Stability and Well-Posedness of Horizontal Stratified Flow: Effect of Velocity Shape

Byoung Jae Kim [a*], Joshua Kim Schimpf [a,b], Kyung Doo Kim [a]
[a]*Thermal-Hydraulic Safety Research Division, Korea Atomic Energy Research Division, Yuseong-gu, Daejeon*
[b]*Department of Advanced Nuclear System Engineering, University of Science and Technology, Yuseong-gu, Daejeon*
[*]*Corresponding author: byoungjae@kaeri.re.kr*

1. Introduction

The Kelvin-Helmholtz instability is closely related to a transition from horizontal stratification to slug/annular flow. The full Navier-Stokes equation can be used for the stability analysis of two-layer laminar flow [1]. The two-fluid model is a practical approach for turbulent flow, since the base flow can be described by averaged values. The drawback is that the stability result depends largely on the reliability of the closure relations. Nevertheless, owing to the practical reason, the two-fluid model is popularly used to develop the flow regime criteria pertaining to horizontal stratification.

Among various criteria for horizontal stratification, we note the viscous Kelvin-Helmholtz instability (VKH) including the terms of the wall and interfacial frictions. Referring to Fig. 1, in terms of the relative velocity between two phases, the neutral stability condition in the VKH is lower than that in inviscid Kelvin-Helmholtz instability (IKH) [2, 3]. Waves occurs at the neutral condition of the VKH. The amplification factor of the VKH is small in the region between the neutral stability lines of the VKH and IKH. For a low liquid level, the interface is wavy-stratified in that region. The amplification factor of the VKH increases sharply near the neutral condition of the IKH. Though the rapid growth of the interface does not guarantee a transition to non-stratified flow, the neutral condition of the IKH is considered as an upper limit of wavy stratified interface. The region between two neutral lines is interpreted as a buffer region between a stable stratified flow and annular flow. For a high liquid level, the neutral condition of the VKH indicates a transition to slug flow.

Various studies adopted the VKH concept to improve the flow transition criteria in a horizontal flow, adding the effects of the traverse curvature of the interface [4], sheltering in the interfacial drag [5], and 1D turbulent viscosity [6].

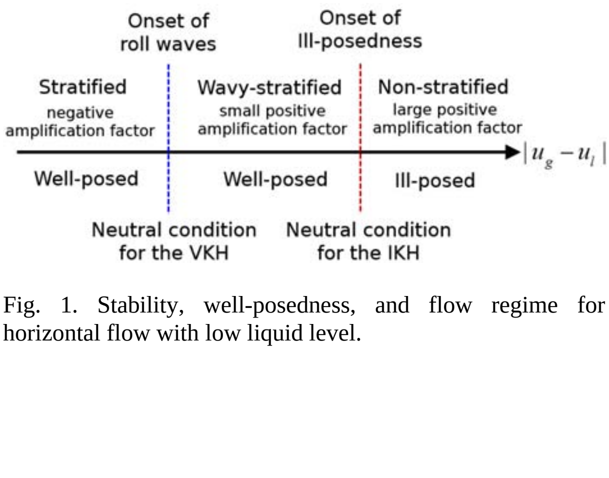

Fig. 1. Stability, well-posedness, and flow regime for horizontal flow with low liquid level.

As mentioned earlier, the transition to non-stratified flow is associated with the neutral condition of the IKH. Interestingly, the characteristic analysis is equivalent to the IKH analysis, since the characteristic analysis is performed neglecting zeroth-order terms such as wall and interfacial frictions. Therefore, the stability of inviscid flow is in phase with well-posedness.

Now, we note the velocity shape parameter in the momentum equation. In many cases, the velocity shape parameter is assumed to be unity for simplicity. Refs. [7, 8] obtained both the neutral stability condition of the VKH and the condition for real characteristics, including the velocity shape parameters. A non-uniform velocity profile had a stabilizing effect in terms of stability and well-posedness. Refs. [7, 8] tested their theory in oil-water flow. Refs. [9-11] also showed that the velocity shape parameter extended the well-posed region for vertical bubbly, slug, and annular flows.

