KNS Fall Meeting

Linear Analysis of X-ray Imaging

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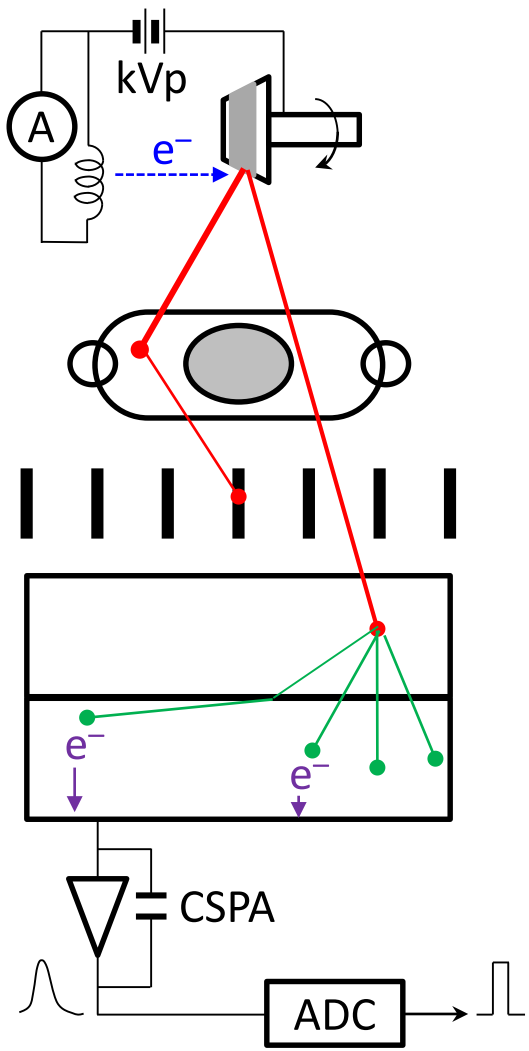
Linear analysis on:

- Image quality
  - Fourier-based image quality
- Detector performance
  - Cascaded linear-systems theory on flat-panel detectors
- System performance
  - Dual-energy CBCT

Imaging chain



- **X-ray tube**
  - Spectrum (polyenergetic)
  - Focal spot size (image blur)
  - Heel effect (nonuniformity)
- **Patient**
  - Scatter (the most harmful)
- **Anti-scatter grid**
  - Transmittance (or selectivity)
- **Detector**
  - Converter
    - Glare
  - Photodiode
  - Amplifier
  - ADC

Image quality

- Contrast
  - Signal difference = $\Delta d = d_b - d_s$
  - Contrast = $\frac{\Delta d}{d_b}$
- Noise
  - *Standard deviation* of signal
  - $\text{CNR} = \frac{\Delta d}{\sqrt{\frac{\sigma_b^2 + \sigma_s^2}{2}}}$
- Spatial resolution
  - FWHM
- Artifact
- Scatter

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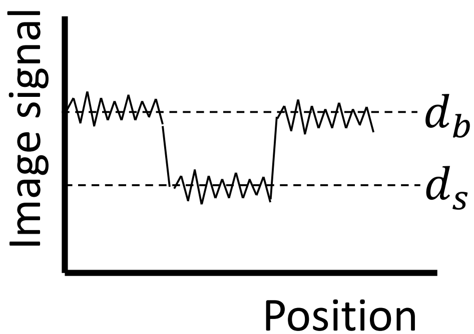

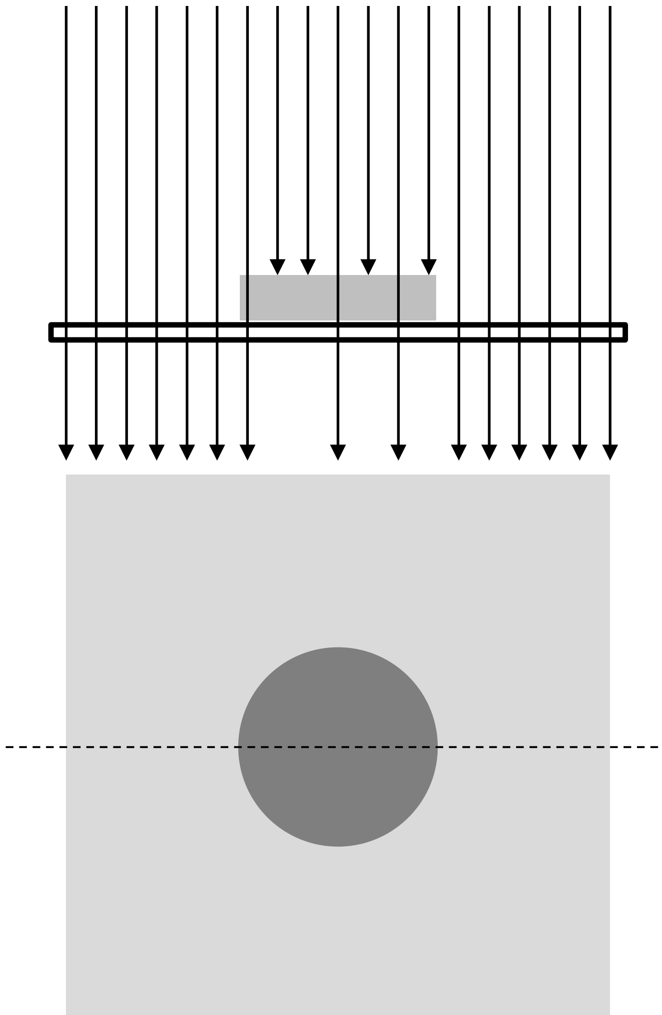

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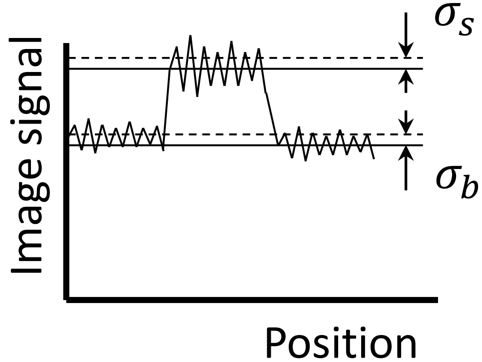

