# A Mechanistic Model of Onset of Flow Instability Due to Mergence of Bubble Layers in a Vertical Narrow Rectangular Channel

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## 1. Introduction

The onset of flow instability (OFI) is the one of important boiling phenomena since it may induce the premature critical heat flux (CHF) at the lowest heat flux level due to sudden flow excursion in a single channel of multichannel configuration[1]. Especially prediction of OFI for narrow rectangular channel is very crucial in relevant to thermal-hydraulic design and safety analysis of open pool-type research reactors (RRs) using plate-type fuels. Even so, simple empirical correlations have been widely used and still considered as reliable way to predict the occurrence of OFI rather than mechanistic model due to the lack of knowledge of the triggering mechanism. Recently, based on high speed video (HSV) technique, the authors[2,3,4] observed and determined that OFI and the minimum premature CHF in a narrow rectangular channel are induced by abrupt pressure drop fluctuation due to the mergence of facing bubble boundary layers (BLs) on opposite boiling surfaces. In this study, new mechanistic OFI model for narrow rectangular channel heated on both sides has been derived, which satisfies with the real triggering phenomena.

### 2. Methods and Results



2.1 Mechanistic model

(b) D(

Fig. 1. Force balance on a bubble for (a) upward flow and (b) downward flow

In observations in the previous literature[4], the maximum bubble layer thickness (BLTs) would be considered to be comparable to the bubble departure

diameter, since the BLs were mostly composed of attached or sliding bubbles in a narrow rectangular channel. Since net forces acing on the bubble in *y*direction should become zero at the bubble departure, force balance in *y*-direction could be used to derive the model for the growth of BLTs:

$$\sum F_{y} = F_{Sy} + F_{D} + F_{Gy} + F_{B} = 0$$
(1)

where  $F_y$  is the forces acting in the y-direction,  $F_{Sy}$  is the surface tension force in the y-direction,  $F_D$  is the drag force,  $F_{Gy}$  is the growth force in the y-direction,  $F_B$ is the buoyancy force. The forces acting on a bubble at nucleation site for upward and downward flow are schematically shown in Fig. 1.

The surface tension force in the *y* direction was given by Klausner [5] as:

$$F_{Sy} = -D_W \sigma \frac{\pi \left(\theta_a - \theta_r\right)}{\pi^2 - \left(\theta_a - \theta_r\right)^2} \left(\sin \theta_a + \sin \theta_r\right) \qquad (2)$$

where,  $D_W$ ,  $\sigma$ ,  $\theta_a$  and  $\theta_r$  are the bubble contact diameter on the heater surface, surface tension, the advancing contact angle, and the receding contact angle. At the moment of bubble departure, the surface tension force can be neglected because the bubble contact area on the wall become negligible. In the present model, therefore, the surface force is set to be zero.

The drag force acting on the bubble is defined using the drag coefficient  $C_D$  as

$$F_D = C_D \left(\frac{1}{2}\rho_f \Delta U^2\right) \left(\frac{\pi}{4}D^2\right) \tag{3}$$

where  $\Delta U$  is the relative velocity between the bubble and liquid, but it assumed to be equal to liquid velocity, in consideration of the moment of bubble departure. Therefore, the equation becomes

$$F_D = \frac{C_D \pi G^2 D^2}{8\rho_f} \tag{4}$$

Drag coefficient is important to determine the drag force on the bubble. Similarly used in a literature, Ishii and Zuber [6] correlation for  $500 \le \text{Re}_b \le 2 \times 10^5$  is adopted in the present model:

$$C_D = \frac{2}{3} \left( \frac{g(\rho_f - \rho_g) D^2}{\sigma} \right)^{0.5}$$
(5)

where  $Re_b = \rho_f \Delta U D / \mu_f$  is the bubble Reynolds number.

The growth force, also called unsteady drag force is the unsteady drag due to asymmetrical growth of the bubble. This force is crucial in the present model, since the growth force is generally governed by the wall temperature, and therefore heat flux. As experimentally observed in the previous study[4], the maximum BLT is increasing with increasing heat flux under given mass flux and inlet subcooling. This can be explained by the fact that the increase of heat flux or wall temperature increases the growth force, which tending to hold the bubble on the surface, and therefore bubble diameter becomes enlarged to compensate the forces acting to detach the bubble such as the drag force. The growth force was derived from the inertial force of virtual added mass for an attached spherical bubble by Situ et al.[7]:

$$F_G = -\rho_f \pi r^2 \left( \frac{11}{2} \dot{r}^2 + \frac{11}{6} r \ddot{r} \right)$$
(6)