Recently, we found that when the liquid level is low in a large horizontal pipe, SPACE and MARS tend to identify the flow regime as an annular flow though the actual flow is stratified. This foible motivated this study. This study investigates the effect of the velocity shape parameter on the transition to non-stratified gas-liquid.

2. Existing Criteria for Horizontal Stratification

We first review existing criteria used in current thermal-hydraulic codes. The relative velocity between two phases alone does not determine whether flow is horizontally stratified, but it is a primary criterion. Table 1 shows the relative velocity criteria used in current thermal-hydraulic codes. K is defined as

$$K = \frac{|u_g - u_l|}{\left[\dfrac{\alpha_l \rho_g + \alpha_g \rho_l}{\rho_g \rho_l} \Delta\rho g \dfrac{A}{S_i}\right]^{1/2}}, \tag{1}$$

which is basically in the form of the classical inviscid Kelvin-Helmholtz instability. HST denotes horizontal stratification

Table 1. Horizontal stratification criteria

TH Code	Transition to HST	Transition to non-HST	Ref.
SPACE	K=0.249	K=0.497	[12]
MARS	0.25	0.5	[13]
CATHARE	0.5	1	IKH
TRACE	$1-h_l/D$	$2(1-h_l/D)$	[14]

In SPACE, for K<0.249, the flow is horizontally stratified, and for K>0.497 the flow is identified as non-stratified flow. The region of 0.249≤K≤0.497 is a buffer region intended for smooth transition. In SPACE, MARS, and CATHARE, the lower limit of the buffer region is set to the half of the neutral condition, whereas in TRACE the upper limit is set to twice the neutral condition.

When the liquid level is low in a large horizontal pipe, SPACE and MARS tend to identify the flow regime as an annular flow though the actual flow is stratified. Ref. [14] suggested $K = 1 - h_l / D$ for the transition to a non-horizontal flow. Meanwhile, according to Ref. [3], for air-water at atmospheric pressure, $K = 1 - h_l / D$ match approximately the neutral condition of the VKH. This implies that $K = 1 - h_l / D$ is the criteria for onset of roll waves rather than a transition to non-stratified flow. In this regard, the TRACE model for transition to HST is reasonable. However, the model for transition to non-HST has no physical background: $K = 2(1 - h_l / D)$ is just twice the criterion of the transition to HST. In addition, $K = 1 - h_l / D$ may not match well the neutral condition of the VKH for steam-water at high pressures.

The present study follows the criteria suggested by Refs. [3, 7]: the neutral conditions of the VKH and IKH indicate, respectively, a transition to horizontal stratification and a transition to non-horizontal stratification. Moreover, the effect of the velocity shape parameter is investigated.

3. Well-posedness and Stability of Horizontally Stratified Flow

We consider a one-dimensional gas-liquid horizontal flow. Fluids are incompressible and flow is stratified. The mass and momentum equations for phase k are given as follows:

$$\frac{\partial \alpha_k}{\partial t} + \frac{\partial(\alpha_k u_k)}{\partial x} = 0 , \qquad (2)$$

$$\rho_k \frac{\partial}{\partial t}(\alpha_k u_k) + \rho_k \frac{\partial}{\partial x}(\gamma_k \alpha_k u_k^2) = -\alpha_k \frac{\partial p_i}{\partial x} \pm \frac{\tau_i S_i}{A} - \frac{\tau_k S_k}{A} - \alpha_k \rho_k g \frac{A}{S_i}\frac{\partial \alpha_l}{\partial x} , \qquad (3)$$

where γ_k, p_i, τ_i, S_i, A, and τ_k are the velocity shape parameter, interfacial pressure, interfacial shear stress, cross-sectional interface length, channel area, and wall shear stress, respectively. In Eqs. (1) and (2), the effect of the surface tension is neglected.

