Uncertainty Analysis with Considering Resonance Self-shielding Effect

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1. Introduction

In core parameter uncertainty analysis induced by nuclear data uncertainty, a sensitivity of a response should be consistently combined with a covariance data. If infinitely diluted multi-group cross sections were used for the sensitivity, the covariance data from the evaluated nuclear data library (ENDL) was directly applied. However, in case of using a self-shielded multi-group cross section, the covariance data should be corrected considering self-shielding effect. Usually, implicit uncertainty can be defined as the uncertainty change by the resonance self-shielding effect as described above.

MUSAD (Modules of Uncertainty and Sensitivity Analysis for DeCART) [1] has been developed for a multiplication factor and cross section uncertainty based on the generalized perturbation theory and it, however, can only quantify the explicit uncertainty by the self-shielded multi-group cross sections without considering the implicit effect. Thus, this paper addresses the implementation of the implicit uncertainty analysis module into the code and the numerical results for the verification are provided.

2. Methodology

MUSAD directly uses the self-shielded multi-group cross sections for DeCART. Simultaneously, it applies the covariance matrix from ENDL using NJOY. In this procedure, however, there is the inconsistency of the product of the sensitivity based on the self-shielded cross section and the covariance matrix for the unshielded cross section. For resolving the inconsistency, Takeda proposed the method applying the self-shielding factor [2] for the implicit uncertainty and Chiba presented infinitely-diluted cross section-based consistent methodology [3]. In this paper, Chiba's method was basically applied for the implicit uncertainty and some formulations were rewritten for MUSAD/DeCART code system.

2.1. Infinitely-Diluted Cross Section-based Consistent Methodology

If an infinitely diluted cross section, σ, was used for calculating the sensitivity, the sensitivity coefficient for a response, R, can be rewritten as:

$$S = \frac{dR}{d\sigma}\frac{\sigma}{R} = \left(\frac{dR}{d\tilde{\sigma}}\frac{\tilde{\sigma}}{R}\right)\left(\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}}\right) = \tilde{S}\left(\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}}\right), \quad (1)$$

where $\tilde{\sigma}$ is an effective cross section and $\tilde{S}$ is the explicit sensitivity coefficient which can be easily obtained with the conventional way from self-shielded multi-group cross section.

Takeda and Chiba proposed the method applying the self-shielding factor as follows:

$$f = \tilde{\sigma}/\sigma \quad (2)$$

Applying the derivative with respect to σ into Eq.(2), it can be expressed as:

$$\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}} = 1 + \frac{df}{d\sigma}\frac{\sigma}{f} \quad (3)$$

Therefore, the sensitivity, Eq.(1), can be transformed as follows:

$$S = \tilde{S}\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}} = \tilde{S}\left(1 + \frac{df}{d\sigma}\frac{\sigma}{f}\right) \quad (4)$$

Also, Eq.(4) can be rewritten with respect to the background cross section, σ_b, as follows:

$$S = \tilde{S}\left(1 + \frac{df}{d\sigma}\frac{\sigma}{f}\right) = \tilde{S}\left(1 + \frac{df}{d\sigma_b}\frac{d\sigma_b}{d\sigma}\frac{\sigma}{f}\right) \quad (5)$$

For obtaining the relation between the background cross section and the infinitely diluted cross section, an approximation is introduced as follows:

$$\sigma(E) \approx \sigma \cdot g(E) \quad (6)$$

It means that a cross section consists of the infinitely diluted cross section and the energy distribution function.

Thus, the effective cross section can be defined using the neutron flux, $\phi(E) = \dfrac{\sigma_b}{\sigma_a(E) + \sigma_b}$, as:

$$\tilde{\sigma} = \frac{\left\langle \dfrac{g(E)}{\sigma_a \cdot g(E) + \sigma_b} \right\rangle}{\left\langle \dfrac{1}{\sigma_a \cdot g(E) + \sigma_b} \right\rangle}\sigma = f\sigma \quad (7)$$

where σ_b is $\lambda\Sigma_p / N_r$ and N_r is the number density of resonant nuclide.

