Weibull 모수 추정치의 불확실성 평가를 통한 균열생성실험 조건 연구

한국원자력학회 2016 춘계학술대회, 핵연료 및 원자력재료 분과
제주 국제컨벤션센터(ICC JEJU), 302
2016. 5. 13. (금)

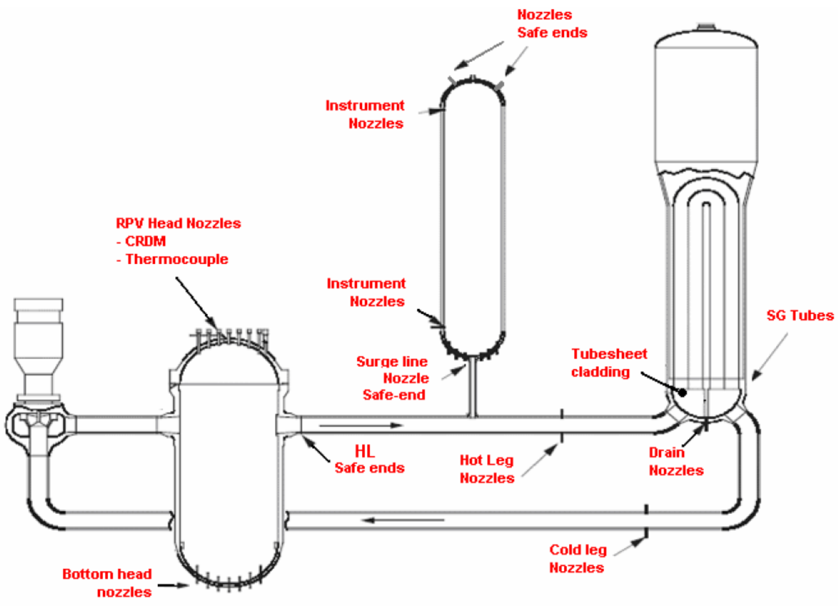
박재필, 반치범*

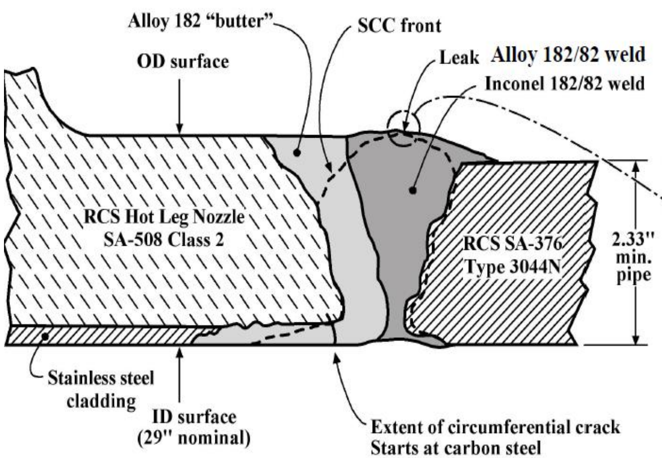
Materials in Nuclear Systems Laboratory
School of Mechanical Engineering
Pusan National University

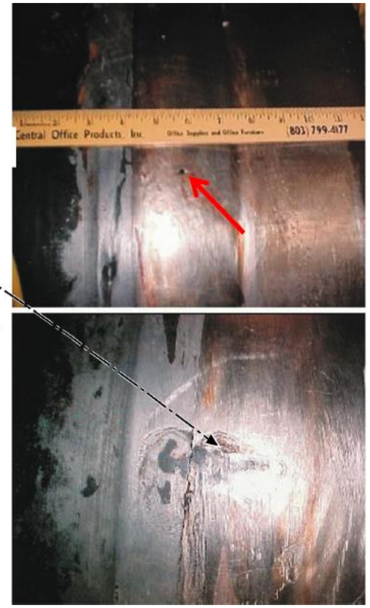
Contents

Introduction

- **Stress Corrosion Cracking (SCC)**
  - ✓ One of the main materials-related issues in operating nuclear reactors.
    - Can cause significant Loss of Coolant Accident (LOCA).







SCC가 발견된 182/82 용접부 위치 개략도

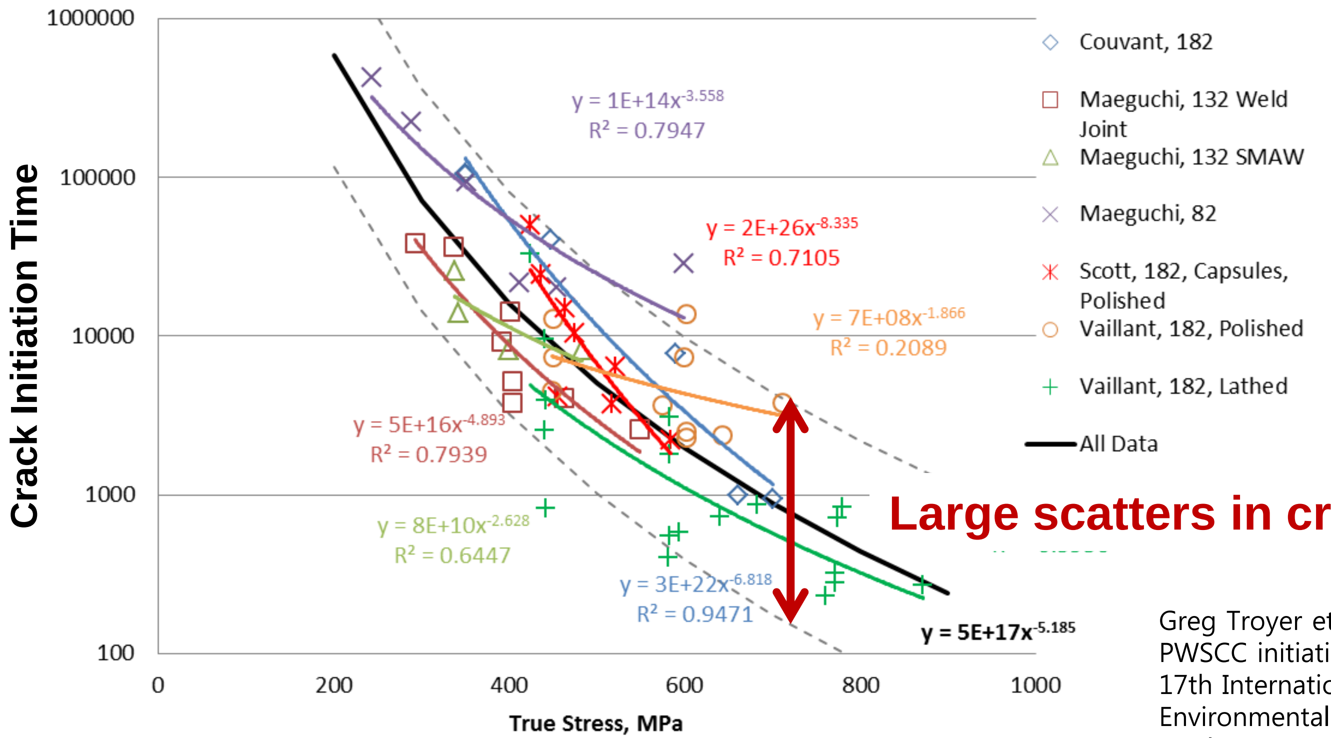
[EPRI MRP-220, 2007]

미국 V.C. Summer 고온관 용접부에서 발견된 PWSCC

"PWSCC/LPSCC in PWRs (+ Steam Generator Corrosion),"
USNRC, Adams No. ML11266A011

Introduction

- **Initiation time of SCC**
  - ✓ To predict the accurate time of SCC is very difficult.
    - The mechanism of SCC initiation is quite complex.
    - Most of SCC experiment show **<u>non-negligible scatters</u>** in cracking time.