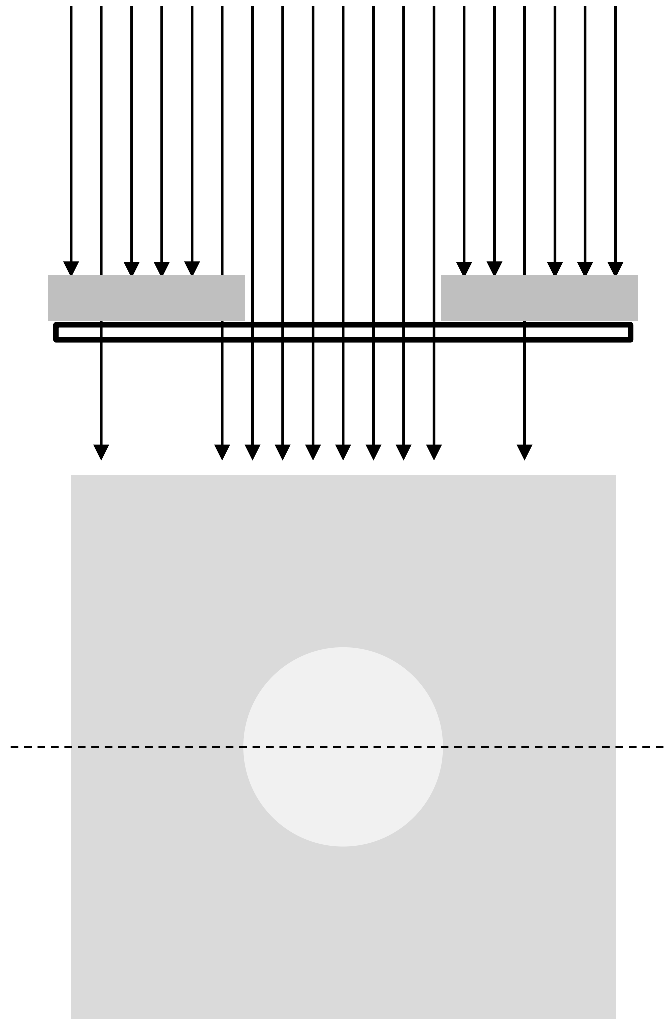

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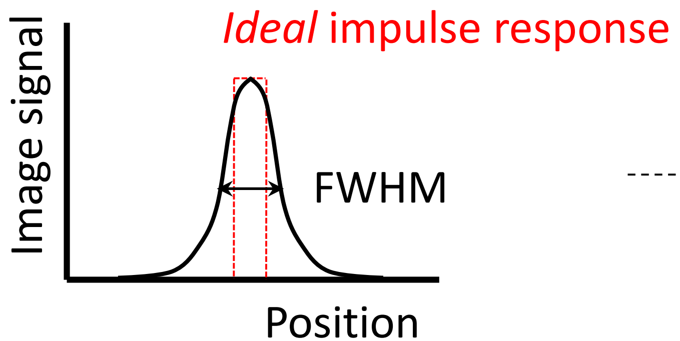

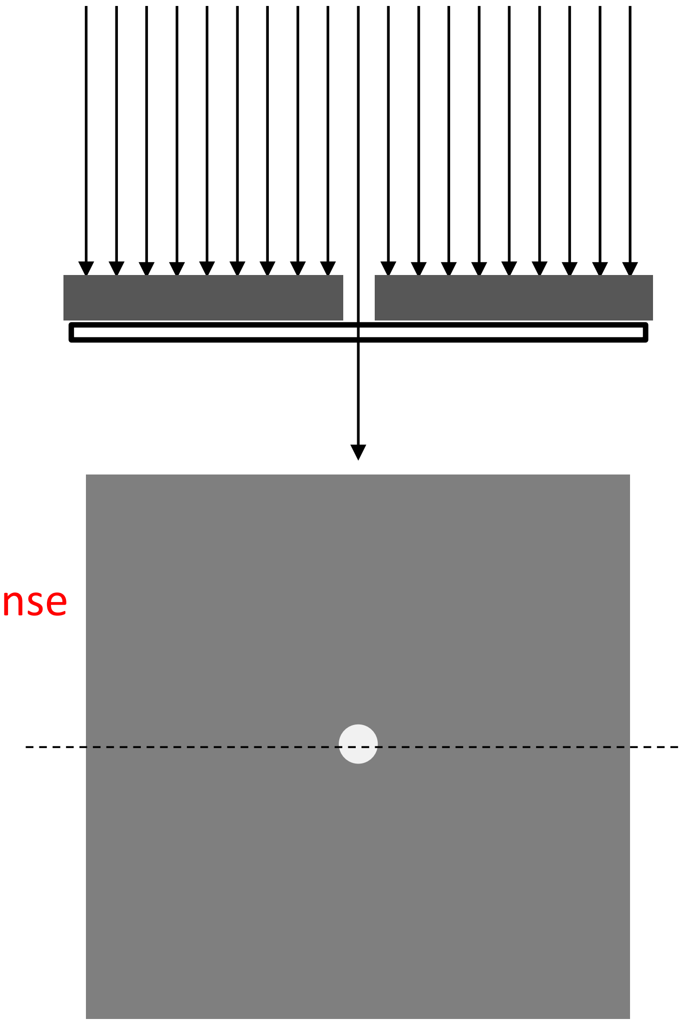

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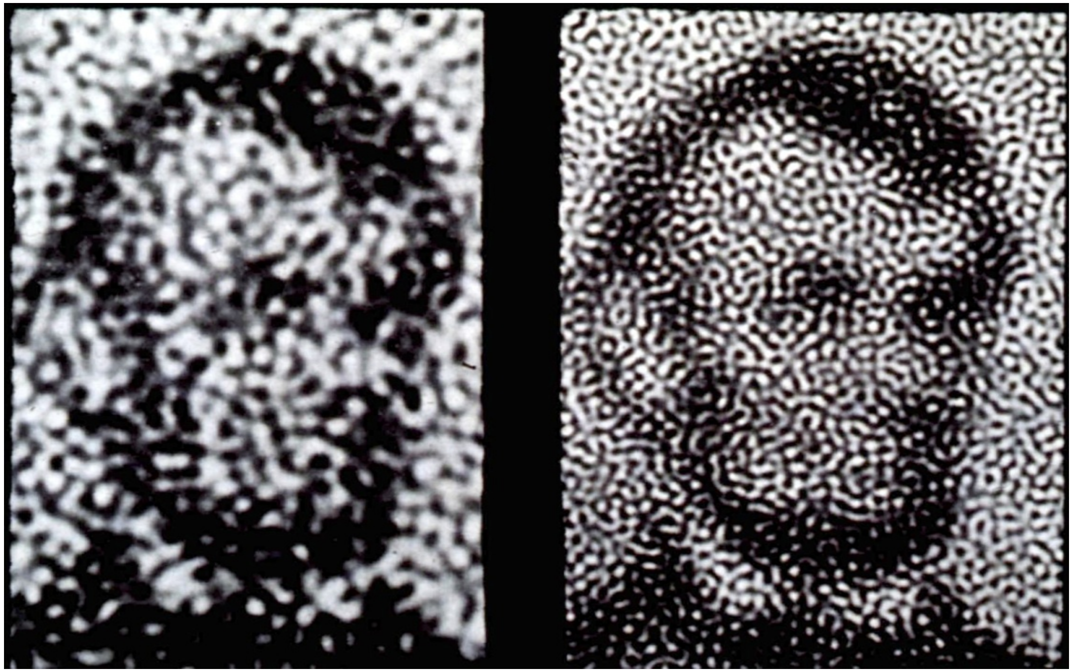
Spatial correlation

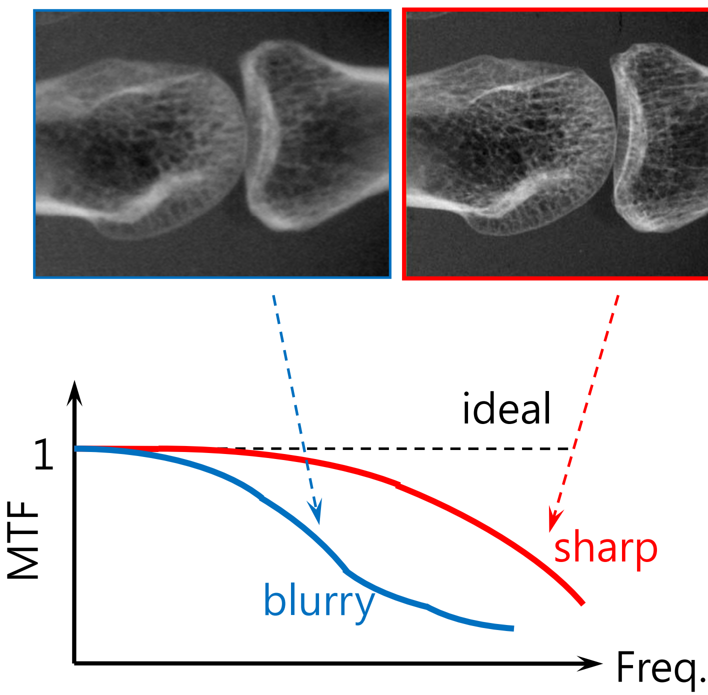


- These two images have the same *pixel variance*, but different *correlation structure* (different textures!)
- Simple image pixel variance ignores second-moment statistics (correlation between pixels)

