Application of Looped Network Analysis Method to Core of Prismatic VHTR

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1. Introduction

The Very High Temperature Reactor (VHTR) is currently envisioned as a promising future reactor concept because of its high-efficiency and capability of generating hydrogen [1]. Prismatic Modular Reactor (PMR) is one of the main VHTR concepts, which consists of hexagonal prismatic fuel blocks and reflector blocks made of nuclear grade graphite. However, their shape could be changed by neutron damage during the reactor operation and the shape change can makes the gaps between the blocks inducing bypass flow. Most of reactor coolant flows through the coolant channel within the fuel block, but some portion of the reactor coolant bypasses to the interstitial gaps. The vertical gap and horizontal gap are called bypass gap and cross gap, respectively as shown in Fig. 1. CFD simulation for the full core of VHTR might be possible but it requires vast computational cost and time. Moreover, it is hard to cover whole cases corresponding to the various bypass gap distribution in the whole VHTR core. In order to solve this problem, in this study, the flow network analysis code, FastNet (Flow Analysis for Steady-state Network), was developed using the Looped Network Analysis Method. The applied method was validated by comparing with SNU VHTR multi-block experiment. A 3-demensional network modeling was conducted representing flow paths as flow resistances. The calculation results with the network analysis code were compared with the experimental data and CFD calculation results and a reasonably good agreement were obtained among them.

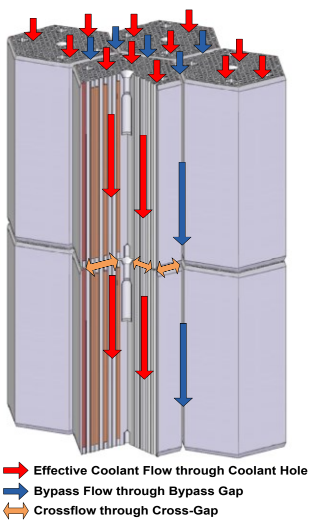

Fig. 1. Bypass flow and Cross flow in PMR core

2. Governing Equations

In the VHTR core, main flows are pipe flows and the pipe flow is usually calculated with Darcy-Weisbach equation. The form of Darcy-Weisbach equation is Eq. (1).

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (1)$$

Where h is head loss, f is friction factor, L is length of the flow path, D is diameter of the flow path, V is flow velocity, and g is the gravity acceleration.

To analyze network, Eq. (1) can be expressed as a relation of head loss and flow rate.

$$h_f = RQ^2, \qquad where\, R = \frac{fL}{2gDA^2} \quad (2)$$

Where A is flow area, Q is the volumetric flow rate, and K is the loss factor that can be thought as flow resistance.

If a network has one or more closed loop, it is called looped network. In a simple pipe network, the flow is unique and can be obtained easily. However, in case of a looped network, the number of pipes is too large to find the flows by merely applying flow continuity equations at nodes. Furthermore, nonlinearity of the head and flow rate makes the problem more difficult. The analysis of looped network consists of the determination of flow rates of the pipes and heads at the nodes. The following laws, called Kirchhoff's circuit laws [2], generate the governing equations.

- The algebraic sum of inflow and outflow discharges at a node is zero.
- The algebraic sum of the head loss around a loop is zero.

2.1 Conservation of Mass

Conservation of mass at a node is established based on the law that the amounts of inflow and outflow are same at the junction where the pipes are connected. In another words, the sum of inflow and outflow at a node is zero. For a junction node *i*, conservation of mass can be written as Eq. (3).

$$F_j = \sum_{n=1}^{j_n} Q_{jn} = 0 \qquad (3)$$

Where Q_{jn} is the inlet flow from *n-th* pipe at node *j*, and j_n is the total number of pipes at node *j*. This mass equation is used at every node in the system and so, it can be referred as nodal equation.