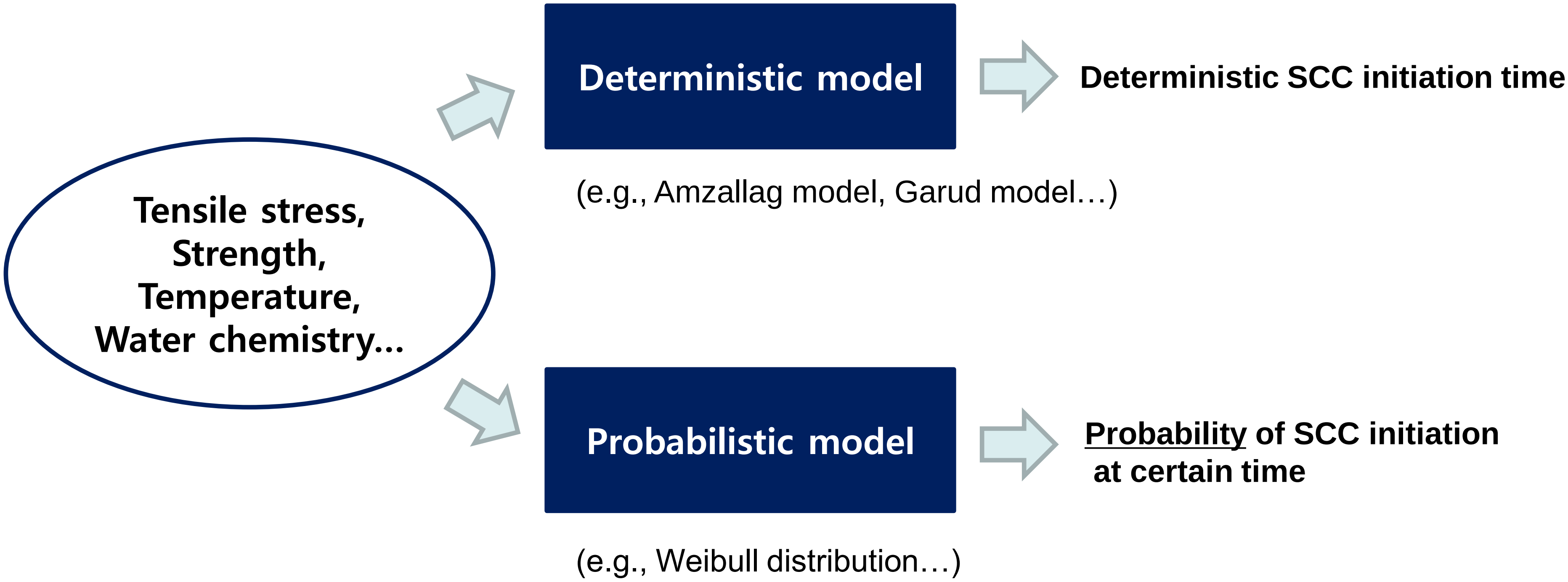


The results of several SCC initiation experiments (normalized to 325℃)

Greg Troyer et al., Dissimilar metal weld PWSCC initiation model refinement for xLPR, 17th International Conference on Environmental Degradation of Materials in Nuclear Power Systems – Water Reactors August 9-13, 2015, Ottawa, Ontario, Canada

Introduction

- **Prediction model of SCC initiation**
  - ✓ Deterministic model or **<u>Probabilistic model</u>**
    - Probabilistic model can quantify the data scatters in SCC experiment.

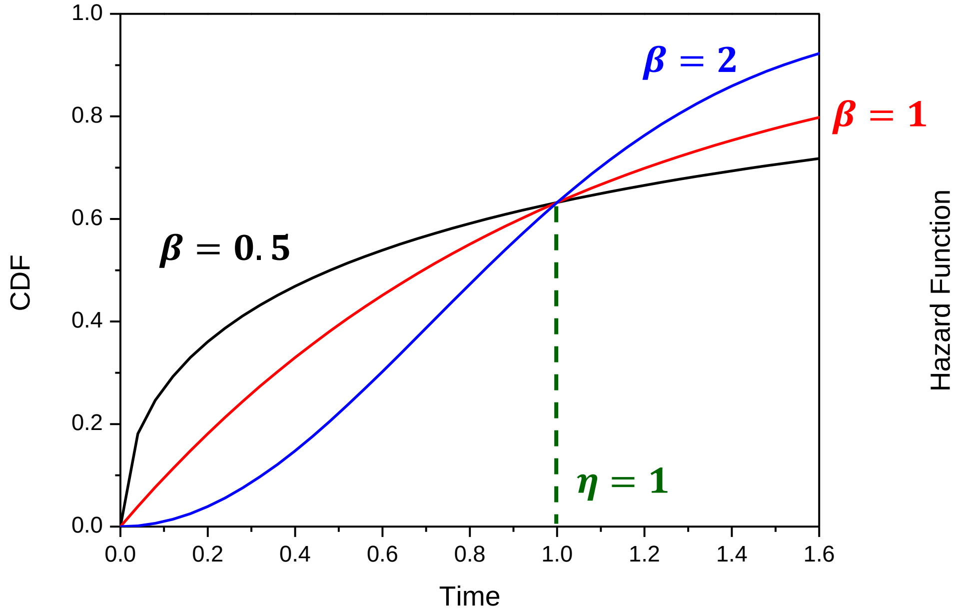


Introduction

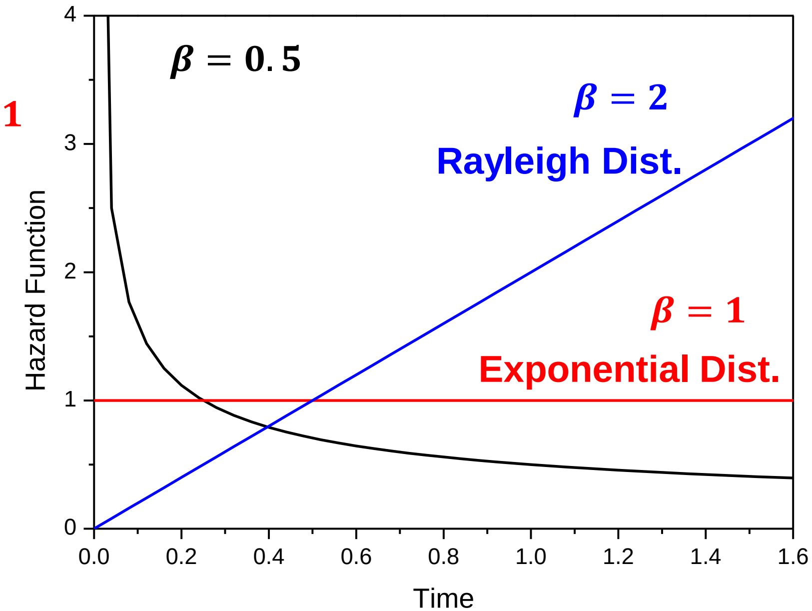
- **The Weibull distribution**
  - ✓ Frequently used as a cracking probability function.
    - Can consider the effect of **<u>time-dependent degradation</u>** of material.
    - Shape parameter (β); scale parameter (η)

$$F(t;\beta,\eta) = 1 - exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

$$\lambda(t;\beta,\eta) = \frac{\beta t^{\beta-1}}{\eta^{\beta}}$$



Cumulative Distribution Function



Hazard Function (Cracking rate)

Introduction

- **Experimental factors for Weibull estimation**
  - ✓ It is possible to estimate the parameters of the Weibull distribution by SCC test.
    - Many specimens
    - Narrow censoring interval
    - Long test duration

→ **Accurate estimators for the SCC initiation model**

What is the 'reasonable' level?

