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Development of Mathematical Model and Analysis Code for Estimating Drop Behavior of the Control Rod Assembly in the Sodium Cooled Fast Reactor

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1. Introduction

In a sodium-cooled fast reactor(SFR), the control rod assembly is used to control the reactor power. If any of the operating limits are exceeded, the control rod assemblies are inserted into the core within a stipulated time to shut down the reactor power as soon as possible. On receiving the scram signal, the control rod assemblies are released to fall into the reactor core by its weight. Thus drop time and falling velocity of the control rod assembly must be estimated for the safety evaluation. There are three typical ways to estimate the drop behavior of the control rod assembly in scram action: Experimental, numerical and theoretical methods. But experimental and numerical(CFD) method require a lot of cost and time. Thus, these methods are difficult to apply to the initial design process. In this study, mathematical model and theoretical analysis code have been developed in order to estimate drop behavior of the control rod assembly to provide the underlying data for the design optimization.

2. Features of Control Rod Assembly

The control assembly consists of two main parts: control rod assembly and its guide duct. The guide duct is composed of handling socket, nose piece, hexagonal duct and damper. Hexagonal duct serves to restrict the falling path of the control rod assembly. And the damper is located at the bottom of the hexagonal duct to reduce the falling velocity of the control rod assembly. The control rod assembly consists of control rods, lower and upper adapter, mounting rail, clamping head, piston head.

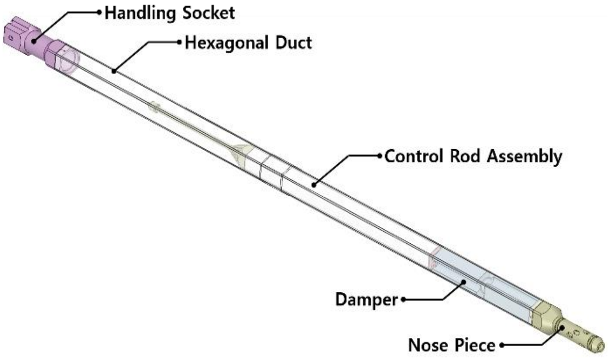
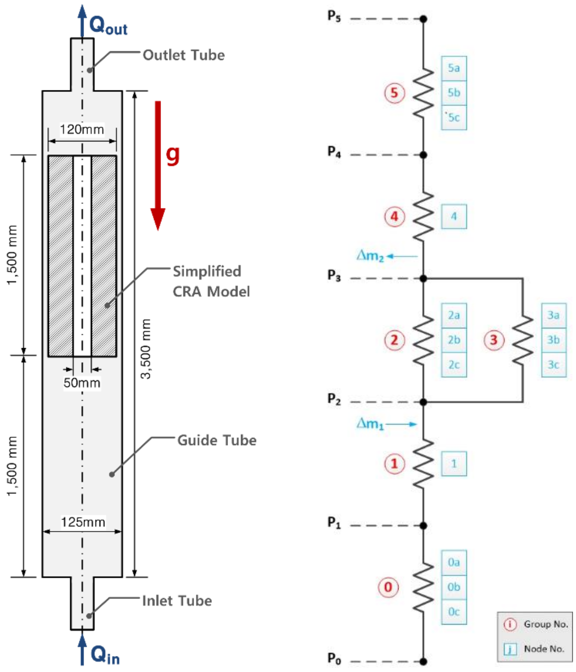


Fig.1 Configuration of control rod assembly and guide duct

3. Simplified Model and Hydraulic Circuit

In order to minimize the uncertainty in the development process, a simplified control rod assembly(CRA) model that can represent the flow path characteristics of the actual CRA is considered. Fig.2 shows schematic diagram of the simplified CRA model and the hydraulic circuit of flow paths. Part of the coolant entering the guide tube passes through the CRA model and the remaining part flows through the annular path between the CRA model and the side wall of the guide tube. By applying the hydraulic circuit analysis techniques, local flow paths in the control assembly can be considered as the network of piping elements. The hydraulic circuit of the CRA model is composed of the following piping elements: circular tube, circular annulus, sudden expansion, sudden contraction and entrance. By performing the hydraulic circuit analysis, it can be evaluate the internal/external flow distribution of the control rod assembly. External forces acting on the CRA model can be calculated with these flow distribution information.