2.2 Conservation of Momentum

The conservation equation of momentum can be represented with head loss. Hence, the momentum equation can be called as head loss equation. While traversing along a loop, as one reaches at the starting node, the net head loss is zero. In short, the sum of the head loss around a loop is zero. It can be written as Eq. (4).

$$F_k = \sum_{n=1}^{k_n} R_{kn} \left|Q_{kn}\right| Q_{kn} = 0$$

Where k_n is the total number of pipes at the *k-th* loop. Since one loop has one head loss equation, it can be referred as loop equation.

3. Application of Linear Theory Method

The linear theory method is a looped network analysis method presented by Wood and Charles (1972) [3]. The entire network is analyzed altogether by calculating matrix. The nodal flow continuity equations are obviously linear but the looped head-loss equations are nonlinear. In this method, the looped momentum equations are modified to be linear for previously known discharges and solved iteratively. The process is repeated until the two solutions are close to the allowable limits. The nodal equations are Eq. (3). It can be generalized in the following form for the entire network:

$$F_j = \sum_{n=1}^{i_L} a_{jn} Q_{jn} = 0 \qquad (5)$$

Where a_{jn} is +1 if positive discharge flows in pipe n, -1 if negative discharge flows in pipe n, and 0 if pipe n is not connected to node *j*. The total pipes in the network are i_L.

The loop head-loss equations are Eq. (4). It can be linearized as Eq. (6).

$$F_k = \sum_{n=1}^{k_n} b_{kn} Q_{kn} = 0 \qquad (6)$$

Where $b_{kn}=R_{kn} \mid Q_{kn} \mid$ for initially known pipe discharges. Eq. (6) can be generalized for the entire network in the following form:

$$F_k = \sum_{n=1}^{i_L} b_{kn} Q_{kn} = 0 \qquad (7)$$

where $b_{kn}=R_{kn} \mid Q_{kn} \mid$ if pipe *n* is in loop *k* or otherwise $b_{kn}=0$. The coefficient b_{kn} is revised with current pipe discharges for the next iteration. This results in a set of linear equations, which are solved by using any standard method for solving linear equations. Thus, the total set of equations required for i_L unknown pipe discharges are

- Nodal continuity equations for $n_L - 1$ nodes
- Loop head-loss equations for k_L loops

The overall procedure for looped network analysis by the linear theory method can be summarized in the following steps:

Step 1: Number pipes, nodes, and loops.
Step 2: Write nodal discharge equations as Eq. (3).
Step 3: Write loop head-loss equations as Eq. (6).
Step 4: Assume initial pipe flows, Q_1, Q_2, Q_3, ... arbitrarily.
Step 5: Generalize nodal continuity and loop equations for the entire network.
Step 6: Calculate pipe discharges. The equation generated is of the form ***Ax=b***, which can be solved for Q_i.
Step 7: Recalculate coefficients b_{kn} from the obtained Q_i values.
Step 8: Repeat the process again until the calculated Q_i values in two consecutive iterations are close to predefined limits.

4. Network Modeling of SNU Multi-Block Experiment

The test facility of SNU multi-block experiment [4] consists of 28 test blocks; 7 columns radially and 4 layers axially. Two types of test blocks are used. One is a standard fuel block and the other is a reflector block. Fig. 2 shows a schematic of the experimental apparatus.