U-bend specimen

# of specimens (6 ea.)

Censoring interval (150 h) — **Test duration (1200 h)**

Specimen	100 hr	250 hr	500 hr	700 hr	900 hr	1200 hr
# 1	x	x	x	x	Crack	-
# 2	x	x	Crack	-	-	-
# 3	x	x	x	x	x	Crack
# 4	x	x	x	x	Crack	-
# 5	x	x	x	x	x	Crack
# 6	x	x	x	x	x	Crack

The example of SCC experiment result (interval censored)

Weibull Estimation

→ **Median Rank Regression (MRR)**
→ Maximum Likelihood Estimation (MLE)

Median rank

- ✓ Median rank is used to estimate **the cracking probability** at the certain time.
  - The number of total specimen : N
  - The number of cracking specimen : j
  - Cracking probability : θ

- ✓ Assume that all the specimens are tested independently (i.e., j is binomially distributed),
  - $$CDF_{bin} = \sum_{i=0}^{j} \binom{N}{j} \theta^{i} (1-\theta)^{N-i}$$
    $$= (N-j)\binom{N}{j} \int_0^{1-\theta} t^{N-j-1}(1-t)^{j}\, dt$$
    $$= I_{1-\theta}(N-j, j+1)$$
    $$= 0.5$$
  - $$\theta_{median} = 1 - I_{0.5}^{-1}(N-j, j+1)$$

Median Rank

※ **Incomplete beta function**

$$\boldsymbol{B(x;\alpha,\beta) = \int_0^{x} t^{\alpha-1}(1-t)^{\beta-1}\, dt}$$

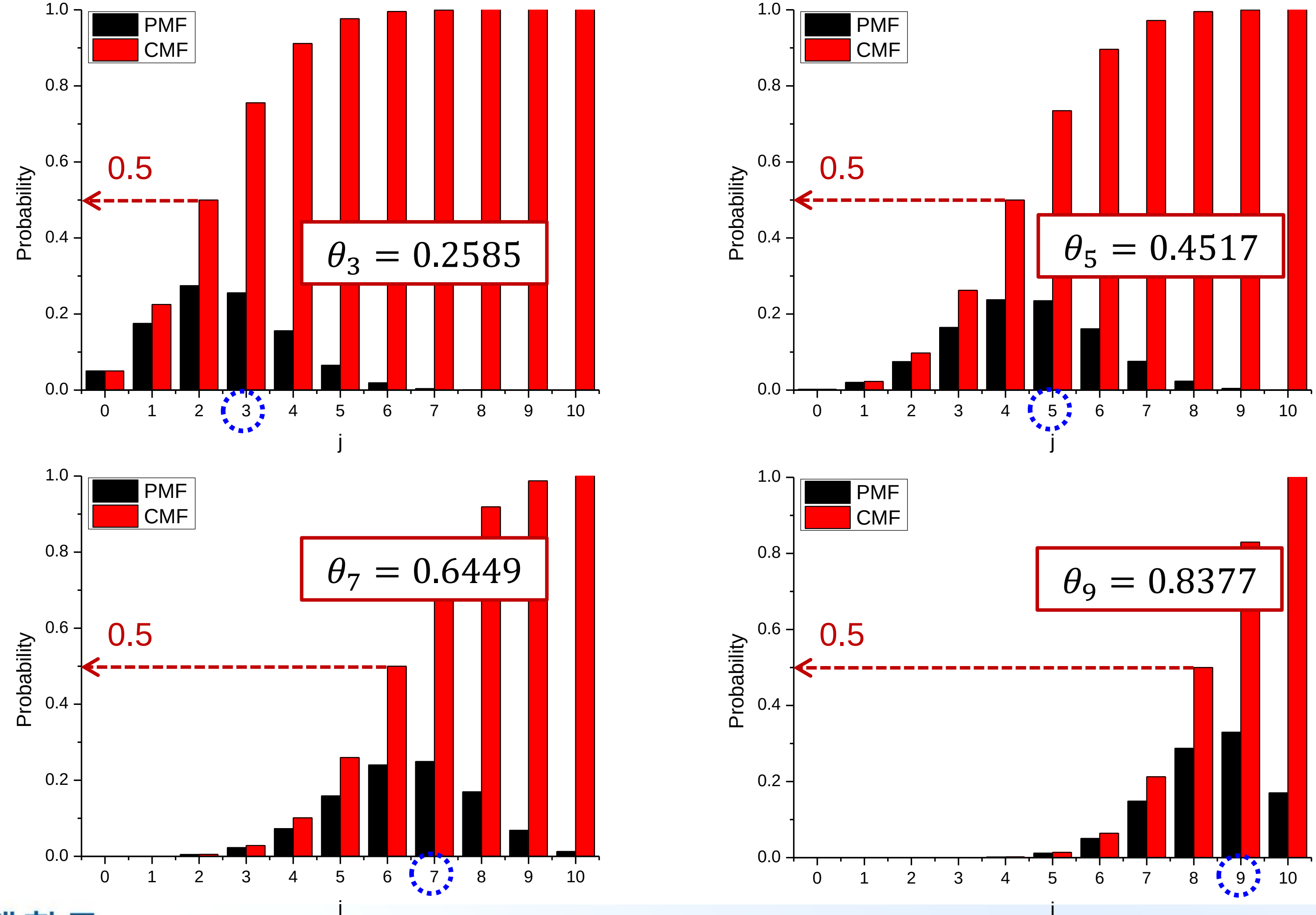
$$\boldsymbol{I_x(\alpha,\beta) = \frac{B(x;\alpha,\beta)}{B(1;\alpha,\beta)}}$$