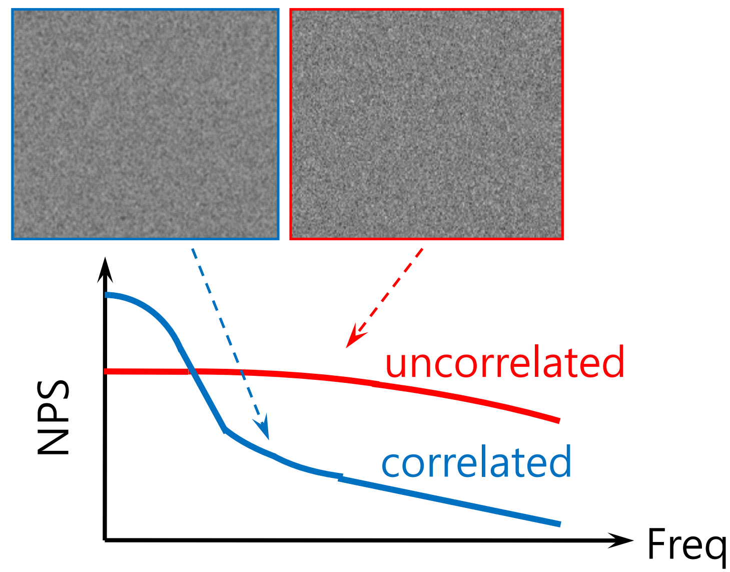
(Images taken from) R. F. Wagner | AAPM | 2004

Fourier-based metrics

- Particle-based metrics
  - contrast transfer
  - noise variance



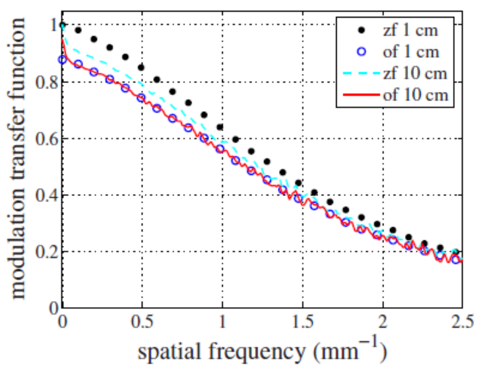
- Fourier-based metrics
  - modulation-transfer function
  - Wiener noise-power spectrum



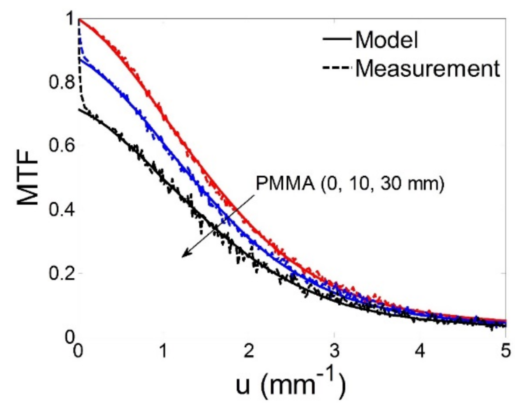
MTF

$$\mathrm{MTF}(\mathbf{u}) = \mathcal{F}\left\{\frac{\mathrm{psf}(\mathbf{x})}{\int_{-\infty}^{\infty}\mathrm{psf}(\mathbf{x})\mathrm{d}\mathbf{x}}\right\} = \frac{|T(\mathbf{u})|}{|T(\mathbf{0})|}$$

- The conventional zero-frequency normalization may result in inflated MTF values [1)]
  - Consider the analyzing ROI size enough to take the optical glare into



- Scatter x-ray photons also reduce the zero-frequency MTF values [2)]



[1)] S. N. Friedman and I. A. Cunningham | Med. Phys. | 2008
[2)] J. Park et al. | SPIE | 2016

NPS

$$\mathrm{NPS}(\mathbf{u}) = \frac{1}{\Delta\mathbf{u}} \langle |\mathcal{F}\{\Delta d(\mathbf{x})\}|^2 \rangle$$

- How to determine $\mathrm{NPS}(\mathbf{0})$; hence $\mathrm{DQE}(\mathbf{0})$?

$$\mathrm{NPS}(\mathbf{0}) = A_{eff}\sigma_d^2$$

$$\mathrm{DQE}(\mathbf{0}) = \frac{\bar{d}^2}{\bar{q}_0 A_{eff}\sigma_d^2}$$

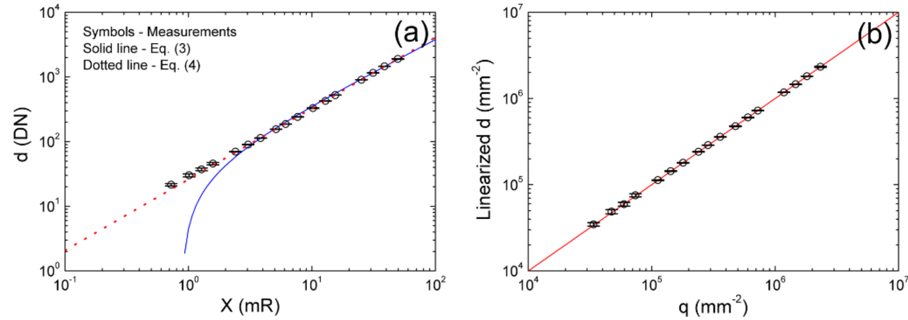
Manuscript in preparation

Requirements

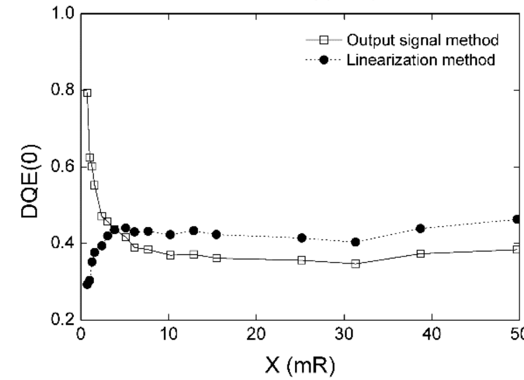
1. Linearity
   - image intensity scales with x-ray input

2. Shift invariance
   - impulse-response function is same over image

3. Stationary noise



- If either fails:
  - linearization
  - small-signal approach
  - regional analysis



J. C. Han *et al.* | JKPS | 2014

Forward model

$$d(\xi; s) = Xka^2(1 + \mathrm{SPR}) \int_0^{\infty} q_0(E) e^{-\int_s \mu(\mathbf{x},E)\mathrm{d}s} R(E)\, \mathrm{d}E$$

where

$$R(E) = \int_0^L e^{-\mu_d(E)z} \mu_E(E) g(E)\, \mathrm{d}z = \left(1 - e^{-\mu_d(E)L}\right) \frac{\mu_E(E)}{\mu_d(E)} g(E)$$

- $g(E)$
  - linear approximation by using the cascaded linear-systems theory

Signal & noise transfer

$$\mathrm{SNR}_{in} = \frac{\bar{q}_{in}}{\sigma_{in}} = \frac{\bar{q}_{in}}{\sqrt{\bar{q}_{in}}} = \sqrt{\bar{q}_{in}}$$

$$\mathrm{SNR}_{out} = \frac{\bar{q}_{out}}{\sigma_{out}} = ?$$

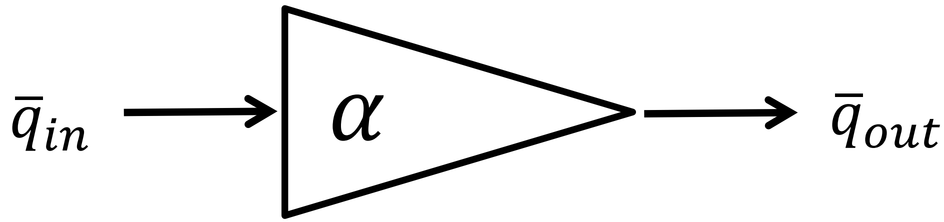
depending on the statistical model

- Detective quantum efficiency
  - working even for different units between *in* and *out*

$$\mathrm{DQE} = \frac{\mathrm{SNR}_{out}^2}{\mathrm{SNR}_{in}^2}$$ (called *conceptual* or SNR^2–transfer form)