2.1 Well-Posedness

The equation system is well-posed if

$$u_g \le C_1 u_l + \left[C_1 (C_1 - C_2) u_l^2 + \frac{C_1}{\gamma_g \gamma_l} \frac{\alpha_l \rho_g + \alpha_g \rho_l}{\rho_g \rho_l} \Delta\rho g \frac{A}{S_i} \right]^{1/2} , \qquad (4)$$

where

$$C_1 = \frac{\gamma_l \alpha_g \rho_l}{\alpha_g \rho_l - (\gamma_g - 1)\alpha_l \rho_g} , \qquad (5)$$

$$C_2 = \frac{\alpha_l \rho_g - (\gamma_l - 1)\alpha_g \rho_l}{\gamma_g \alpha_l \rho_g} . \qquad (6)$$

For $\gamma_g = \gamma_l = 1$, Eq. (4) exactly reduces to the stable condition for classical inviscid Kelvin-Helmholtz instability:

$$u_g - u_l \le \Delta u_{IKH} = \left[\frac{\alpha_l \rho_g + \alpha_g \rho_l}{\rho_g \rho_l} \Delta\rho g \frac{A}{S_i} \right]^{1/2} . \qquad (7)$$

Ref. [7] insisted that $\gamma_g, \gamma_l > 1$ had a stabilizing contribution, but it is not always true. A simple test was performed for saturated steam-water in a 5 cm diameter pipe under atmospheric pressure. We fixed $u_l = 0.1$ m/s and $\gamma_g = 1.016$. Figure 2 shows the relative velocity between two phases required for onset of ill-posedness, which is computed by Eq. (4). The relative velocity is normalized by the classical inviscid result (Δu_{IKH}). The well-posed region is considerably extended for low α_l and high γ_l. It means that for low liquid level, a gas velocity required for annular flow is higher than that with uniform velocity profiles ($\gamma_g = \gamma_l = 1$). One can see that for $\gamma_l = 1.05$, the well-posed region can reduce for high liquid level. Though this behavior may occur when the liquid velocity is low in a large pipe, the well-posed region is generally extended for real flow conditions: the amount of liquid is small for horizontal annular flows, especially for a large pipe.

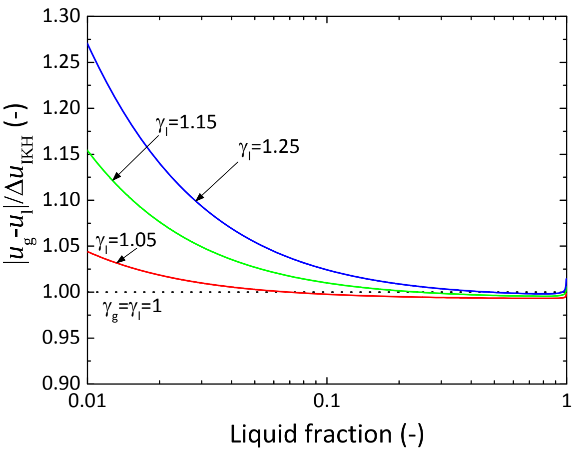

Fig. 2. Effect of the velocity shape parameter on the condition for onset of ill-posedness.

2.2 Stability

The neutral condition of the VKH represents a transition to wavy interfacial structure in the case of low liquid level. Conversely, for high liquid level, it indicates a transition to slug flow. Ref. [7] provided the neutral line of the VKH including the velocity shape parameters:

$$(C_V - C_{IV})^2 + \frac{\rho_g \rho_l}{\rho^2 \alpha_g \alpha_l}(\gamma_l u_l - \gamma_g u_g)^2 - \frac{\Delta\rho}{\rho} g \cos\beta \frac{A}{S_i} + \frac{1}{\rho}\left(\gamma_l (1-\gamma_l)\frac{\rho_l u_l^2}{\alpha_l} + \gamma_g (1-\gamma_g)\frac{\rho_g u_g^2}{\alpha_g}\right) = 0 ,$$

(8)

where

$$\rho = \rho_l / \alpha_l + \rho_g / \alpha_g , \tag{9}$$

$$C_V = \frac{\alpha_g u_l \partial F / \partial u_l - \alpha_l u_g \partial F / \partial u_g - \alpha_g \alpha_l \partial F / \partial \alpha_l}{\alpha_g \partial F / \partial u_l - \alpha_l \partial F / \partial u_g} , \tag{10}$$

$$C_{IV} = \frac{\gamma_l \alpha_g \rho_l u_l + \gamma_g \alpha_l \rho_g u_g}{\alpha_g \rho_l + \alpha_l \rho_g} , \tag{11}$$

$$F = \frac{\tau_i S_i}{A}\left(\frac{1}{\alpha_g} + \frac{1}{\alpha_l}\right) + \frac{\tau_g S_g}{\alpha_g A} - \frac{\tau_l S_l}{\alpha_l A} . \tag{12}$$

One can see that the stability depends on the reliability of the closure relations for interface and wall drag.