※ **Regularized incomplete beta function**

Weibull Estimation

- **Meaning of median rank**

※ **For the case of $N = 10$**



Weibull Estimation

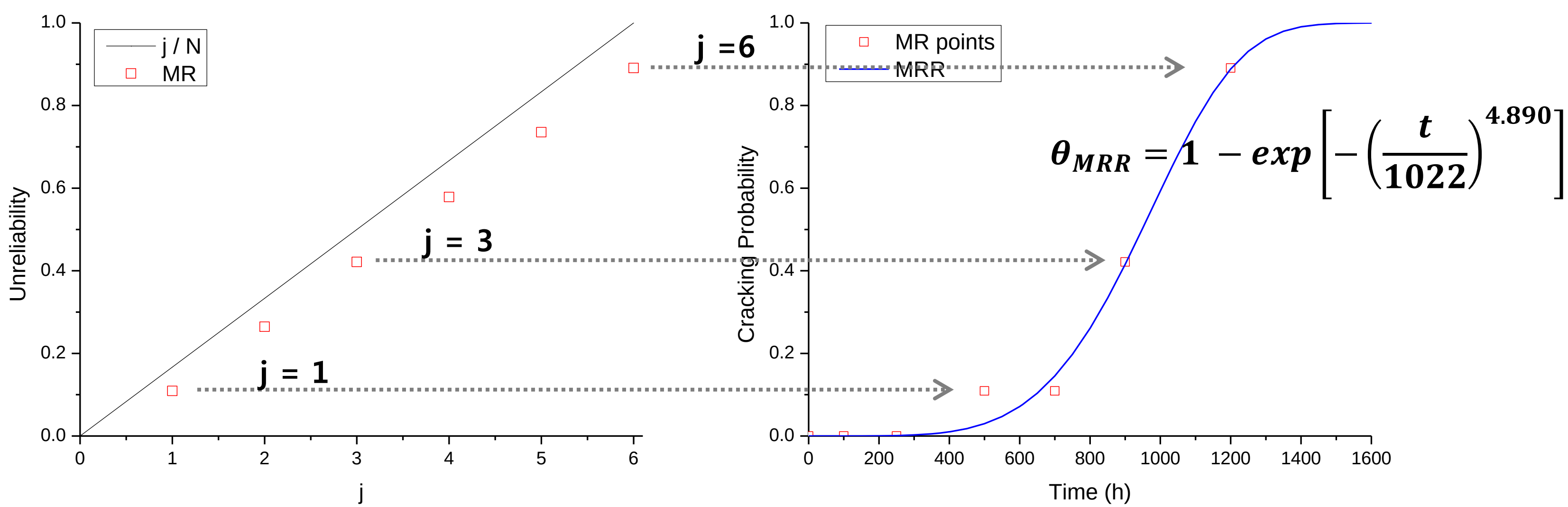
- **Median Rank Regression (MRR)**
  - ✓ Weibull parameters are determined by regression.
    - Linearization technique is not recommended*.

$$ln\left[ln\left[\frac{1}{1-F(t)}\right]\right] = \beta ln(t) - \beta ln(\eta)$$

Not recommended for regression

* U. Genschel, W.Q. Meeker, A comparison of maximum likelihood and median-rank regression for Weibull estimation, Quality Engineering, 22.4 (2010) 236-255

Time (hr)	Cracking fraction	Cracking probability
100	0 / 6	0
250	0 / 6	0
500	1 / 6	0.1091
700	1 / 6	0.1091
900	3 / 6	0.4214
1200	6 / 6	0.8909



Median rank (for N=6) **Cracking probability curve**

Weibull Estimation

→ Median Rank Regression (MRR)
→ **Maximum Likelihood Estimation (MLE)**

- **Likelihood function**

✓ MLE method estimates the Weibull parameters directly by using the likelihood function.

✓ For interval censored data*;

* Likelihood function

$$L(\beta,\eta) = \prod_{i=1}^{S}[1 - F(s_i;\beta,\eta)] \cdot \prod_{j=1}^{C}[F(c_{j_U};\beta,\eta) - F(c_{j_L};\beta,\eta)]$$

* Log-likelihood function

$$\Lambda(\beta,\eta) = \sum_{i=1}^{S} \ln[1 - F(s_i;\beta,\eta)] + \sum_{j=1}^{C} \ln[F(c_{j_U};\beta,\eta) - F(c_{j_L};\beta,\eta)]$$

* The condition of **maximum likelihood** point

$$\begin{cases} \dfrac{\partial}{\partial\beta}\Lambda(\beta,\eta) = 0 \\ \dfrac{\partial}{\partial\eta}\Lambda(\beta,\eta) = 0 \end{cases}$$

⇒ **Solve these two simultaneous non-linear equations**

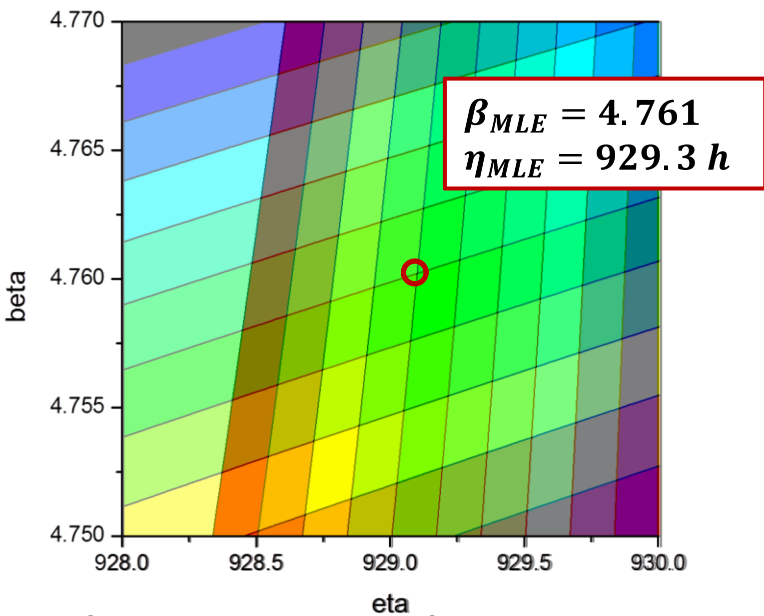
* ReliaSoft Corporation, Life Data Analysis Reference Book, Retrieved Jan. 1, 2014, available under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License

Weibull Estimation

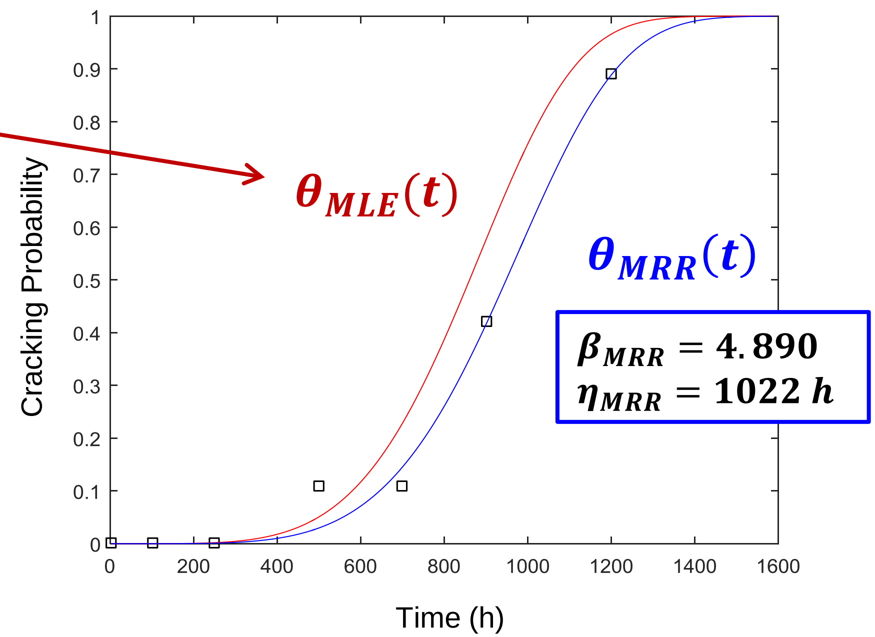
- **Maximum Likelihood Estimation (MLE)**
  - ✓ Numerical (or graphical) approach is used for finding the maximum likelihood point.
  - ✓ It is interesting that '$\theta_{MRR}(t) \neq \theta_{MLE}(t)$'.

→ **Which method is accurate?**

Time (hr)	Cracking fraction	Unreliability
100	0 / 6	0
250	0 / 6	0
500	1 / 6	0.1091
700	1 / 6	0.1091
900	3 / 6	0.4214
1200	6 / 6	0.8909



$$\frac{\partial}{\partial \eta} Log\big(L(\eta,\beta)\big) \;\&\; \frac{\partial}{\partial \beta} Log\big(L(\eta,\beta)\big)$$



Comparison of MRR and MLE

Monte Carlo Simulation

* John. I. McCool, Using the Weibull distribution: reliability, modeling and inference, John Wiley & Sons, New Jersey, 2012

- **Motivation of the simulation**
  - ✓ There is no MLE theory yet available to set the estimation confidence for **interval censored data***.
    - Monte Carlo simulation could be used to evaluate uncertainties of each estimation methods quantitatively.