Binomial quantum-detection detector

(*detection* or *not*)



$$\bar{q}_{out} = \alpha \bar{q}_{in}$$

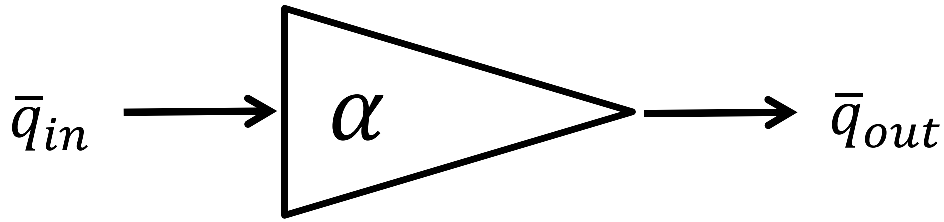
$$\sigma_{out}^2 = \alpha^2 \sigma_{in}^2 = \alpha^2 \bar{q}_{in}$$

$$\mathrm{SNR}_{out} = \frac{\bar{q}_{out}}{\sigma_{out}} = \frac{\alpha \bar{q}_{in}}{\alpha \sqrt{\bar{q}_{in}}} = \sqrt{\bar{q}_{in}}$$

$$\mathrm{DQE} = \frac{\mathrm{SNR}_{out}^2}{\mathrm{SNR}_{in}^2} = 1$$

Binomial quantum-detection detector

(*detection* or *not*)



$$\bar{q}_{out} = \alpha \bar{q}_{in}$$

$$\sigma_{out}^2 = \alpha^2 \sigma_{in}^2 + \bar{q}_{in} \sigma_g^2 \,{}^{1)} = \alpha^2 \bar{q}_{in} + \bar{q}_{in} \alpha (1 - \alpha) = \alpha \bar{q}_{in}$$

$$\mathrm{SNR}_{out} = \frac{\bar{q}_{out}}{\sigma_{out}} = \frac{\alpha \bar{q}_{in}}{\sqrt{\alpha \bar{q}_{in}}} = \sqrt{\alpha \bar{q}_{in}}$$

$$\mathrm{DQE} = \frac{\mathrm{SNR}_{out}^2}{\mathrm{SNR}_{in}^2} = \alpha$$

[1)] M. Rabbani, R. Sahw, and R. L. Van Metter | JOSA | 1987

Various forms of DQE

- Descriptive
  - In terms of parameters determined from measured images

$$\mathrm{DQE}(u) = \frac{\bar{q}^2|G\mathrm{MTF}(u)|^2/\mathrm{NPS}(u)}{\bar{q}} = \frac{\bar{q}G^2\mathrm{MTF}(u)^2}{\mathrm{NPS}(u)} = \frac{\mathrm{MTF}(u)^2}{\bar{q}[\mathrm{NPS}(u)/d^2]}$$

- Stochastic (most general)
  - NPS by a *deterministic* syst. relative to an actual *stochastic* syst.

$$\mathrm{DQE}(u) = \frac{\mathrm{NPS}_{ideal}(u)}{\mathrm{NPS}_{actual}(u)} = \frac{\mathrm{NPS}_{in}(u)|G\mathrm{MTF}(u)|^2}{\mathrm{NPS}(u)} = \frac{\bar{q}G^2\mathrm{MTF}(u)^2}{\mathrm{NPS}(u)}$$

- Predictive
  - In terms of known design parameters

$$\mathrm{DQE}(u) = \frac{1}{1+\sum_{j=1}^{M}\frac{1+\varepsilon_{g_j}\mathrm{MTF}_j^2(u)}{\prod_{i=1}^{j}\bar{g}_i\mathrm{MTF}_i^2(u)}}$$

I. A. Cunningham | AAPM Summer School | 1997

Flat-panel detector

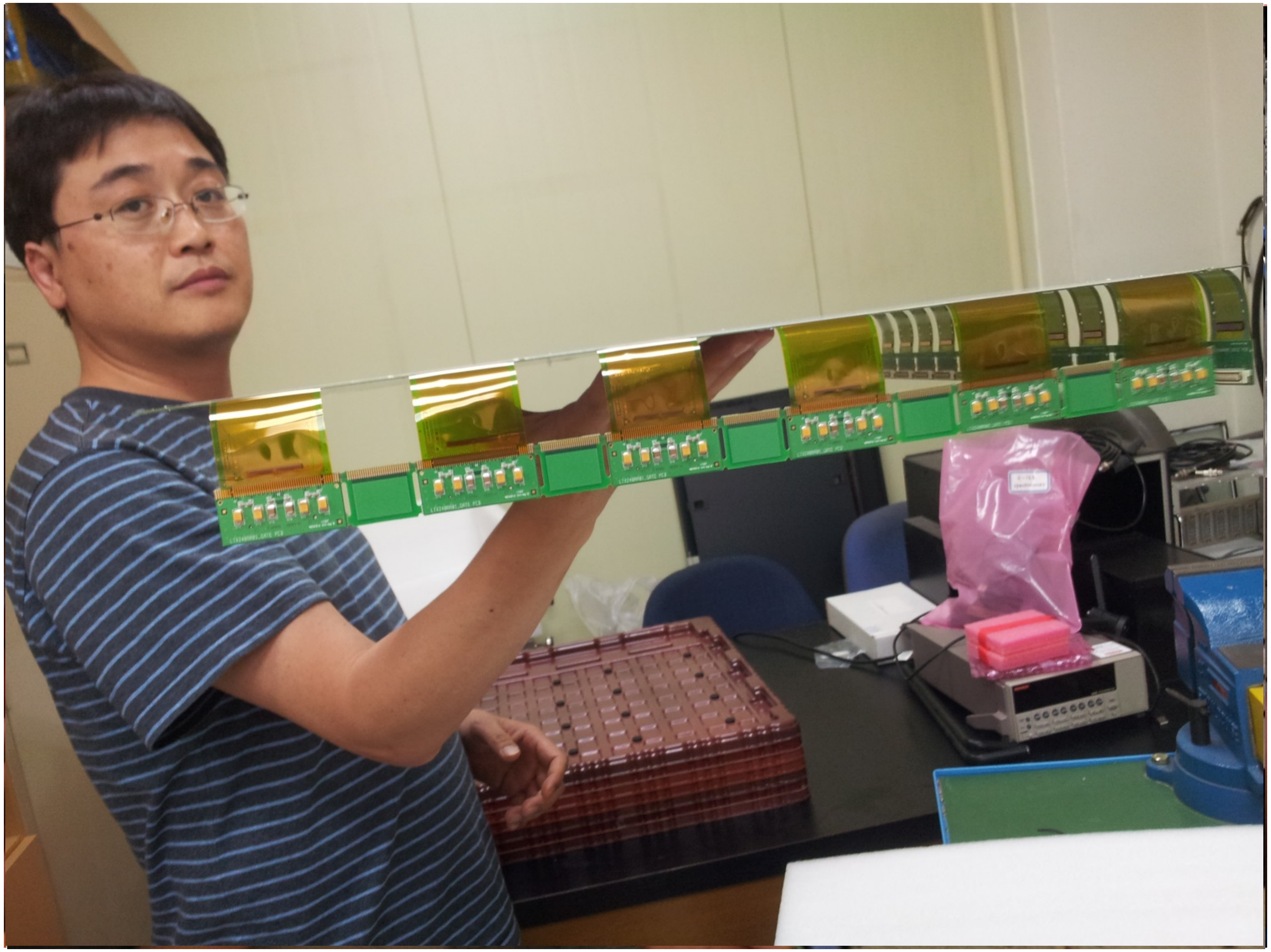
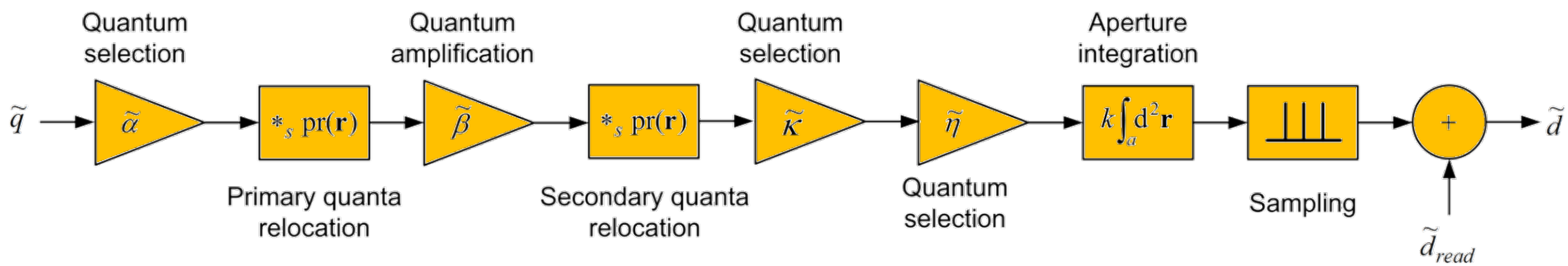
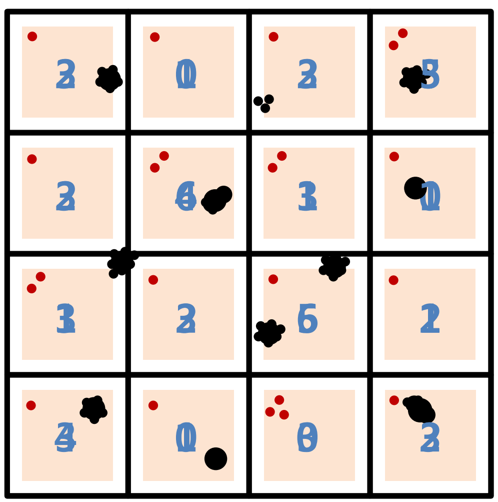