Zuber's bubble-growth model [8] was used to predict the growth rate of a bubble, which is expressed as:

$$r = \frac{2b_G}{\sqrt{\pi}} Ja \sqrt{\alpha_f t} \tag{7}$$

where  $b_G$  is a constant suggested as 1.73 by Zeng et al. [9] and  $\alpha_f$  is the thermal diffusivity. The Jacob number is defined as

$$Ja = \frac{\rho_f C_{pf} \left( T_w - T_{sat} \right)}{\rho_g h_{fg}} \tag{8}$$

From Eqs. (6) and (7), the growth force could be expressed as

$$F_G = -\frac{44\rho_f b_G^4 J a^4 \alpha_f^2}{3\pi} \tag{9}$$

Therefore, the growth force in *y*-direction is finally given by

$$F_{Gy} = -\sin\theta_i \frac{44\rho_f b_G^4 J a^4 \alpha_f^2}{3\pi}$$
(10)

where  $\theta_i$  is the inclination angle, as shown in Fig. 1. The value of 10° was suggested for inclination angle in a literature[5], but much lower inclination angle was observed in the previous study[4].

The buoyancy force is importantly included in the present model, since it was also observed that the flow direction highly influence on the maximum BLT and OFI. The buoyancy force is given by

$$F_B = \mp \left(\rho_f - \rho_g\right) g V = \mp \frac{\pi D^3}{6} g \left(\rho_f - \rho_g\right) \quad (11)$$

where, a negative sign is for downward flow, and a positive sign is for upward flow. Since the buoyancy force is acting to hold the bubble on the boiling surface for downward flow, bubble departure diameter should be enlarged under same fluid conditions in compared with that for upward flow, which agrees well with the experimental observations[4], i.e. more rapid growth of maximum BLT for downward flow.

Substituting the drag force (Eq. (4)), growth force (Eq. 10)) and buoyancy force (Eq. (11)) into the force balance (Eq. (1)) leads to the following expression for Jacob number, Ja:

$$Ja = \left[\frac{3\pi}{44(\sin\theta_i)\rho_f b_G^4 \alpha_f^2} \left\{\frac{C_D \pi G^2 D_d^2}{8\rho_f} \mp \frac{\pi g \left(\rho_f - \rho_g\right) D_d^3}{6}\right\}\right]^{1/4}$$
(12)

where  $D_d$  is the bubble departure diameter. Since  $C_D$  is proportional to  $D_d$  in the present model, Ja is proportional to  $D_d$ <sup>3/4</sup> in this model.

We determined that the onset of mergence of facing BLs is the key triggering mechanism of OFI for narrow rectangular channel heated from both sides, and the maximum BLTs are comparable to the bubble departure diameters. Therefore, following criterion is mainly used to develop the present mechanistic model for prediction of OFI:

$$D_d \approx \delta_{max} = \frac{b}{2}$$
 at OFI (13)

Combining with Eq.(12), we can have the wall temperature at the onset of mergence of facing BLs for given mass flux and pressure conditions.

In order to calculate the OFI heat flux from the derived wall temperature, Chen correlation [10] is used, which would be extended in the subcooled boiling region [11], as given form:

$$q'' = h_{NB} \left( T_w - T_{sat} \right) + h_c \left( T_w - T_f \right)$$
(14)

where,  $h_c$  is the contribution due to convection, which is calculated by Dittus-Boelter correlation in the present model:

$$h_c = 0.023 \operatorname{Re}_f^{0.8} \operatorname{Pr}_f^{0.4} \left( \frac{k_f}{D_e} \right)$$
 (15)

and,  $h_{NB}$  is the contribution due to nucleate boiling, which is expressed as

$$h_{NB} = 0.00122 \left( \frac{k_f^{0.79} C_{pf}^{0.45} \rho_f^{0.49}}{\sigma^{0.5} \mu_f^{0.29} h_{fg}^{0.24} \rho_g^{0.24}} \right) \Delta T_{sat}^{0.24} \Delta P_{sat}^{0.75} S$$
(16)

and

$$S = \left(1 + 2.53 \times 10^{-6} \,\mathrm{Re}_{TP}^{1.17}\right)^{-1} \tag{17}$$

where  $Re_{TP}$  is the two-phase Reynolds number calculated by setting vapor quality, *x* and the factor *F* as zero and unity, respectively. Since the coalescence of facing bubbles always occurred at the end of heated channel, bulk temperature,  $T_f$  is equal to outlet temperature,  $T_o$ . Based on the heat balance equation, Eq.(14) could be expressed using inlet temperature,  $T_i$ such as

$$q'' = \left(1 + h_c \frac{A_H}{AGC_{p,f}}\right)^{-1} \left\{ \left(h_{NB} + h_c\right) \left(T_w - T_{sat}\right) + h_c \left(T_{sat} - T_i\right) \right\}$$
(18)

It was verified that wall temperature are fairly well predicted by the Eq. (18) over saturation temperature at the wall, as shown in Fig. 2.