The perturbed effective cross section by the perturbation of the infinitely diluted cross section, $\Delta\sigma$, can be expressed as:

$$\tilde{\sigma}' = \frac{\left\langle \dfrac{g(E)}{(\sigma_a + \Delta\sigma)\cdot g(E) + \sigma_b} \right\rangle}{\left\langle \dfrac{1}{(\sigma_a + \Delta\sigma)\cdot g(E) + \sigma_b} \right\rangle} \sigma' = f(\sigma_b')\sigma' \quad (8)$$

In Eq.(8), if the self-shielding factor was perturbed with respect to the background cross section, the perturbed background cross section can be written as:

$$\sigma_b + \Delta\sigma_b = \frac{\sigma_b \sigma_a}{\sigma_a + \Delta\sigma} \quad (9)$$

Therefore, the sensitivity, Eq.(1), can be transformed using Eq.(9) as follows:

$$S = \tilde{S}\left(1 + \frac{df}{d\sigma_b}\frac{d\sigma_b}{d\sigma}\frac{\sigma}{f}\right) \approx \tilde{S}\left(1 - \frac{\Delta f}{\Delta\sigma_b}\frac{\sigma_b}{\sigma_a}\frac{\sigma}{f}\right) \quad (10)$$

3. Numerical Results

The sensitivity calculation module of MUSAD was modified to consider the implicit uncertainty using the above method and the verification calculation was performed on MHTGR 350 Ex.I-1a proposed by IAEA CRP HTGR UAM [4]. It is a homogeneous fuel compact pin cell problem. Also, MUSAD code uses the cross sections originated from the ENDF/B-VII.0 and the covariance matrix processed from the ENDF/B-VII.1. The reference results were made from McCARD [5] based on Monte Carlo method. It can product the complete uncertainty including the implicit uncertainty, because it uses the unshielded continuous energy group cross section directly from the nuclear data.

Table 1 shows the comparisons between the reference and the uncertainty by MUSAD including the implicit uncertainty on Ex.I-1a CZP problem. The explicit uncertainty by U238 abs-abs of MUSAD was overestimated to about 40% when comparing with McCARD result. But, in case of the complete uncertainty considering the implicit effect, the difference with the reference is decreases from 0.141 to 0.001. Also, the similar trend shows in Table 2 for Ex.I-1a HFP problem. The total uncertainty of keff from MUSAD is very similar to the reference when considering implicit uncertainty.

Table 1. Ex.I-1a CZP keff Uncertainty (%*dk/k*)

	McCARD	MUSAD	MUSAD
	Complete	Explict	Complete
^{235}U ν-ν	0.615	0.615	0.615
^{235}U abs-abs	0.269	0.280	0.280
^{235}U fis-fis	0.066	0.067	0.067
^{238}U abs-abs	**0.354**	**0.495**	**0.355**
^{238}U fis-fis	0.002	0.002	0.002
Total	**0.761**	**0.831**	**0.770**

Table 2. Ex.I-1a HFP keff Uncertainty (%*dk/k*)

	McCARD	MUSAD	MUSAD
	Complete	Explict	Complete
^{235}U ν-ν	0.609	0.610	0.610
^{235}U abs-abs	0.267	0.275	0.275
^{235}U fis-fis	0.073	0.073	0.073
^{238}U abs-abs	**0.438**	**0.618**	**0.426**
^{238}U fis-fis	0.003	0.002	0.002
Total	**0.799**	**0.905**	**0.800**

4. Conclusion

The implicit uncertainty analysis module has been implemented into MUSAD based on infinitely-diluted cross section-based consistent method. The verification calculation was performed on MHTGR 350 Ex.I-1a and the differences with McCARD result decrease from 40% to 1% in CZP case and 3% in HFP case.

From this study, it is expected that MUSAD code can reasonably produce the complete uncertainty on VHTR or LWR where the resonance self-shielding effect should be significantly considered.

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