- **Simulation study range**

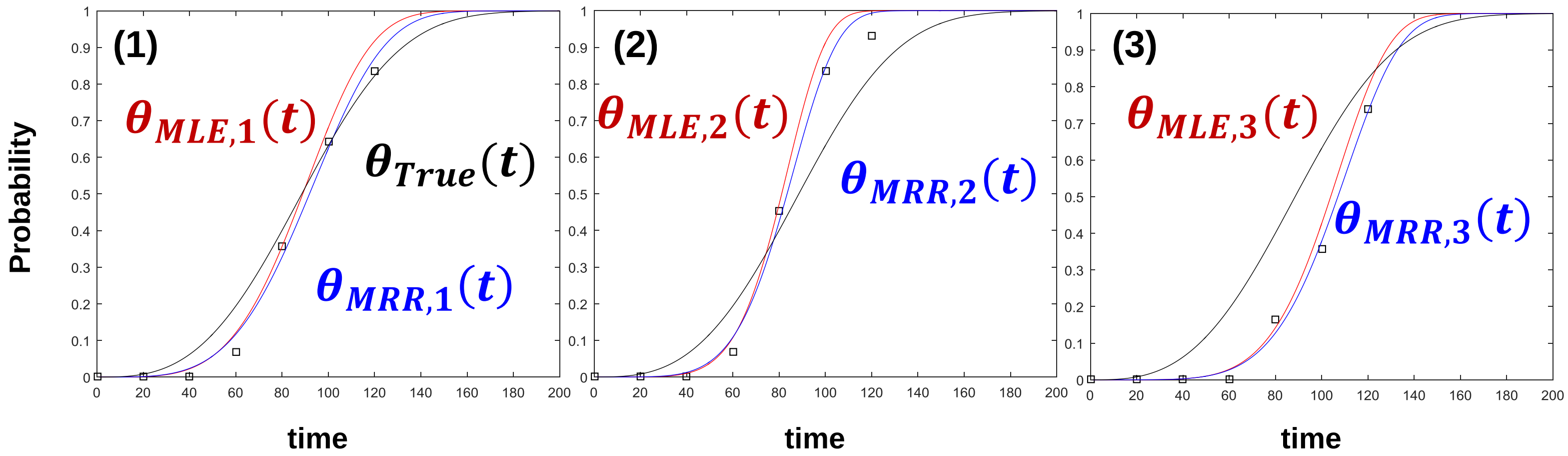
※ Select 63 combinations of simulation cases

True Weibull parameter		The number of specimen	Test duration [% of η_{true}]	Censoring interval [% of η_{true}]
η_{true}	β_{true}			
100	**2**	5	80	5
	3	**10**	100	10
	4	15	**120**	15
		20	140	**20**
		25	160	30
		50	180	40
		100	200	60

※ Baseline case of the simulation study

Monte Carlo Simulation

- **Method of the simulation**
  - ✓ 10,000 times of **random simulations** for each combination cases.



...

10,000 simulations

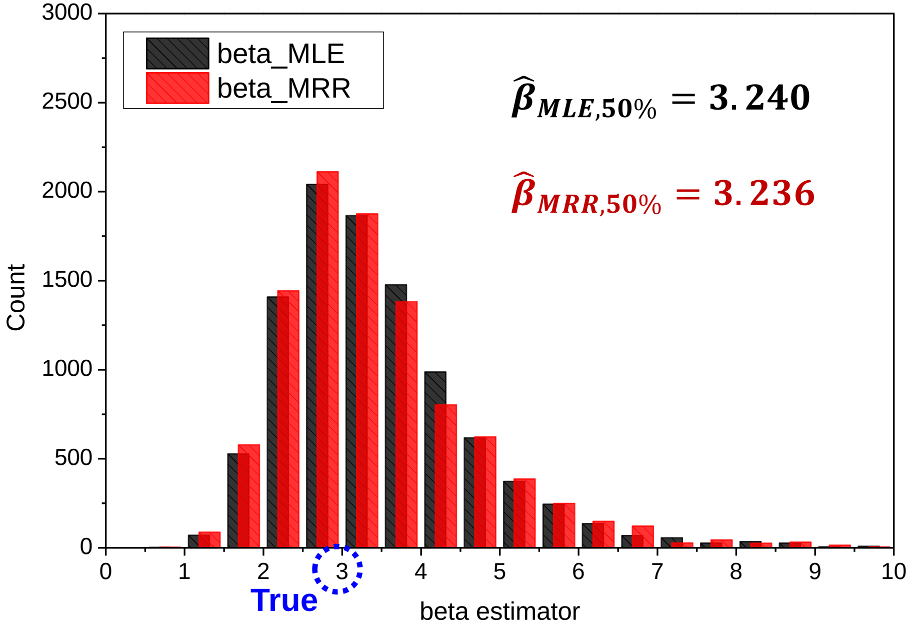
Case 1	Sim. 1	Sim. 2	Sim. 3	...	Sim. 10,000
β_{True}	$\hat{\beta}_{MLE,1}$ $\hat{\beta}_{MRR,1}$	$\hat{\beta}_{MLE,2}$ $\hat{\beta}_{MRR,2}$	$\hat{\beta}_{MLE,3}$ $\hat{\beta}_{MRR,3}$	...	$\hat{\beta}_{MLE,10000}$ $\hat{\beta}_{MRR,10000}$
η_{true}	$\hat{\eta}_{MLE,1}$ $\hat{\eta}_{MRR,1}$	$\hat{\eta}_{MLE,2}$ $\hat{\eta}_{MRR,2}$	$\hat{\eta}_{MLE,3}$ $\hat{\eta}_{MRR,3}$	...	$\hat{\eta}_{MLE,10000}$ $\hat{\eta}_{MRR,10000}$

⋮ 63 cases

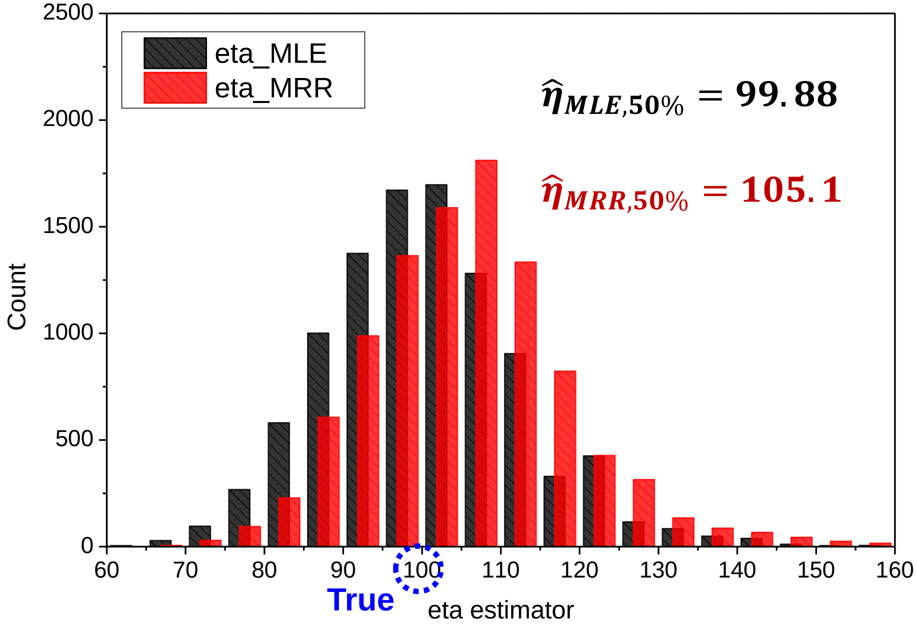
Monte Carlo Simulation

- **Distribution of estimators**
  - ✓ For baseline simulation case,

True Weibull parameter		The number of specimen	Test duration [% of η_{true}]	Censoring interval [% of η_{true}]
η_{true}	β_{true}			
100	3	10	120	20