Fig. 2. Validation of Chen's correlation for narrow rectangular channel

# 2.2 Model Validation

KAIST OFI database was used to validate the proposed mechanistic model, which is constructed by OFI data for upward flow and downward flow in a narrow rectangular channel having gap sizes of 2.35-4.1

mm (see Table I). Among 130 OFI data, 21 data were obtained for upward flow conditions.

Table I: Problem Description

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<i>b</i> (mm)	2.35-4.1
L <sub>H</sub> /D <sub>e</sub>	47.1-78.8
G (kg/m <sup>2</sup> s)	190-1430
$T_i$ (°C)	24-58
$P_o$ (bar)	1.0-1.3
Flow direction	Upward (21) / Downward (109)
Total # of OFI data	130

Since the inclination angle of experiments is quite smaller than the postulated value in the observation, the inclination angle is used to optimize the present model. As a result, it was found that  $1.7 \circ$  of the inclination angle minimizes root-mean-squared error (RMSE) of prediction, and which is in good agreement with experimental observation. Therefore,  $1.7 \circ$  is adopted for the value of indication angle in the present model.

Comparison between the model predictions and measured OFI heat fluxes are shown in the Fig. 3. The present mechanistic model shows fairly good agreement with KAIST OFI database: 95% of the data fall within - 33% and +50% of the prediction. It also found that the model provides good predictions for all range of mass flux and inlet subcooling enthalpy. In addition, the RMSE and MAE of the mechanistic model are assessed by 23.4% and 17.8%, respectively.



Fig. 3. Predicted vs. Measured OFI heat flux

Since the present model also predicts the bubble departure diameter, and therefore maximum BLT, 74 experimental maximum thickness data (for  $Re_b \ge 500$ ) which were obtained by image processing[4] were compared to the predictions from the model (Fig. 4). As shown in the figure, the maximum BLTs are predicted fairly well by the current model, which providing RMSE of 25.4% and MAE of 20.8% while most of the experimental data (90% of 74 data) can be estimated with a maximum error of 43%.



Fig. 4. Comparison between predicted and measured maximum thickness

Lastly, the extensibility of the mechanistic model to the wider range of conditions was checked by using Whittle and Forgan's OFI database [12]. 66 OFI data were selected from the database, which are obtained with narrow rectangular channel heated from both sides, for high mass flux and large  $L_H/D_e$  conditions (Table II). As shown in the Fig. 5, the present model is slightly overprecting Whittle and Forgan's OFI data, but still 95% of the data among 196 data falls within the error bands. In addition, the performance of the mechanistic model is confirmed not to be significantly changed (RMSE: 25.4%, MAE: 20.8%). Therefore, it is expected that the present mechanistic model based on predicting the point of mergence of facing BLs (that is, occurrence of mergence of BLs) using force balance could be utilized for prediction of OFI heat flux, even for high mass flux and large  $L_H/D_e$  conditions.



Fig. 5. Comparison with Whittle and Forgan's OFI database

#### 3. Conclusions

Pressure drop fluctuation which is initiated by mergence of facing BLs on opposite boiling surfaces, was recently pointed out as the main triggering mechanism of OFI in a narrow rectangular channel heated from both-side. In this regard, a new mechanistic model for OFI, based on predicting the mergence point of BLs in the narrow channel has been proposed. Force balance approach was used for modeling of the maximum BLT since the quantity is comparable to the bubble departure diameter. From the validation with OFI database, it was shown that the new model fairly well predicts OFI heat flux for wide range of conditions.

Table II: Problem Description

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<i>b</i> (mm)	1.4-3.23
$L_{H}/D_{e}$	91.3-201
G (kg/m <sup>2</sup> s)	818-9105
$T_i$ (°C)	35-75
$P_o$ (bar)	1.17-1.72
Flow direction	Upward (59) / Downward (7)
Total # of OFI data	66

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