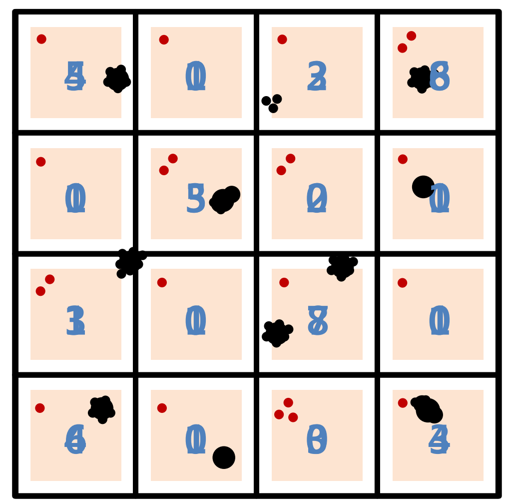
Image Courtesy of Samsung Electronics, Co., Ltd. & PNU

Cascaded linear-systems model





Long-range scattering



Short-range scattering

I. A. Cunningham | Ch. 2. *Applied linear-system theory*
Handbook of Medical Imaging | SPIE | 2000

Predictive DQE based on CSA

$$\mathrm{DQE}(\mathbf{k}) = \frac{T^2(\mathbf{k})\mathrm{sinc}^2(a\mathbf{k})}{\frac{1}{\alpha\beta\kappa\eta}\left[\frac{1}{\gamma} + \kappa\eta\left(\frac{\beta}{I}-1\right)\sum_{j=0}^{\infty}\left\{T^2\left(\mathbf{k} \pm \frac{j}{p}\right)\mathrm{sinc}^2\left(a\left(\mathbf{k} \pm \frac{j}{p}\right)\right)\right\}\right] + \frac{\sigma_{add}^2}{\gamma\bar{q}a^2(\alpha\beta\kappa\eta)^2}}$$

- Dose-independent if only if the additive noise can be ignored
- Additive noise is harmful to DQE at high frequencies where the number of secondary quanta lessens

Implications

$$\frac{\sigma_{add}^2}{\gamma \bar{q} a^2 (\alpha\beta\kappa\eta)^2} \rightarrow 0$$

- σ_{add} ↓
  - new metal line process
- γ ↑
  - limited by the TFT design rule
  - critical to high-resolution FPD (e.g. *a*-Se)
    - Electrostatic lens design
- $\bar{q} a^2$ ↑
  - wrong approach (∵ patient dose ↑)
- α ↑
  - high Z converters
  - thick converters $\Rightarrow \mathrm{MTF}(u)$ ↓
- β ↑
  - converters having a lower W-value
    - e.g. CdZnTe, HgI_2 $> 10 \times$ *a*-Se
- $\kappa\eta$ ↑
  - block small leakages (optical and charge leakages)
  - optical mismatch, poor charge-collection efficiency …

H. K. Kim *et al.* | Int. J. Precs. Eng. Manuf. | 2008

Forward model, again

$$d(\xi; s) = Xka^2 \int_0^\infty q_0(E) e^{-\int_s \mu(\mathbf{x},E)\mathrm{d}s} R(E)\, \mathrm{d}E$$

- $\mu(\mathbf{x}, E)$ is implicitly carved in projection signal
  - *spatial averaging*
    - hiding lesions
    - resulting in the background noise clutter ($\sigma_{anat} \geq 10 \times \sigma_q$)
  - *energy averaging*
    - same $\hat{\mu}$ from different materials (ρ, Z)