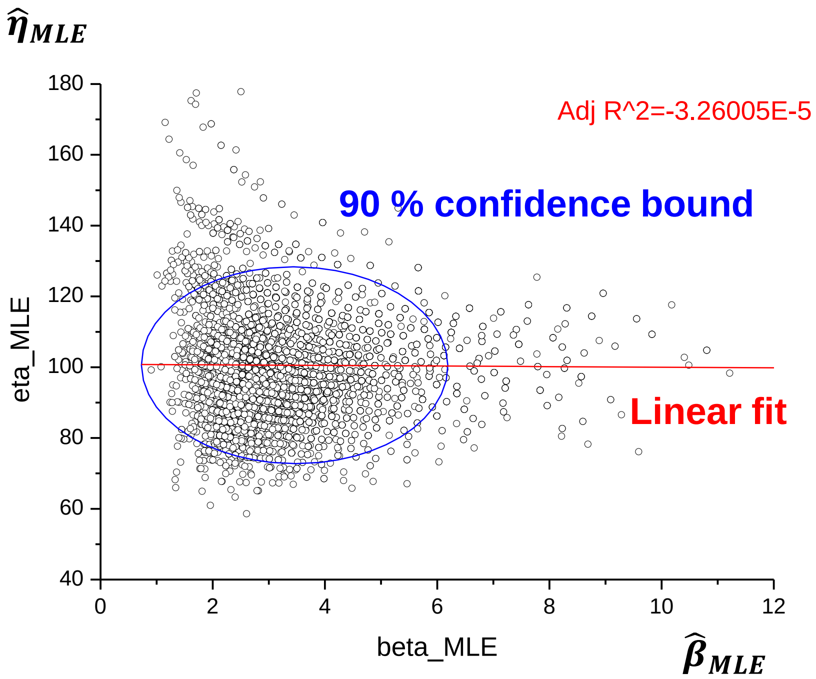
Distribution of $\hat{\beta}$ for baseline case



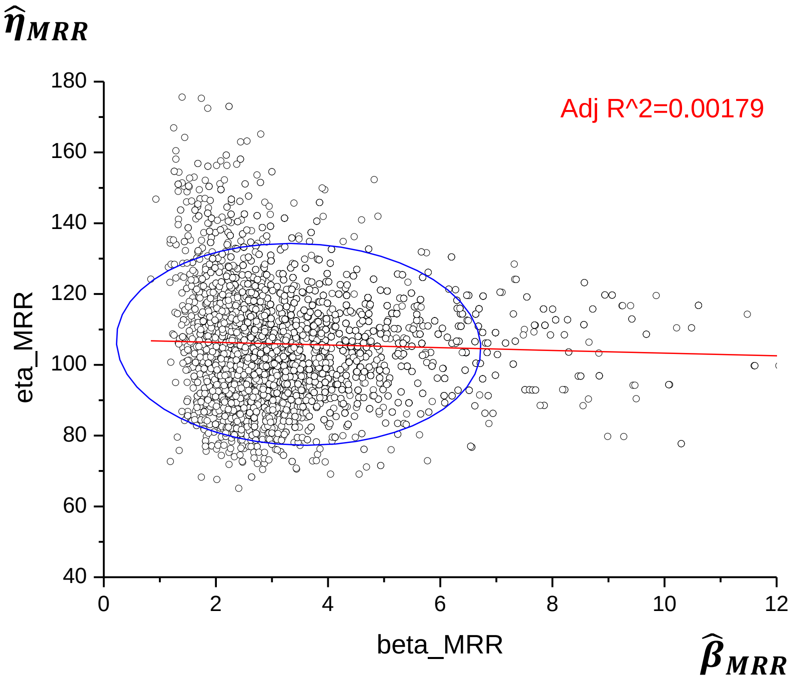
Distribution of $\hat{\eta}$ for baseline case

Monte Carlo Simulation

- **Correlation between $\hat{\beta}$ and $\hat{\eta}$**
  - ✓ There is no correlation between $\hat{\beta}$ and $\hat{\eta}$ for both MLE and MRR estimation.
    - because, $R^2 \cong 0$.



Scatter plot of MLE estimators



Scatter plot of MRR estimators

Monte Carlo Simulation

- **Evaluation of estimation uncertainty**
  - ✓ 5 %, 50 % and 95 % percentiles of Weibull estimators were compared to the true parameters.

※ **Standard Error** (SE):

$$\mathrm{SE}(\hat{\beta}) = \frac{\hat{\beta} - \beta_{true}}{\beta_{\mathrm{true}}}$$

$$\mathrm{SE}(\hat{\eta}) = \frac{\hat{\eta} - \eta_{true}}{\eta_{\mathrm{true}}}$$

※ **Standardized Confidence Interval** (SCI)

$$\mathrm{SCI}_{90\%}(\hat{\beta}) = \mathrm{SE}(\hat{\beta}_{95\%}) - \mathrm{SE}(\hat{\beta}_{5\%})$$