(a) Simplified C.R.A. Model (b) Hydraulic Circuit

Fig.2 Schematic of the simplified control rod assembly model and its hydraulic circuit diagram

4. Mathematical Modelling

4.1 Equations of Motion

The motion of the CRA model is governed by the following force balance equation (Eq.1).

$$m\frac{dV}{dt} = mg - \left[\rho_f V g + F_{D,friction} + F_{D,pressure}\right] \qquad (1)$$

the drop velocity of the CRA model is determined by the four external force terms(gravity, buoyancy, frictional drag, pressure drag). Each external force terms are calculated with local flow characteristics. Therefore, flow distribution in the internal/external flow paths of the CRA model should be estimated.

4.2 Governing Equations

The flowing governing equations are formulated based on mass balance and pressure balance in the hydraulic circuit shown in Fig.2(b).

$$F_1 = m_1 + \Delta m_1 - m_2 - m_3 = 0 \qquad (2)$$

$$F_2 = \Delta P_2 - \Delta P_3 = 0 \qquad (3)$$

Eq.2 represents the mass balance of the parallel flow paths. Where m_i is mass flowrate of each flow paths, and Δm_i is the equivalent flow rate due to drop of the CRA model. Eq.3 is the pressure balance equation which means that the pressure drops of the parallel pipes are same, regardless of the flow path.

4.3 Pressure Distribution in the flow paths

Eq.4 and Eq.5 represent the Navier-Stokes equation and its integration(over the length of the flow path) form. Eq.5 has been re-arranged to get the total pressure drop of the flow path.

$$\partial/\partial t(\rho U) + \partial/\partial t(\rho U^2) = \rho g - \partial P/\partial x - F_{vis} \qquad (4)$$

$$\Delta P_i = \Delta P_f + \Delta P_h + \rho(\partial U/\partial t)L + 0.5\rho(U_o^2 - U_i^2) \qquad (5)$$

Total pressure drop in flow path is evaluated as the sum of following terms: frictional pressure drop, pressure difference due to elevation, pressure drop due to temporal acceleration and pressure drop due to spatial acceleration. Since the buoyancy term is included in Eq.1, hydrostatic pressure term(ΔP_h) can be negligible.

In order to calculate the simultaneous equations consisting of Eq.2 and Eq.3, Eq.3 should be converted as a function of flowrate. During scram action of CRA, certain flow paths are also in motion. Therefore, flow velocity in the certain flow paths is to be considered as the relative velocity of the drop velocity of the CRA model. Hence, for the path No. 2, frictional pressure drop can be expressed as Eq.6.

Meanwhile, for the flow path No.3 which has stationary outer sheath and moving inner sheath, frictional pressure drop can be expressed as Eq.7. And the frictional pressure drop in the stationary flow path, such as the path No. 0, 1, 4, 5 can be expressed as Eq.8. Where K_L is a resistance coefficient, V is drop velocity of the CRA model, m_i is mass flowrate and A_i is cross-sectional area of flow paths.

$$\Delta P_f = \frac{K_L}{2\rho A_i^2} m_i^2 + \frac{K_L V}{A_i} m_i + \frac{K_L \rho V^2}{2} \qquad (6)$$

$$\Delta P_f = \frac{K_L}{2\rho A_i^2} m_i^2 + \frac{K_L V}{2A_i} m_i + \frac{K_L \rho V^2}{8} \qquad (7)$$

$$\Delta P_f = \frac{K_L}{2\rho A_i^2} m_i^2 \qquad (8)$$

As above, temporal acceleration term and spatial acceleration term in the pressure drop equation(Eq.5) can be defined in terms of the mass flowrates.

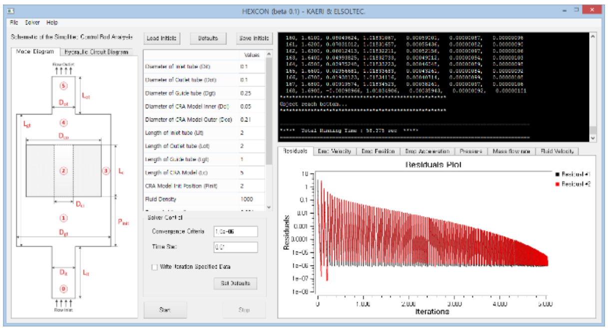
4.4 Numerical Processing

Hence the non-linear simultaneous equations(Eq.2 and Eq.3) are in terms of two unknown variable(m_2, m_3). These equations are solved using iterative method (Newton-Raphson method). Eq.9 represents the relation between mass flowrate of *k*th iteration and (*k+1*)th iteration.