- Consequently, projection radiography provides poor lesion conspicuity

Space discrimination

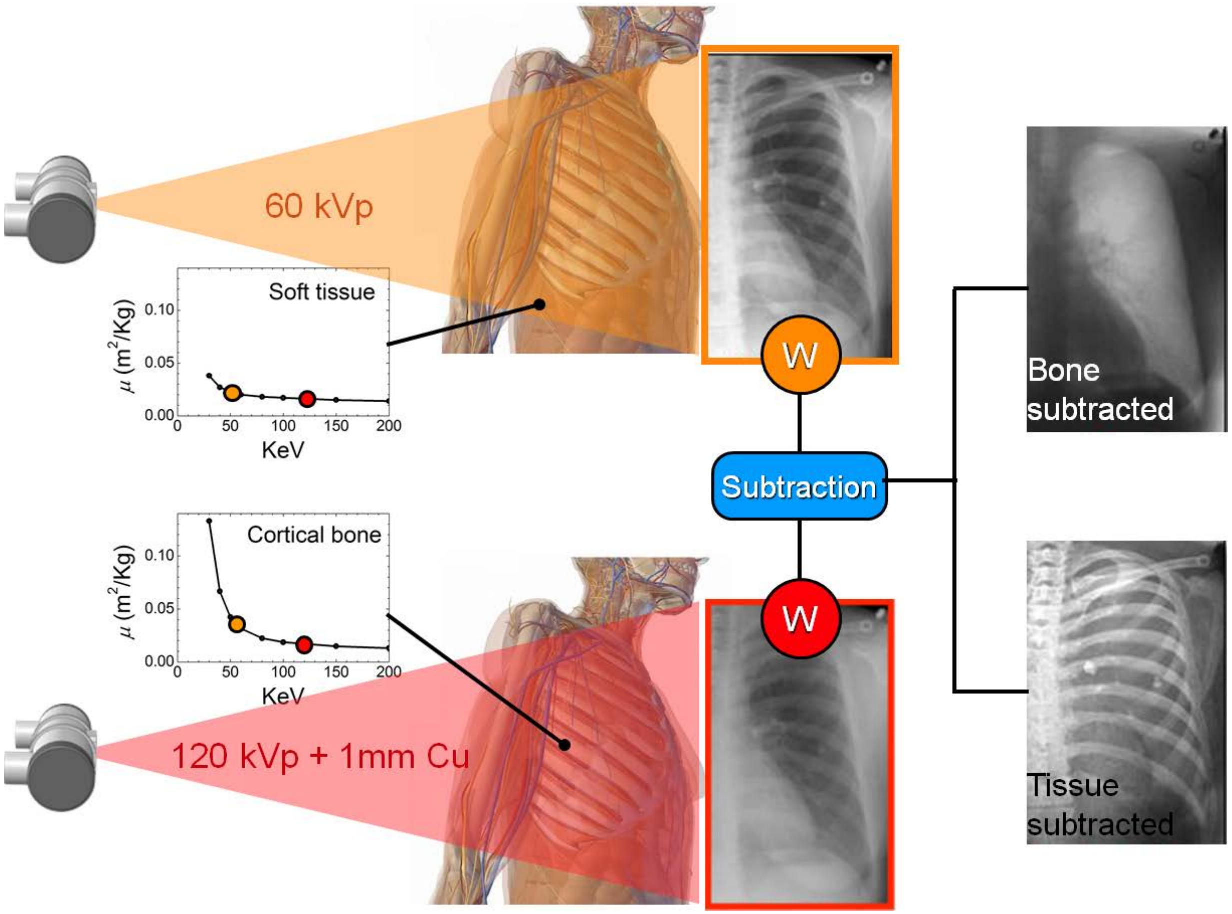
Obtained from the SMC

Monochromatic approximation

$$p(\xi) = -\log \frac{d(\xi)}{\bar{d}_0} \approx \int_s \mu(\mathbf{x}, E_{eff})\, \mathrm{d}s$$

$$\hat{f}(\mathbf{x}) = a \int_{\theta_{min}}^{\theta_{max}} \hat{p}(\xi; \theta)\Big|_{\xi = \mathbf{x} \cdot \boldsymbol{\theta}}\, \mathrm{d}\theta$$

Energy discrimination



Linear approximation

$$p(\boldsymbol{\xi}; s; \mathrm{kVp}) \approx \sum_j \hat{\mu}_j(\mathrm{kVp}) \int_s f_j(\mathbf{x})\, \mathrm{d}s$$

where

$$\hat{\mu}(\mathrm{kVp}) = \mathbb{E}\left\{ \frac{\int_0^{\mathrm{kVp}} q_0(E) R(E) \mu(E)\, \mathrm{d}E}{\int_0^{\mathrm{kVp}} q_0(E) R(E)\, \mathrm{d}E} \right\}$$

- Two-basis (i.e., $j = 2$) material analysis (e.g., bone & soft tissue)

$$p(\mathrm{kVp}) = \hat{\mu}_b(\mathrm{kVp}) t_b + \hat{\mu}_s(\mathrm{kVp}) t_s$$

$$t_j = \mp p\left(p_{\mathrm{kVp}_H}\right) \pm w_j p\left(p_{\mathrm{kVp}_L}\right)$$

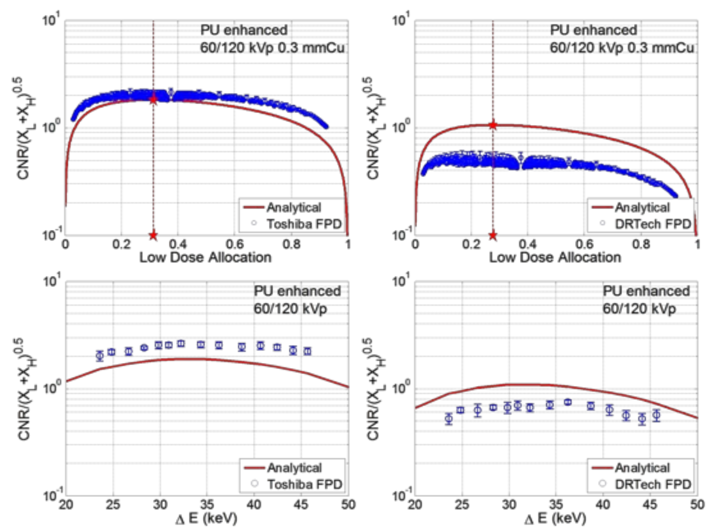
Optimization

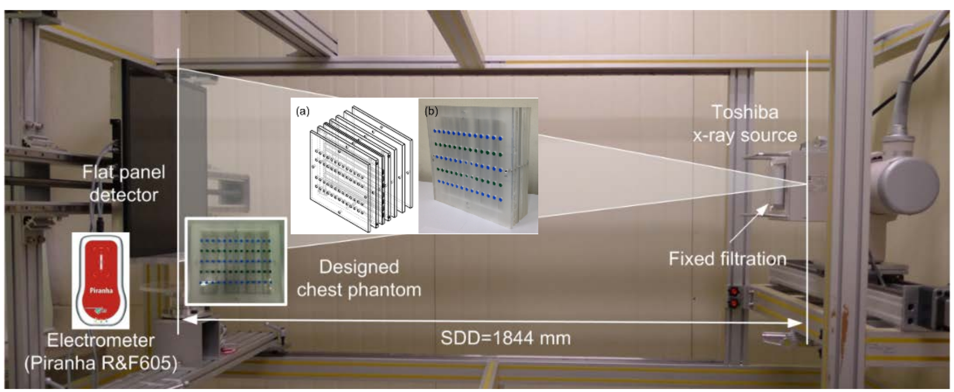
$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} f(\boldsymbol{\theta})$$

- Ex) Determine the optimal exposure fraction in low-energy imaging which maximizing the benefit-to-cost performance

$$f(A_{XL}) = \frac{\mathrm{CNR}_j^2}{X} = C_j^2 \left[\frac{1}{\bar{q}_{H0}(1 - A_{XL})\mathrm{DQE}_H} + \frac{w_j^2}{\bar{q}_{L0} A_{XL} \mathrm{DQE}_L} \right]^{-1}$$

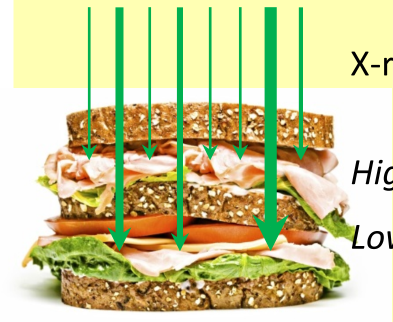
$$A_{XL}^* = \left[1 + \frac{1}{w_j} \sqrt{\frac{\bar{q}_{L0}\mathrm{DQE}_L}{\bar{q}_{H0}\mathrm{DQE}_H}} \right]^{-1}$$





