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Estimation of POL-iteration methods in fast running DNBR code

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1. Introduction

Online core monitoring and protection system in SMART core estimates the core status based on a real time calculation of operating margin. The operating margin that is commonly called to POL, power operating limit, is defined as a distance between the present local power at a present operating condition and the limiting power that occurred at limit DNBR. The POL requires the iteration until present local power reach limit power. The process to search the limiting power is equivalent with a root finding of nonlinear equation.

POL iteration process involved in online monitoring system used a variant bisection method that is the most robust algorithm to find the root of nonlinear equation. The method including the interval accelerating factor and escaping routine out of ill-posed condition assured the robustness of SCOMS system.

POL iteration module in SCOMS shall satisfy the requirement which is a minimum calculation time. For this requirement of calculation time, non-iterative algorithm, few channel model, simple steam table are implemented into SCOMS to improve the calculation time. MDNBR evaluation at a given operating condition requires the DNBR calculation at all axial locations. An increasing of POL-iteration number increased a calculation load of SCOMS significantly. Therefore, calculation efficiency of SCOMS is strongly dependent on the POL iteration number.

In this study, various root finding methods are applied to the POL-iteration module in SCOMS and POL-iteration efficiency is compared with reference method. On the base of these results, optimum algorithm of POL iteration is selected.

2. Methods and Results

2.1 Root finding method of nonlinear equations

Root finding methods of nonlinear equations are frequently used such as Bisection, Regula falsi, and secant method[1]. These methods are based with the interval search procedure that is looking for an interval to meet the condition f(a)f(b) < 0 if a close interval [a,b] is given. After this interval search, the linear interpolation is then used to find an approximate value of the true root ζ . Next approximate solution is estimated with an interpolation in Eq (1). The solution is used to determine the subinterval [a,x] or [x,b], which contain the true root ζ by checking the sign of f(x). If f(x)f(a) < 0 the [a,x] contain the root, and x becomes the new b] for the next iteration. The iteration is repeated until one stopping criterion is attained.

$$x = \frac{af(b) - bf(a)}{f(b) - f(a)} \tag{1}$$

These methods generate a sequence x_i that converges to the bracketed root ζ of the equation f(x) = 0. The sequence x_i is obtained through the recurrence formula in Eq. (2).

$$x_{k+1} = x_k - \frac{f(x_k)}{f(x_k) - f(x_{k-1})}(x_k - x_{k-1}),\ k = 1,2,3,\ldots \tag{2}$$

The $[x_{k-1}, f(x_{k-1})]$ and $[x_k, f(x_k)]$ points are chosen so that $f(x_{k-1})f(x_k)$ always have negative sign, guaranteed $\zeta \in [x_{k-1}, x_k]$. The calculated value $f(x_{k-1})$ is reduced by the factor $\phi[f(x_{k-1}), f(x_k), f(x_{k+1})]$ as shown in Table 1. The various methods are derived with the factor ϕ .

Table 1. Approximate solution factors of root finding method

Method	$\phi[f_a, f_b, f_x]$
Pegasus	$\dfrac{f_a f_b}{f_b + f_x}$
Illinois	$\dfrac{f_a}{2}$
Anderson& Bjork	$f_a \cdot m,\ m = \left(1 - \dfrac{f_x}{f_b}\right)$ if $\left(1 - \dfrac{f_x}{f_b}\right) < 0,\ m = 1/2$

2.2 Modified Brent's Model

Brent's method is one of hybrid root finding algorithm that combines interpolation, bisection method, and the secant method of root finding[2]. The benefits of Brent's approach to root finding are a combination of the best aspects of interpolation, bisection method, and the secant method. By combining the these methods,

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Brent's method was able to incorporate the guarantee of convergence as in the bisection method, but also take advantage of the rapid rate of convergence of the less reliable methods.

This method can be used to determine a root x in the interval [a,b] as long as f(a) and f(b) have different signs that is precondition of intermediate value theorem. Algorithm to find a root is following as:

i) Given 3 points, x_1, x_2, x_3, first fits x as a quadratic function of y, then uses the following Lagrange interpolation method

$$x = L_2(x_1, x_2) + L_2(x_2, x_3) + L_2(x_3, x_1)$$

,where $L_2(x_1, x_2) = \dfrac{(y - f(x_1))(y - f(x_2))}{(f(x_3) - f(x_1))(f(x_3) - f(x_2))}$

ii) To find successive root estimates set y=0, which gives

$$x = x_2 + \frac{P}{Q}$$

,where

$$P = S\left[T(R-T)(x_3 - x_2) - (1-R)(x_2 - x_1)\right]$$

$$Q = (T-1)(R-1)(S-1)$$

$$R = \frac{f(x_2)}{f(x_3)}, S = \frac{f(x_2)}{f(x_1)}, R = \frac{f(x_1)}{f(x_3)}$$

iii) The point at which y=0 becomes the new x_2 and the old x_2 becomes x1, but if x_3 and the new x_2 no longer bound the root, discard x_3 and use x_1 instead.

2.3 Case study of POL iteration efficiency

POL-iteration efficiency at nominal operating condition is estimated with the ten cases of axial power shapes(APS) as shown in Fig. 1. Limit DNBR is 1.5 and convergence tolerance is 0.005[3].

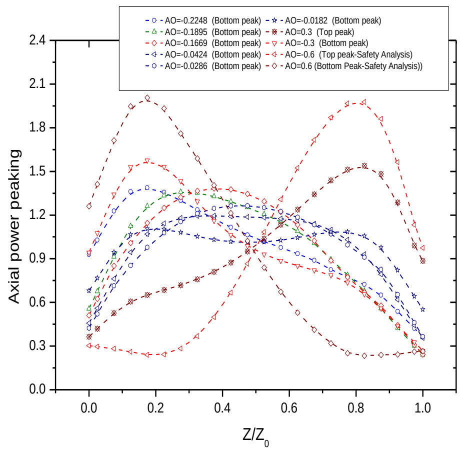

Fig.1. Axial power shape for estimation of POL-iteration

The result of POL-iteration is shown in Table II, where maximum iteration number is restricted with 200. If iteration number of a case is over 200, it is marked with 200+. The comparison with methods shows that efficient and robust method is modified Brent method.

Table II: Fuel assembly geometry and nominal operating condition

AO	Number of iterations				
	Bisection (SCOMS)	Pegasus	Illinois	A&B	Brent
-0.0248	43	8	8	9	4
-0.1895	51	15	14	23	5
-0.1669	36	42	42	200+	12
-0.0424	60	60	60	200+	10
-0.0286	29	11	27	5	9
-0.0182	27	10	18	9	9
0.3	200+	51	48	18	20
-0.3	181	59	57	32	18
-0.6	196	42	39	11	11
0.6	200+	48	48	200+	15
Total iterations	1023	346	361	707	113

3. Conclusions

The study presented a class of root finding algorithm on DNBR with various approximation solution factor. In case study, the iterations of the methods have a superlinear convergence for finding limiting power but Brent method shows a quardratic convergence speed. These methods are effective and better than the reference bisection algorithm. It is expected that the reduction of iteration number is directly affected to the calculation speed and robustness of algorithm of SCOMS.

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