$$\begin{bmatrix} m_2^{k+1} \\ m_3^{k+1} \end{bmatrix} = \begin{bmatrix} m_2^{k} \\ m_3^{k} \end{bmatrix} + \begin{bmatrix} \dfrac{\partial F_1}{\partial m_2} & \dfrac{\partial F_1}{\partial m_3} \\ \dfrac{\partial F_2}{\partial m_2} & \dfrac{\partial F_2}{\partial m_3} \end{bmatrix}^{-1} \cdot \begin{bmatrix} F_1(m_2^k, m_3^k) \\ F_2(m_2^k, m_3^k) \end{bmatrix} \qquad (9)$$

5. Development of the Analysis Code

The theoretical analysis code(named as HEXCON) has been developed based on the above mathematical model to estimate drop behavior of the control rod assembly. HEXCON was developed based on the C++ language. In this analysis code, transient analysis for the equation of motion and steady state iterative calculation for fluid flow equations are performed alternately. Fig.3 shows the user interface layout of the HEXCON, and Fig.4 represents the calculation process of the developed analysis code.



	Values
Diameter of Inlet tube (Dit)	0.1
Diameter of Outlet tube (Dot)	0.1
Diameter of Guide tube (Dgt)	0.25
Diameter of CRA Model Inner (Dci)	0.05
Diameter of CRA Model Outer (Dco)	0.21
Length of Inlet tube (Lit)	2
Length of Outlet tube (Lot)	2
Length of Guide tube (Lgt)	1
Length of CRA Model (Lc)	5
CRA Model Init Position (Pinit)	2
Fluid Density	1000

Solver Control
Convergence Criteria: 1.0e-06
Time Step: 0.01
☐ Write Iteration Specified Data

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Object reach bottom...
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Fig.3 User interface layout of the analysis code(HEXCON)

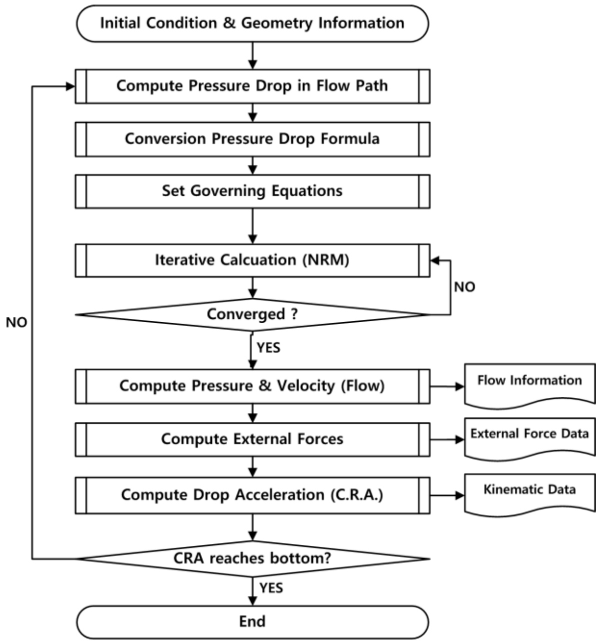

Fig.4 Calculation process of the analysis code(HEXCON)

6. Validation of Developed Analysis Code

To verify the reliability of the developed code, CFD analysis has been conducted on the drop behavior of the simplified control rod assembly model shown in Fig.2(a). In the CFD analysis, 3D unsteady calculation was performed. To calculation the flow field, continuity equation and momentum equation have been used. And standard K-ε model is adopted to consider turbulence effect. About 500,000 grids are used in the entire calculation domain. Mesh deformation method and sliding mesh method are used to deal with the mesh deformation during the scram action of the control rod assembly model. In this calculation, the coolant regarded as water(Table 1). Weight of the control rod assembly model is 49.0 kg. Entering velocity of the coolant is 0.5 m/s. Calculation using the developed code(HEXCON) was carried out under the same condition, and both results were compared.

Fig.5 shows variation of the drop position during scram action. In the beginning of the drop, results of the analysis using the two methods appears to be almost same. After 0.2 s, it starts to grow the difference between two results gradually. In the theoretical analysis using developed code, drop time is estimated to be 1.68 s. The drop time of the CFD analysis is estimated to be 1.57 s. The relative error in the theoretical analysis for the CFD analysis is about 7.0 %.