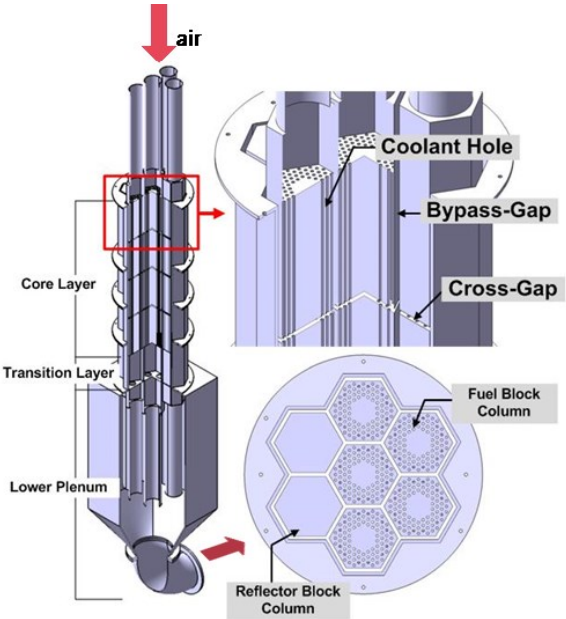

Fig. 2. Configuration of SNU multi-block experimental apparatus

The flow paths of the experiment were simulated with 2-D network model first in the lateral direction as shown in Fig. 3. It represents cross flow and the lateral bypass flow. Then, the 2-D network was extended to 3-D network model as depicted in Fig. 4. Vertical flow paths consist of coolant channels and bypass gaps. Coolant channels in a single block was represented as one flow resistance for simplicity because of their same length and flow area.

Figure 5 shows the flow diagram of developed network code. At first, the geometry information and the boundary conditions are specified. Node equations and loop equations are obtained from these inputs. Then, linearized matrix is constructed by the linear theory matrix. LU-decomposition and back substitution algorithms (H.P. William et al., 2007) were used to solve the linearized matrix.

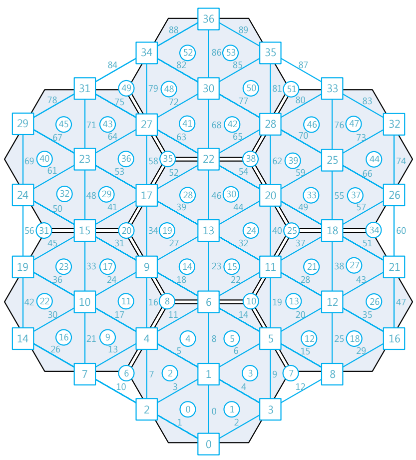

Fig. 3. 2-D network model of the experimental facility

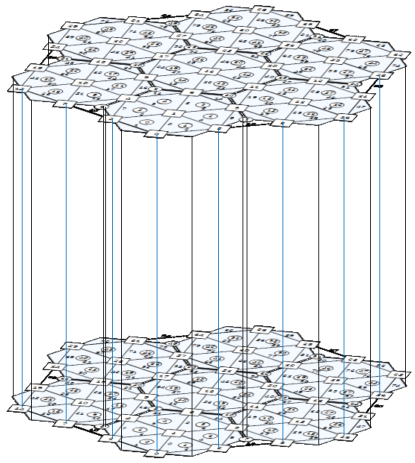

Fig. 4. 3-D network model of the experimental facility.

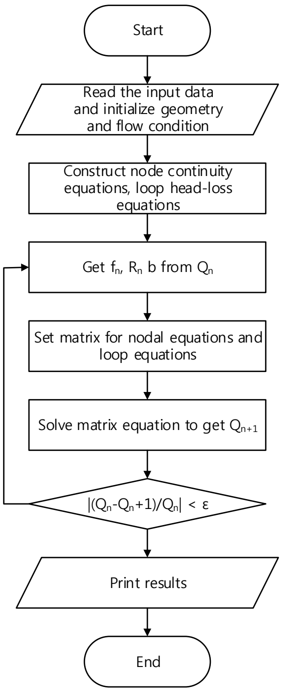

Fig. 5. Flow chart of developed flow network analysis code

5. Results and Discussion

5.1 Code verification with Empirical Correlations

The FastNet code uses the Darcy-Weisbach equation to calculate the pressure drop in the flow path. To verify Darcy-Weisbach equation implemented in the developed code, the pressure drop of the coolant hole by the developed code was compared with independent calculation result by Blasius equations.

The calculation results by the FastNet code and the analytic solution using Blasius equation show good agreement as listed in Table I. Consequently, Darcy-Weisbach equation was coded in the developed code correctly.