Manuscript in preparation

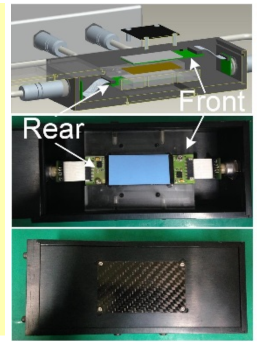
Single-shot DEI

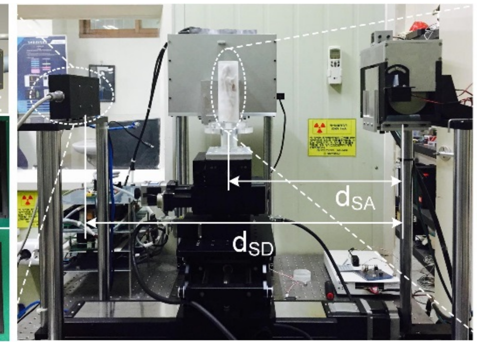


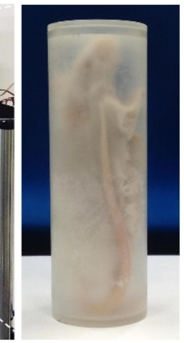
X-ray photons with different energies

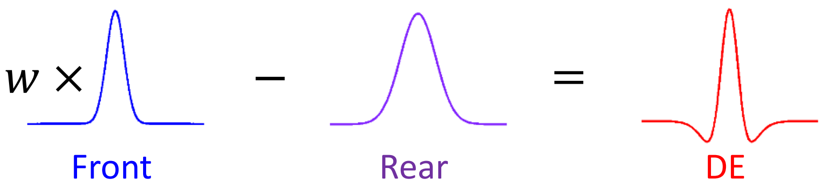
High-resolution *front* detector measures *low*-E

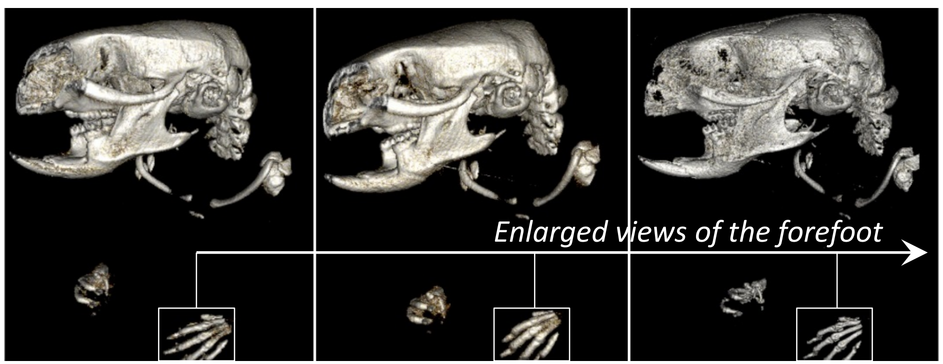
Low-resolution *rear* detector measures *high*-E











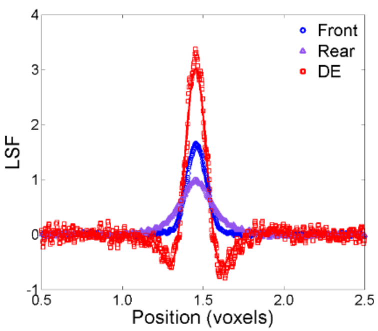
Imaging theory

$$p(\xi) = \bar{\lambda} q(\xi) * g(\xi) = \bar{\lambda}\bar{q}\bar{g}L(\xi)$$

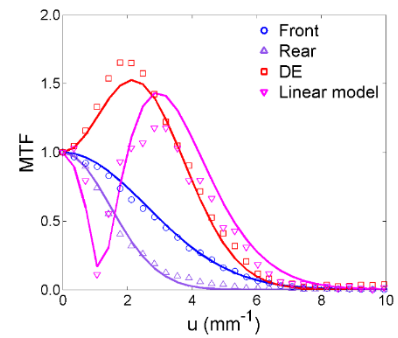
$$\hat{p}(\xi) = \bar{\lambda}\bar{q}\bar{g}L(\xi) * h(\xi) = \bar{\lambda}\bar{q}\bar{g}\hat{L}(\xi)$$

$$\hat{f}(\mathbf{x}) = a\int_0^{2\pi} \hat{p}(\xi;\theta)\Big|_{\xi=\mathbf{x}\cdot\boldsymbol{\theta}} \,\mathrm{d}\theta = a\bar{\lambda}\bar{q}\bar{g}\int_0^{2\pi}\hat{L}(\mathbf{x}\cdot\boldsymbol{\theta};\theta)\mathrm{d}\theta$$

$$\mathrm{MTF}(\mathbf{u}) = \mathcal{F}\left\{\frac{\hat{f}(\mathbf{x})}{\int_{-\infty}^{\infty}\hat{f}(\mathbf{x})\,\mathrm{d}\mathbf{x}}\right\} = \mathcal{F}\left\{\int_0^{2\pi}\hat{L}(\mathbf{x}\cdot\boldsymbol{\theta};\theta)\mathrm{d}\theta\right\}$$



$$\mathrm{MTF}_{DE}(\mathbf{u}) = \frac{\bar{w}\mathrm{MTF}_{DE}(\mathbf{u}) - \mathrm{MTF}_{DE}(\mathbf{u})}{\bar{w}-1}$$



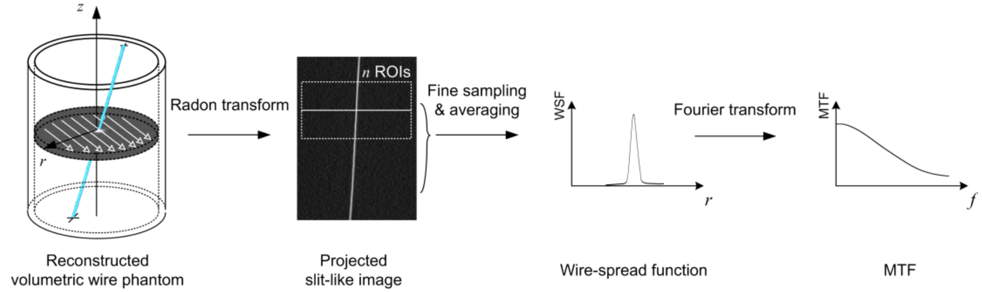
$$\mathrm{NPS}_{DE}(\mathbf{u}) = \bar{w}^2\mathrm{NPS}_L(\mathbf{u}) + (1+\mathrm{SPR})\mathrm{NPS}_H(\mathbf{u})$$

$$\mathrm{NEQ}_{DE}(\mathbf{u}) = \pi\mathbf{u}\frac{MTF_{DE}^2(\mathbf{u})}{\mathrm{NPS}_{DE}(\mathbf{u})}$$

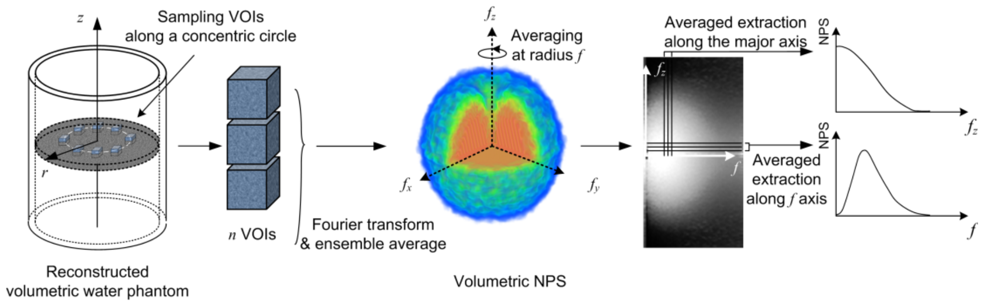
Manuscript in preparation

Imaging performance

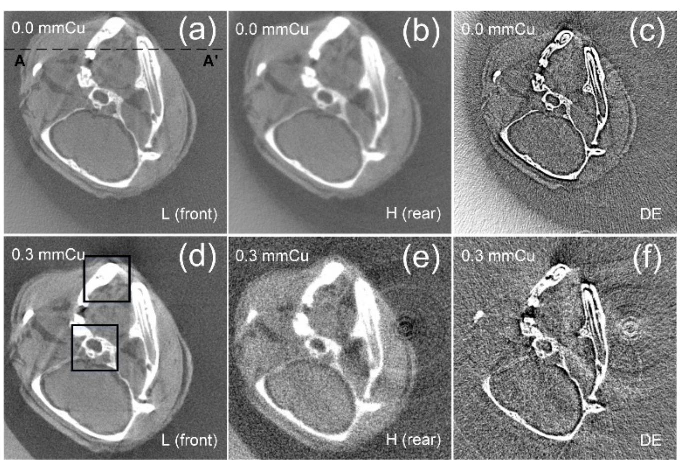
S. Y. Jang *et al.* | IEEE TBME | in press | 2016

