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A Study on Crack Initiation Test Condition by Uncertainty Evaluation of Weibull Estimation Methods

Jae Phil Park [a], Chi Bum Bahn [a*]
[a] *Pusan National University, 2 Busandaehak-ro 63beon-gil, Geumjeong-gu, Busan 609-735, Republic of Korea*
[*] *Corresponding author: bahn@pusan.ac.kr*

1. Introduction

Stress Corrosion Cracking (SCC) is one of the main materials-related issues in operating nuclear reactors. Many researchers have endeavored to develop the prediction model of SCC initiation time. Weibull distribution, which can consider the effect of time-dependent degradation of material, is widely accepted as a statistical model of the SCC initiation.

Typical experimental procedure of a SCC initiation test is an interval censored reliability test. From the result of the test, experimenters can estimate the parameters of Weibull distribution by Maximum Likelihood Estimation (MLE) or Median Rank Regression (MRR).

However, in order to obtain the sufficient accuracy of the estimated Weibull model, it is hard for experimenters to determine the proper number of test specimens and censoring intervals. In this work, a comparison of MLE and MRR is performed by Monte Carlo simulation to quantify the effect of total number of specimen, test duration, censoring interval and shape parameter of the assumed true Weibull distribution.

2. Weibull Estimation

The Cumulative Distribution Function (CDF) of the two parameter Weibull distribution is frequently used as a cracking probability function and given by:

$$\mathrm{F}(\mathrm{t};\beta,\eta) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right];\ \mathrm{t} \geq 0;\ \ \beta,\eta > 0 \quad (1)$$

Where t is time, β is the shape parameter and η is the scale parameter of the Weibull distribution. MRR and MLE are widely accepted methods to estimate Weibull parameters from the results of cracking tests.

For MRR method, the median rank is computed by Benard's approximation [1] which is easy to calculate the median rank value with competent precision. And a nonlinear curve fitting solver in a least squared sense is employed for the regression procedure.

For MLE method, the likelihood function for interval and right censored data [2] is used. Numerical approach was adopted to find out maximum likelihood point which related to the MLE estimators.

3. Monte Carlo Simulation

The estimated Weibull distribution by MRR and MLE methods are different, even though both estimators were derived from the same experiment result. Then, it is in curious that which estimator is more precise as compared to the true behavior of the cracking probability.

Unfortunately, there is no MLE theory yet available to setting the estimation confidence for interval censored data [3]. Therefore, Monte Carlo simulation is used to evaluate uncertainties of each estimation methods quantitatively by conducting crack initiation test simulations. Considered experimental condition factors of the simulation study is as follows:

1) True Weibull parameters $(\beta_{\mathrm{true}}, \eta_{true})$
2) The number of specimen
3) Total test duration
4) Censoring interval

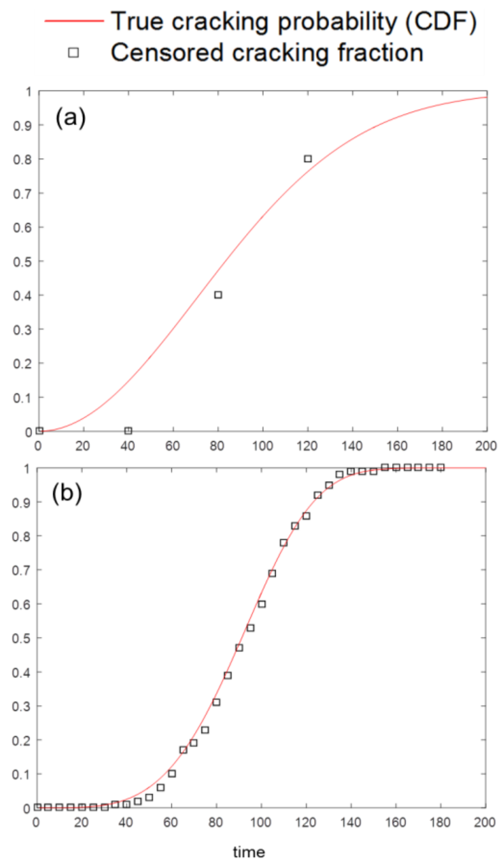

Fig. 1. Two examples of simulation experiment with different combination of condition factors

Table I: The combinations of experimental condition factors applied in Fig. 1.

Experimental factors	Fig. 1 (a)	Fig. 1 (b)
η_{true} (Dimensionless time)	100	100
β_{true}	2	4
The number of specimen	10	100
Total test duration (% of η_{true})	120	180
Censoring interval (% of η_{true})	40	5

Fig. 1 shows the examples of simulation experiment which have different combination of condition factors. Table 1 describes the applied condition factors in Fig. 1.

By combination of the considered experimental condition factors, total 63 simulation cases were studied. Each case has 10,000 times of crack initiation test simulations.

From the results of the simulations, 5%, 50% and 95% percentiles of Weibull estimators were driven from each case. And these estimators were converted to the standard error. The standard error of Weibull estimators were defined as follows:

$$\mathrm{SE}(\hat{\beta}) = \frac{\hat{\beta}-\beta_{true}}{\beta_{true}};\ \mathrm{SE}(\hat{\eta}) = \frac{\hat{\eta}-\eta_{true}}{\eta_{true}} \qquad (2)$$

Where, $\hat{\beta}$ and $\hat{\eta}$ are estimated Weibull parameters by MRR or MLE.