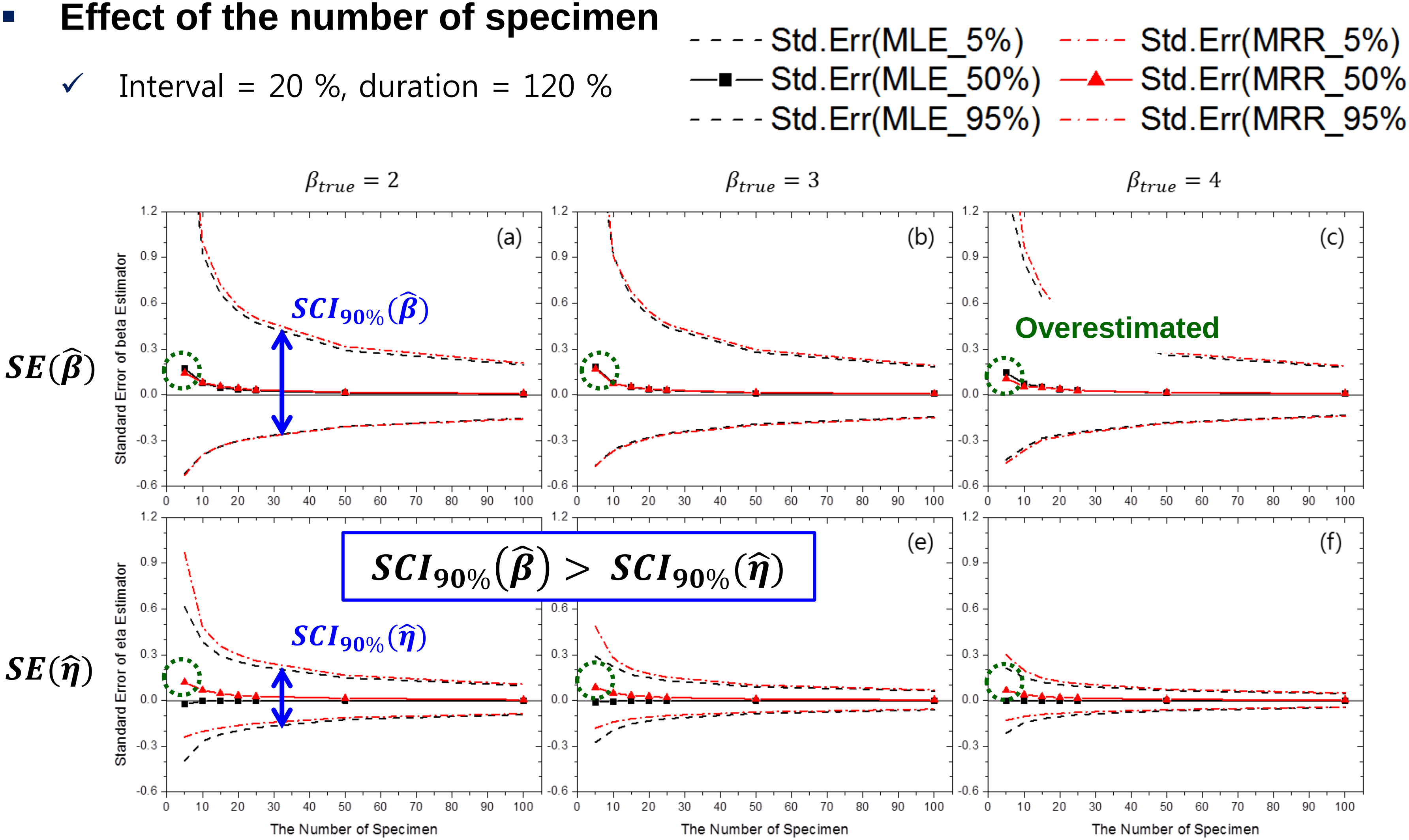
$$\mathrm{SCI}_{90\%}(\hat{\eta}) = \mathrm{SE}(\hat{\eta}_{95\%}) - \mathrm{SE}(\hat{\eta}_{5\%})$$

The estimation method would be reliable when $SE_{50\%}$ and $SCI_{90\%}$ approached to 'zero'.

~ bias ~ deviation

Results and Discussion

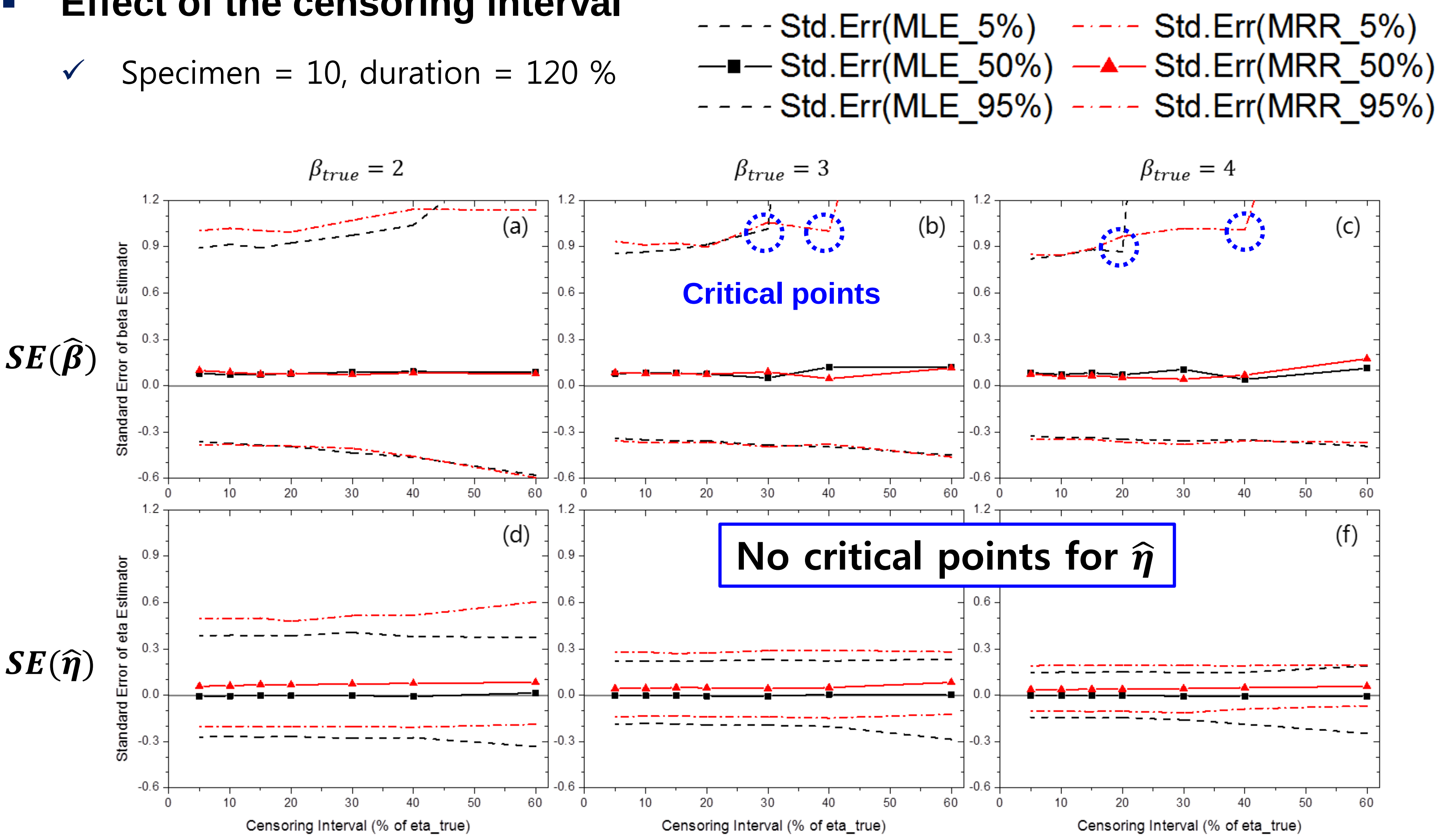
- **Effect of the number of specimen**
  - ✓ Interval = 20 %, duration = 120 %