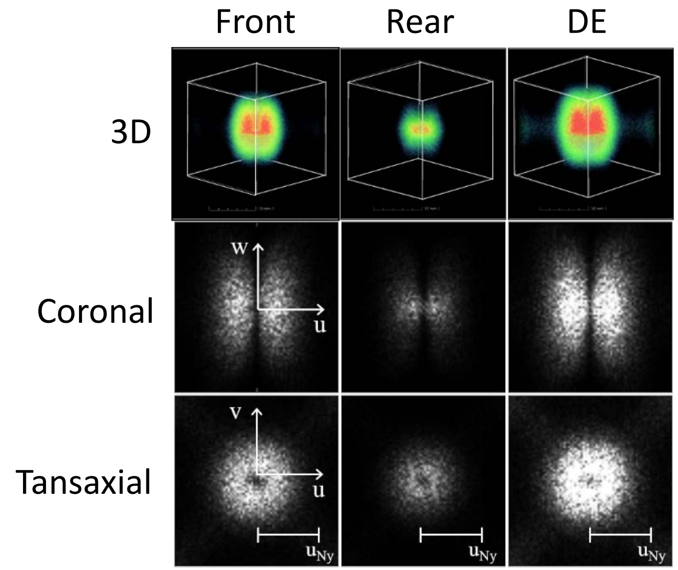


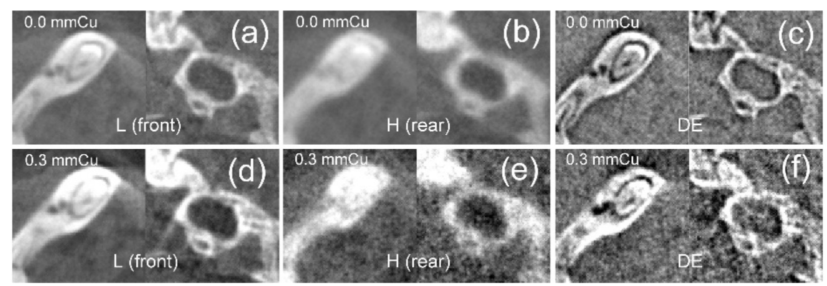
MTF evaluation



NPS evaluation

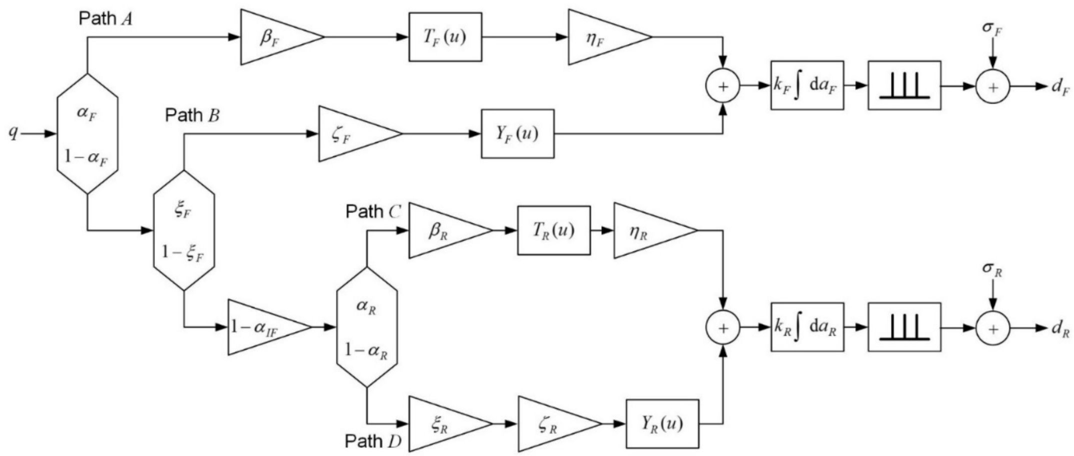


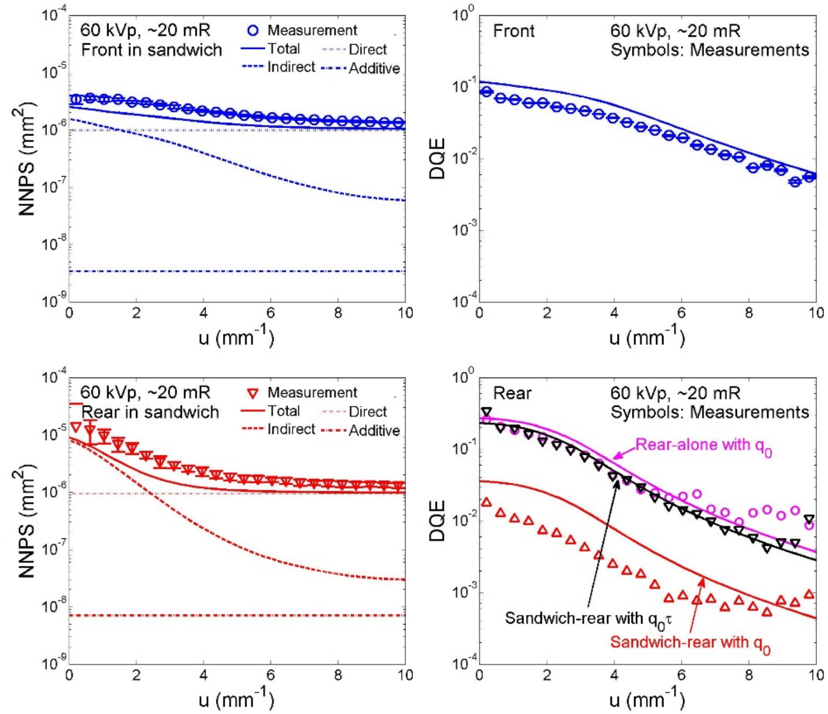




Manuscript in preparation

Analysis & optimization





$$\mathrm{FOM}_k \approx \left(\Delta\mu_{kM}^R - w_{\hat{k}}\Delta\mu_{kM}^F\right)^2 t_k^2 \tau \left[\frac{w_{\hat{k}}^2 \tau}{A_{eff,F}\mathrm{DQE}_F(0)} + \frac{1}{A_{eff,R}\mathrm{DQE}_R(0)}\right]^{-1}$$

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Wrap-up

- Unfortunately, detectors and systems are neither LSI nor stationary

- Nevertheless, the linear analysis (with reasonable assumptions) is useful to understand the working principle, and it can describe the actual performance in some limited extents

- The linear analysis can provides objective functions, $f(\boldsymbol{\theta})$, appropriate for optimizing design and technique parameters, $\boldsymbol{\theta}$

$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} f(\boldsymbol{\theta})$$

- Further consideration of nonlinear effects will result in better optimal parameters

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