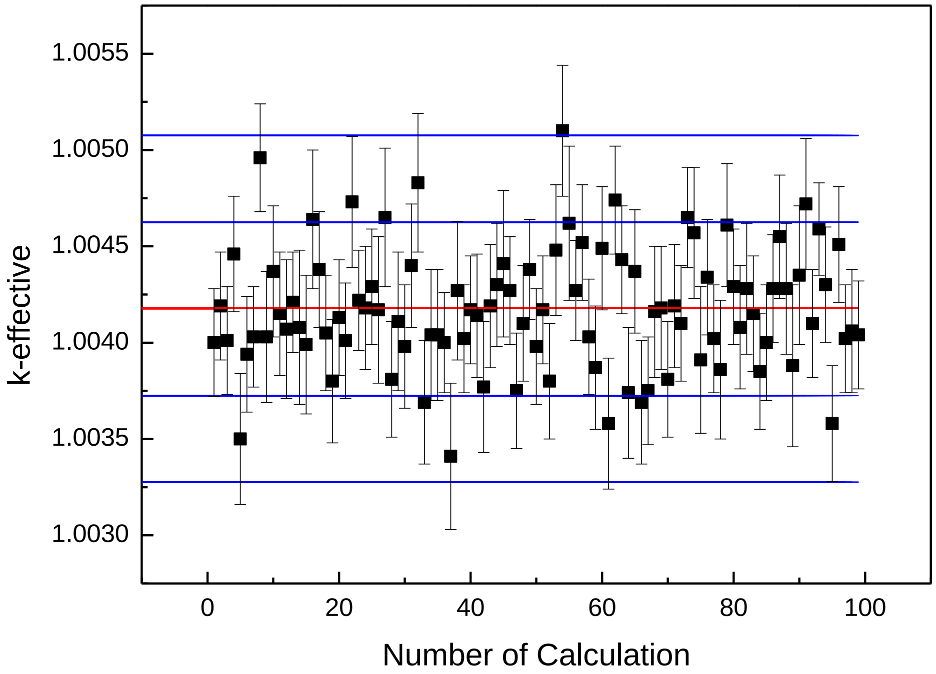
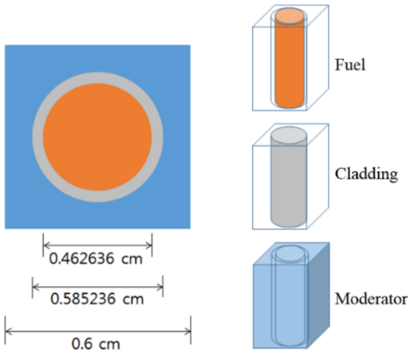
100 Calculations



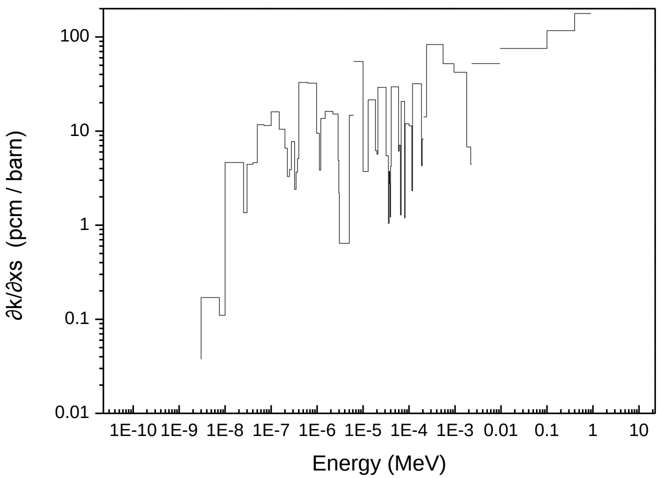
- $\sigma_{pr}(k_{eff}) \approx 42\ pcm$