4. Results and Discussion

To examine the effect of the number of specimen, all other factors were fixed except the number of specimen. In this case, test duration was fixed to 120 (% of η_{true}) and censoring interval was also fixed to 20 (% of η_{true}).

When the number of specimen is large, there is high probability of 'precise estimation' for both MRR and MLE. This effect is well expressed in Fig. 2.

For shape parameter (β) estimation, MRR and MLE show similar estimation uncertainty level (see (a), (b) and (c) in Fig. 2). It appears that the shape parameter is overestimated with high probability by both MRR and MLE when the number of specimen is less than 25 (i.e., $\hat{\beta} > \beta_{true}$).

For scale parameter (η) estimation, smaller magnitude of standard errors were observed for all range of the specimen number case as compared to the shape parameter estimation (see (d), (e) and (f) in Fig. 2), especially for the case of high β_{true}. The noticeable point is that the MLE scale parameter estimators have little bias as compared to the MRR estimators for the range of small number of specimen.

The other effects by the rest factors will be reviewed in later research.

5. Conclusion

The goal of this work is to suggest proper experimental conditions for experimenters who want to develop probabilistic SCC initiation model by cracking test. Widely used MRR and MLE are considered as

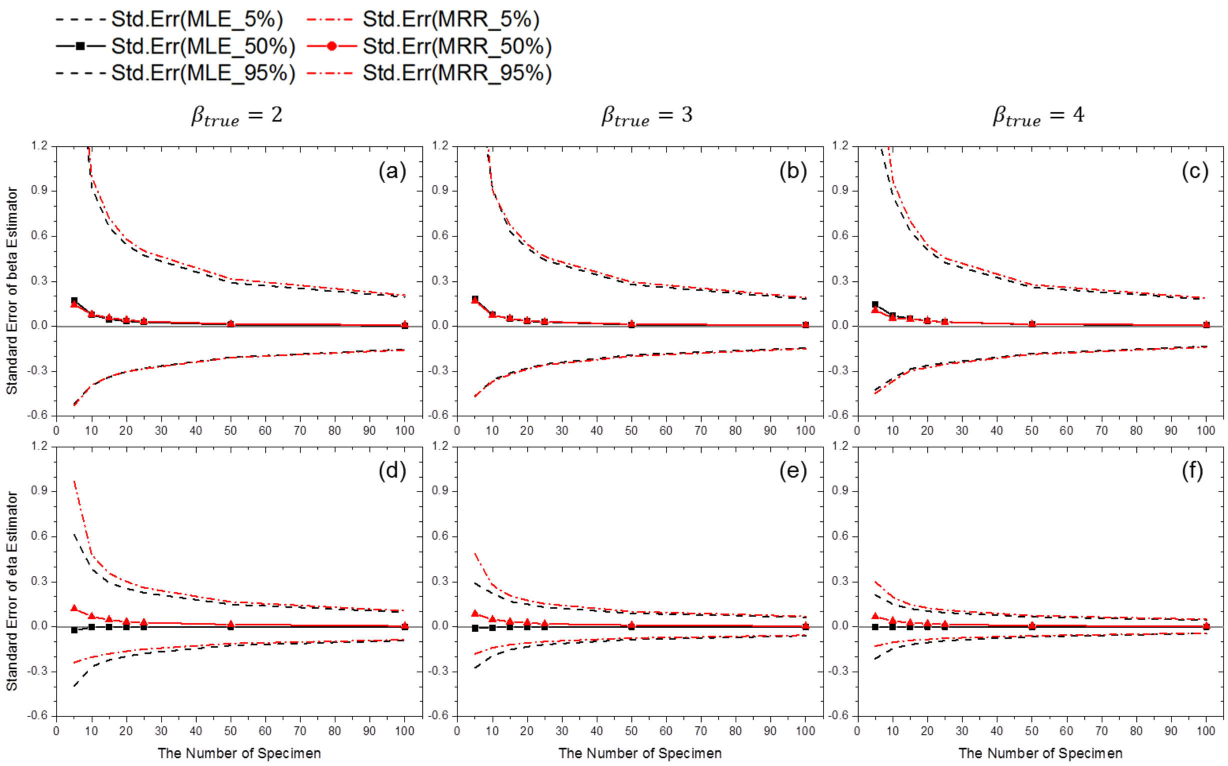

Fig. 2. Effect of the number of specimen for estimation uncertainty of Weibull parameters

estimation methods of Weibull distribution. By using a Monte Carlo simulation, uncertainties of MRR and MLE estimators were quantified in various conditions of experimental cases. The following conclusions could be informative for the experimenters:

1) For all range of the simulation study, estimated scale parameters were more reliable than estimated shape parameters, especially for at high β_{true}.

2) It is likely that the shape parameter is overestimated when the number of specimen is less than 25. For scale parameter estimation, MLE estimators have small bias as compared to the MRR estimators.

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