Graph of the drop velocity is shown in Fig.6. In the theoretical analysis using developed code, terminal velocity is estimated to be 1.07 m/s. In the CFD analysis, terminal velocity of the CRA model is estimated to be 1.10 m/s. Calculation result of the theoretical analysis is evaluated 2.7 % lower than the calculation result of the CFD analysis.

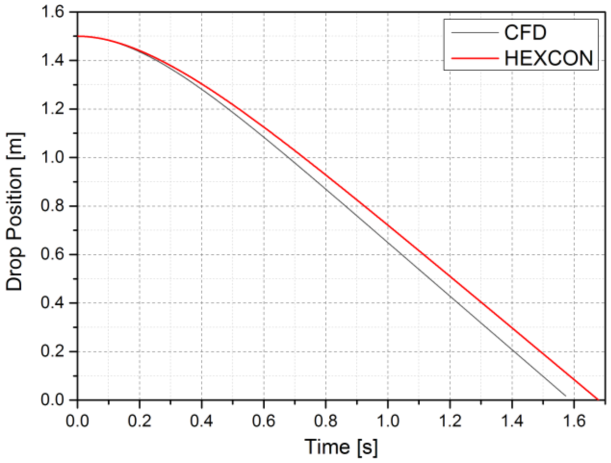

Fig.5 Drop position of the control rod assembly model

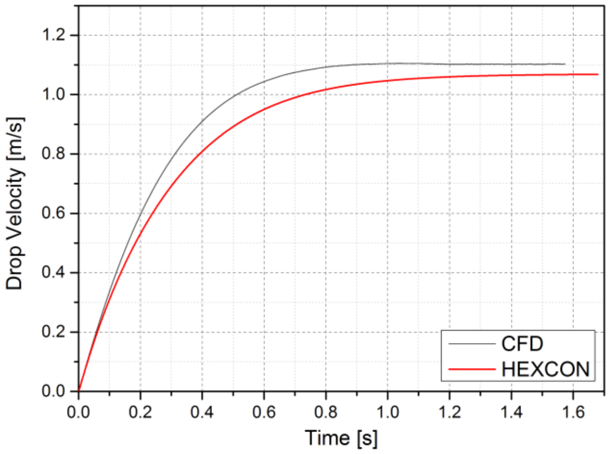

Fig.6 Drop velocity of the control rod assembly model

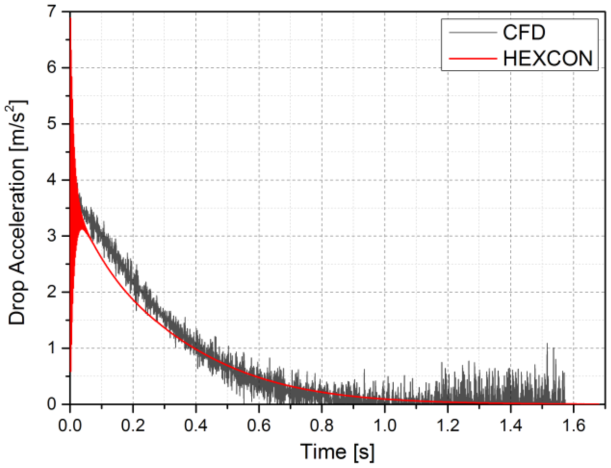

Fig.7 Drop acceleration of the control rod assembly model

Table 1. Material properties of the coolant

	Water (25℃)
Density	997.0 kg/m^3
Viscosity	8.899 × 10^{-4} Pa·s

Fig.7 shows the graph of drop acceleration of the CRA model. Before 0.4 s, theoretical analysis result is lower than CFD result. Thereafter, result of theoretical analysis is relatively slightly higher than CFD result.

7. Conclusion

Mathematical model and theoretical analysis code have been developed in order to estimate drop behavior of the control rod assembly to provide the underlying data for the design optimization.

A simplified control rod assembly model is considered to minimize the uncertainty in the development process. And the hydraulic circuit analysis technique is adopted to evaluate the internal/external flow distribution of the control rod assembly. Finally, the theoretical analysis code(named as HEXCON) has been developed based on the mathematical model.

To verify the reliability of the developed code, CFD analysis has been conducted. And a calculation using the developed analysis code was carried out under the same condition, and both results were compared. It is figured out that the evaluation results of the CRA drop behavior using developed code is similar to the CFD analysis results. However, the developed code estimates CRA drop time conservatively compared to the CFD analysis.

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