Table I: Problem Description

Case	Flow path	Pressure (Pa)		Diff.
		FastNet	Blasius	
BG2-CG0	Coolant channel	1236	1202	2.7%
	Bypass gap	1190	1138	4.3%
BG6-CG0	Coolant channel	524	508	3.1%
	Bypass gap	413	405	1.9%

5.2 Code V&V with Experimental and CFD Simulation Results

The FastNet code was validated by the experimental and CFD simulation results for the multi-block experiment [4]. Figs. 6 and 7 are the comparative results of the experiment, CFD simulation and FastNet code for bypass gap size 2 mm and cross gap size 0 mm case and bypass gap size 6 mm and cross gap size 0 mm case. Calculated result of the FastNet code shows a good agreement with the experimental and CFD simulation results for the uniform bypass gap test. Both the pressure distribution and mass flow distribution are predicted accurately.

The comparative results for non-uniform bypass gap test were plotted in Fig. 8. In this condition, the test section has variant bypass gap size along axial direction. From the top, bypass gap size of the 4th layer, 3rd layer, 2nd layer and 1st layer is 6 mm, 2 mm, 4 mm and 2 mm. To investigate effect of cross flow, the cross gap of 2 mm is assumed because the bypass gap change can cause the cross flow phenomenon significantly. And it was observed that the calculation results of FastNet code, CFD analysis and experimental data show good agreement.

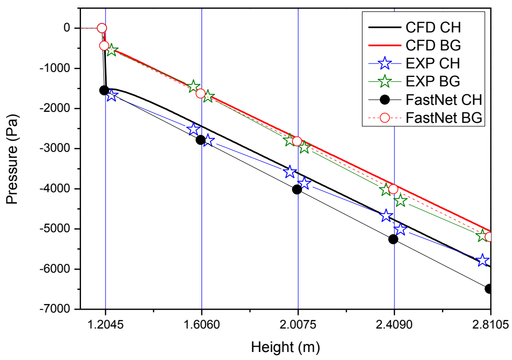

Fig. 6. Comparison results of BG2-CG0 case

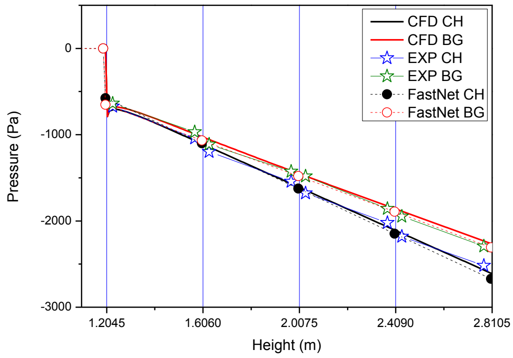

Fig. 7. Comparison results of BG6-CG0 case

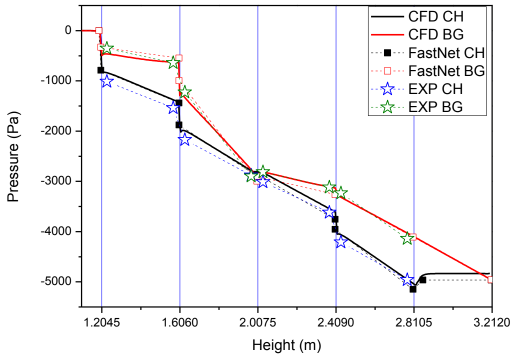

Fig. 8. Comparison results of BG6242-CG2 case

6. Conclusions

Flow network analysis code, FastNet, was developed to evaluate the core bypass flow distribution by using looped network analysis method. Complex flow network could be solved simply by converting the non-linear momentum equation to the linearized equation. The FastNet code predicted the flow distribution of the SNU multi-block experiment accurately. It can be expected that the developed network code can contribute to assure the core thermal margin by predicting the bypass flow in the whole core of VHTR.

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