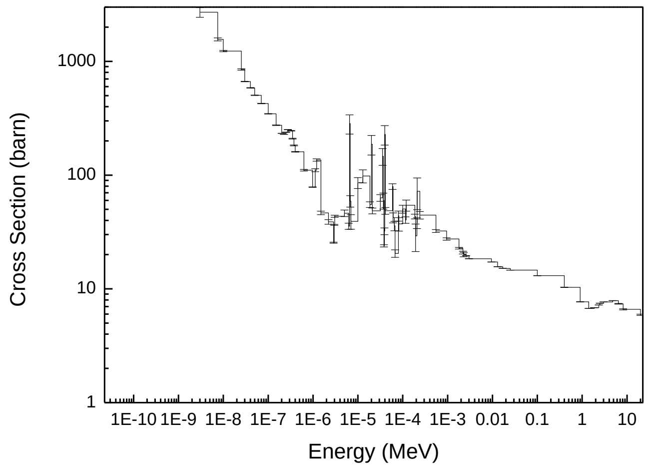
Result

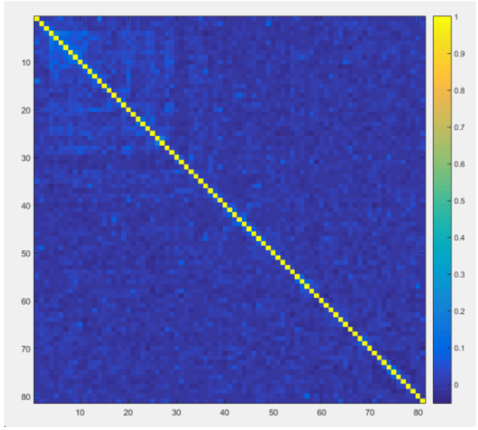
U-Zr Fuel Pin cell



Fuel Material	U-Zr (u-235 19.75w/o)
Density	7.86 g/cc
Number of Particle	3750



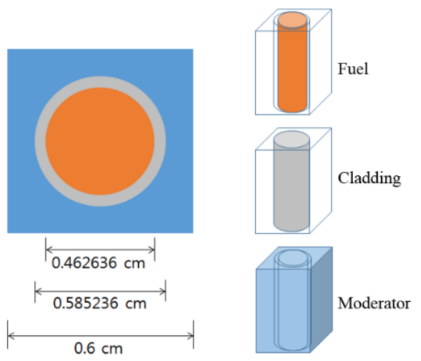




- $\sigma_{pr}(k_{eff}) \approx 330\ pcm$

Result

U-Zr Fuel Pin cell



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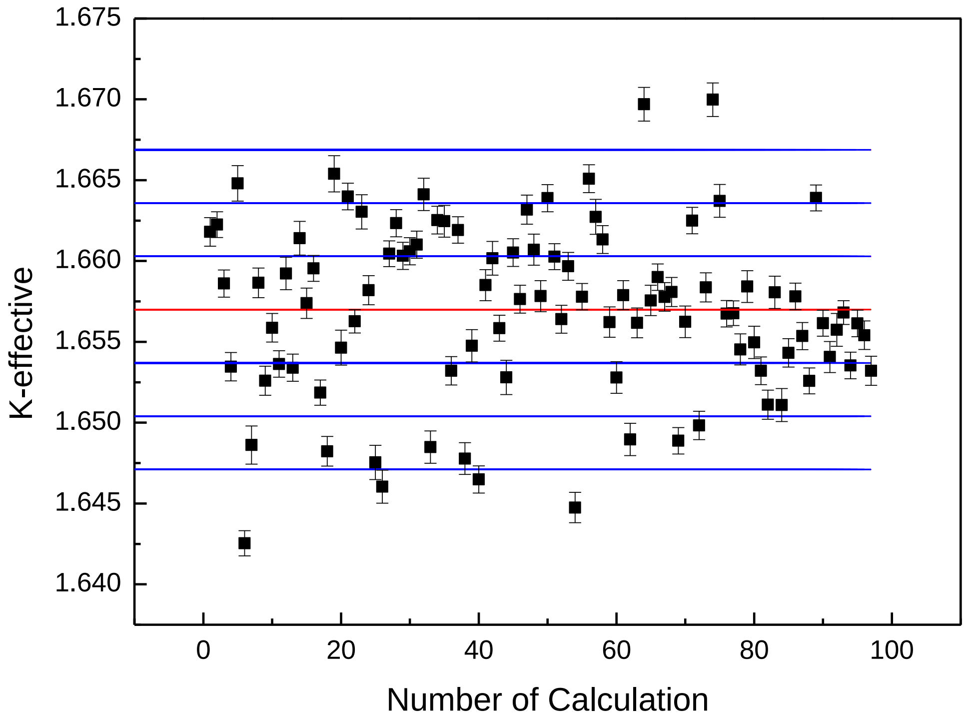

- $\sigma_{pr}(k_{eff}) \approx 330\ pcm$

CONCLUSION

Conclusion

Goal of this study

- Evaluate Multiplication Uncertainty caused by MC uncertainty of Group Xs

Used Method

- Monte Carlo Method to Calculate Multi-group Cross Section
- Correlated Sampling Method
- Uncertainty Propagation

Conclusion

- Using methods introduced in this study, it is possible to guess uncertainty of eigenvalue caused by monte Carlo uncertainty of group cross section.
- Using this study, it is expected that efficient calculation can be performed to generate Xs via MC method