4. Summary

We showed that the velocity shape parameters extended the well-posed region in low liquid level. This means that the gas velocity required for the transition from the horizontal stratified flow to annular flow is higher than is expected by the classical Kelvin-Helmholtz instability. While the velocity shape for gas is very close to unity because of flat velocity profile, the velocity shape for liquid should be determined by experimental data. We are going to find the values of the velocity shape parameters to match the experimental data [15, 16].

REFERENCES

[1] R. Govindarajan, K.C. Sahu, Instabilities in Viscosity-Stratified Flow, *Annual Review of Fluid Mechanics*, 46 (1) (2014) 331-353.

[2] D. Barnea, On the effect of viscosity on stability of stratified gas—liquid flow—application to flow pattern transition at various pipe inclinations, *Chemical Engineering Science*, 46 (8) (1991) 2123-2131.

[3] D. Barnea, Y. Taitel, Kelvin-Helmholtz stability criteria for stratified flow: viscous versus non-viscous (inviscid) approaches, *International Journal of Multiphase Flow*, 19 (4) (1993) 639-649.

[4] O.M.H. Rodriguez, M.S. Castro, Interfacial-tension-force model for the wavy-stratified liquid–liquid flow pattern transition, *International Journal of Multiphase Flow*, 58 (2014) 114-126.

[5] Y. Salhi, E.-K. Si-Ahmed, J. Legrand, G. Degrez, Stability analysis of inclined stratified two-phase gas–liquid flow, *Nuclear Engineering and Design*, 240 (5) (2010) 1083-1096.

[6] W.D. Fullmer, V.H. Ransom, M.A. Lopez de Bertodano, Linear and nonlinear analysis of an unstable, but well-posed, one-dimensional two-fluid model for two-phase flow based on the inviscid Kelvin–Helmholtz instability, *Nuclear Engineering and Design*, 268 (Complete) (2014) 173-184.

[7] N. Brauner, D. Moalem Maron, Stability analysis of stratfied liquid-liquid flow, *International Journal of Multiphase Flow*, 18 (1) (1992) 103-121.

[8] N. Brauner, D. Moalem Maron, Flow pattern transitions in two-phase liquid-liquid flow in horizontal tubes, *International Journal of Multiphase Flow*, 18 (1) (1992) 123-140.

[9] J.H. Song, M. Ishii, On the stability of a one-dimensional two-fluid model, *Nuclear Engineering and Design*, 204 (1–3) (2001) 101-115.

[10] J. Song, M. Ishii, The one-dimensional two-fluid model with momentum flux parameters, *Nuclear Engineering and Design*, 205 (1–2) (2001) 145-158.

[11] J.H. Song, A remedy for the ill-posedness of the one-dimensional two-fluid model, *Nuclear Engineering and Design*, 222 (1) (2003) 40-53.

[12] K. Mishima, M. Ishii, Theoretical Prediction of Onset of Horizontal Slug Flow, *Journal of Fluids Engineering*, 102 (4) (1980) 441-445.

[13] G.B. Wallis, J.E. Dodson, The onset of slugging in horizontal stratified air-water flow, *International Journal of Multiphase Flow*, 1 (1) (1973) 173-193.

[14] Y. Taitel, A.E. Dukler, A model for predicting flow regime transitions in horizontal and near horizontal gas-liquid flow, *AIChE Journal*, 22 (1) (1976) 47-55.

[15] I. Mantilla, Mechanistic modeling of liquid entrainment in gas in horizontal pipes, in, University of Tulsa, 2008.

[16] M. Valette, F. Henry, Droplet entrainment over a stratified flow in a PWR Hot Leg : Results of the REGARD experiment, in, CEA, France, 2015.