The Number of Specimen

Results and Discussion

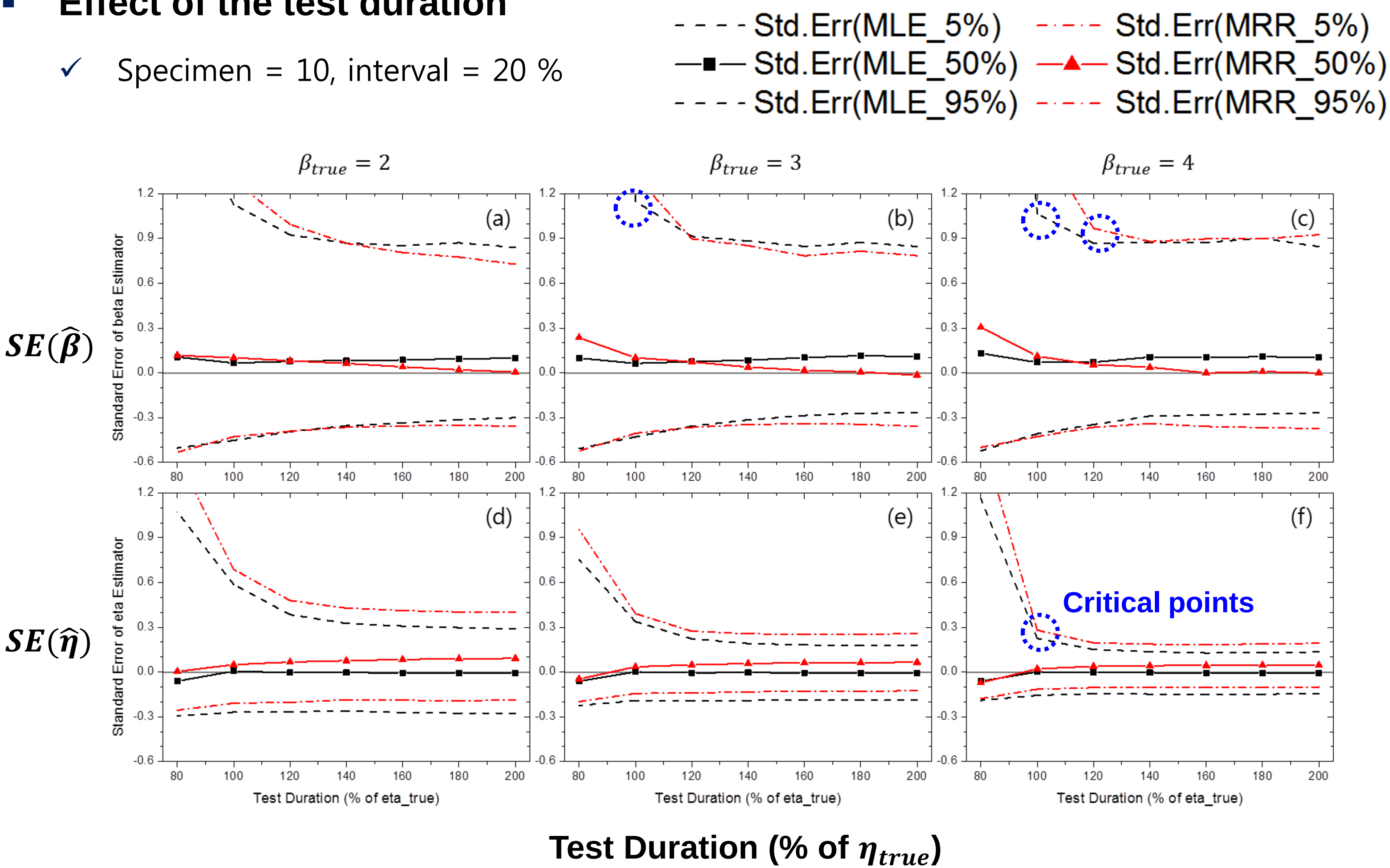
- **Effect of the censoring interval**
  - ✓ Specimen = 10, duration = 120 %



Censoring Interval (% of η_{true})

Results and Discussion

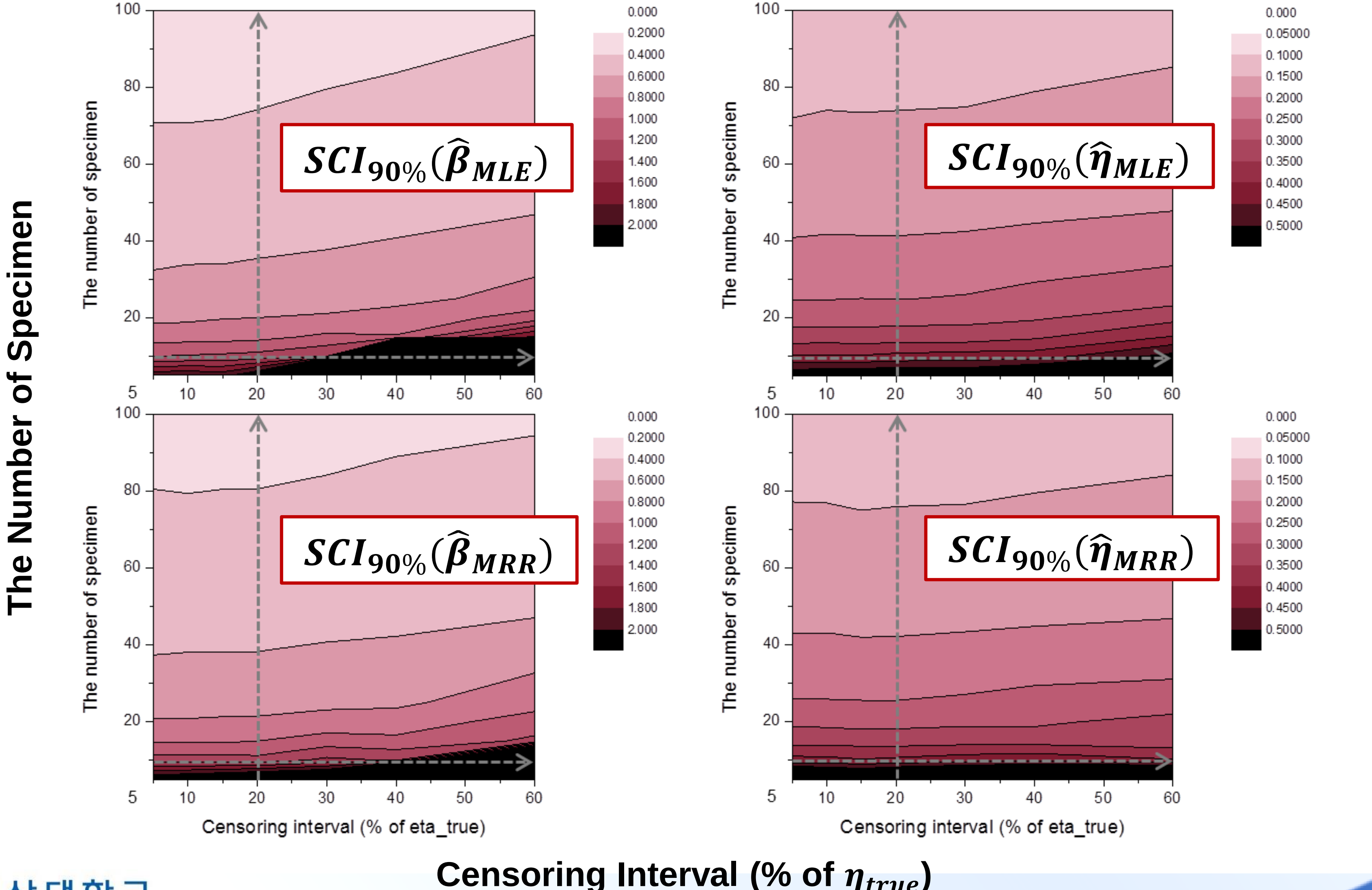
- **Effect of the test duration**
  - ✓ Specimen = 10, interval = 20 %



Results and Discussion

※ For the case of
'$\beta_{true} = 3$'
'$test\ duration = 120\ \%\ of\ \eta_{true}$'

- **Results of SCI evaluation**



Summary

- **Summary**
  - ✓ The goal of this work is
    - to quantify the estimation uncertainty of Weibull estimation.
    - to suggest **reasonable experimental conditions** for SCC test.
  - ✓ $\hat{\eta}_{MLE}$ is reliable even if the number of specimen is relatively small (Esp. at high β_{true}).
    - Estimation of β from the test is not recommended.
  - ✓ There is a critical censoring interval and test duration.
    - Beyond the critical condition is undesirable.
    - Too short censoring interval (< 20 % of η_{true}) is not effective for reducing the estimation uncertainty.
    - Too long test duration (> 160 % of η_{true}) is also not effective.

THANK YOU FOR YOUR ATTENTION