Adjoint-based Mesh Optimization Method: The Development and Application for Nuclear Fuel Analysis

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1. Introduction

Widely used system codes, such as MARS-KS, solve discretized partial differential equations by using method of uniform or empirical meshing method based on user's judgement solely. Meshing still depends highly on the users' experiences, although the number of nodes and size of each node are important parameters that affect both computation resources and the accuracy of the results. If we can optimize the node distribution that can compute the most precise numerical solution of some problems under limited computing resource, positive effects such as increment of numerical stability, minimized user dependency, and increased reliability of the system code can be expected.

In this research, methods for optimizing mesh distribution is proposed. The proposed method uses adjoint base optimization method (adjoint method) [1, 2]. Also, by applying the proposed methodology to a 1-D steady state cylindrical nuclear fuel rod, the authors confirmed whether the suggested method can give more accurate solution than that of the conventional method of uniform meshing produces. Furthermore, the optimized result will be obtained by applying this meshing technique to the existing code input deck and will be compared to the results produced from the uniform meshing method.

Numerical solutions are calculated form an in-house 1D Finite Difference Method code while neglecting the axial conduction. The fuel radial node optimization was first performed to match the Fuel Centerline Temperature (FCT) the best. This was followed by optimizing the axial node which the Peak Cladding Temperature (PCT) is matched the best. After obtaining the optimized radial and axial nodes, the nodalization is implemented into the system analysis code and transient analyses were performed to observe the optimum nodalization performance.

2. Methodology

2.1. Adjoint based optimization method

In this paper, the authors are proposing to use an adjoint based optimization method (adjoint method) [1, 2] to optimize the problem, which is based on the sensitivity of value function over design parameters.

Sensitivity vector is given as equation (1), where $\boldsymbol{p}$ is the design parameter, $g(\boldsymbol{x},\boldsymbol{p})$ is object function and $\boldsymbol{x}$ satisfies $A\boldsymbol{x}=\boldsymbol{b}$. Dimension of $\boldsymbol{x}$, $\boldsymbol{b}$, $\boldsymbol{p}$, A are (1, M), (M, 1), (1, P) and (M, M).

$$\frac{dg(\boldsymbol{x},\boldsymbol{p})}{d\boldsymbol{p}} = g_{\boldsymbol{p}} + g_{\boldsymbol{x}} x_{\boldsymbol{p}} \;\; \ldots(1)$$

$$\left(A_{\boldsymbol{B}} = \left(\frac{\partial A}{\partial B_i}\right), \qquad i \in \boldsymbol{B}\right)$$

$$g_{\boldsymbol{x}} x_{\boldsymbol{p}} = g_{\boldsymbol{x}}\left[A^{-1}\left(\boldsymbol{b}_{\boldsymbol{p}} - A_{\boldsymbol{p}}\boldsymbol{x}\right)\right] \;\; \ldots(2)$$

Equations (2) can be re-written as equation (3)

$$g_{\boldsymbol{x}} x_{\boldsymbol{p}} = \lambda^T\left[\left(\boldsymbol{b}_{\boldsymbol{p}} - A_{\boldsymbol{p}}\boldsymbol{x}\right)\right], \qquad A^T\lambda = g_{\boldsymbol{x}}^T \;\; \ldots(3)$$

To obtain $\frac{dg(\boldsymbol{x},\boldsymbol{p})}{d\boldsymbol{p}}$, Equation (3) requires only one inverse-matrix calculation. This is much less than the traditional calculation method of using the finite differences over the elements of **p**.

This method is known as the adjoint method.

2.2. 1D FDM fuel rod numerical scheme

The problem can be modeled as shown in Figure 1. The 1D nuclear fuel is composed of UO_2 fuel pellet with a uniform heat generation radially, Zircaloy cladding, and the gap between them. There are four boundary and interfacial matching conditions: Centerline symmetric boundary, fuel-gap convection boundary, gap-cladding convection boundary, and cladding-coolant convection boundary. The coolant bulk temperature is specified for the radial mesh optimization.

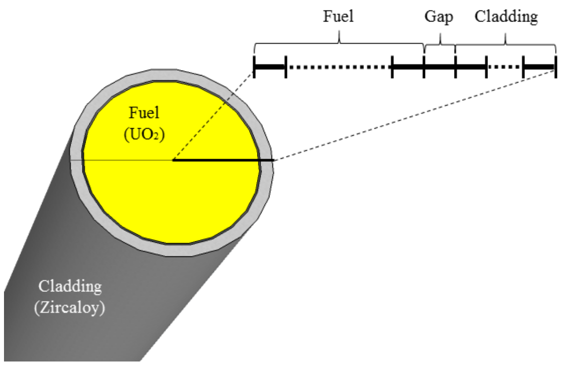

Figure 1. 1D lumped fuel-cladding system problem concept picture

2.3. 1D FDM coolant channel numerical scheme

Cladding temperature was obtained from channel thermal-hydraulic analysis. Assumptions of the analysis are as following:

1: Pressure drop inside the channel is negligible.
2: Heat transfer effect due to boiling can be neglected.
3: Surface of the cladding is ideally clean.
4: At steady state, there is no heat loss.
5: Volumetric heat generation profile of the fuel is assumed to be a chopped cosine shape.

2.4. Optimization Scheme

The optimization process starts from calculating $\frac{dg(\boldsymbol{x},\boldsymbol{p})}{d\boldsymbol{p}}$ using adjoint method and updating the design parameter vector toward the most sensitive direction.

$\boldsymbol{V}$ can be calculated as equation (4) as $g_{\boldsymbol{p}}$ is zero vector, where $\boldsymbol{V}$ is the sensitivity vector from adjoint method.

$$\boldsymbol{V} = \frac{dg(\boldsymbol{x},\boldsymbol{p})}{d\boldsymbol{p}} = g_x x_{\boldsymbol{p}} = \lambda^T[(\boldsymbol{b}_{\boldsymbol{p}} - A_{\boldsymbol{p}}\boldsymbol{x})],$$
$$A^T\lambda = g_x^T \quad \ldots(4)$$

From the given sensitivity, iteration proceeds using equation (5).

$$g_{next} = g_{prev} - C_{step}\frac{\boldsymbol{V}}{norm(V)} \quad \ldots(5)$$

Figure 2 shows a flow chart of the algorithm.

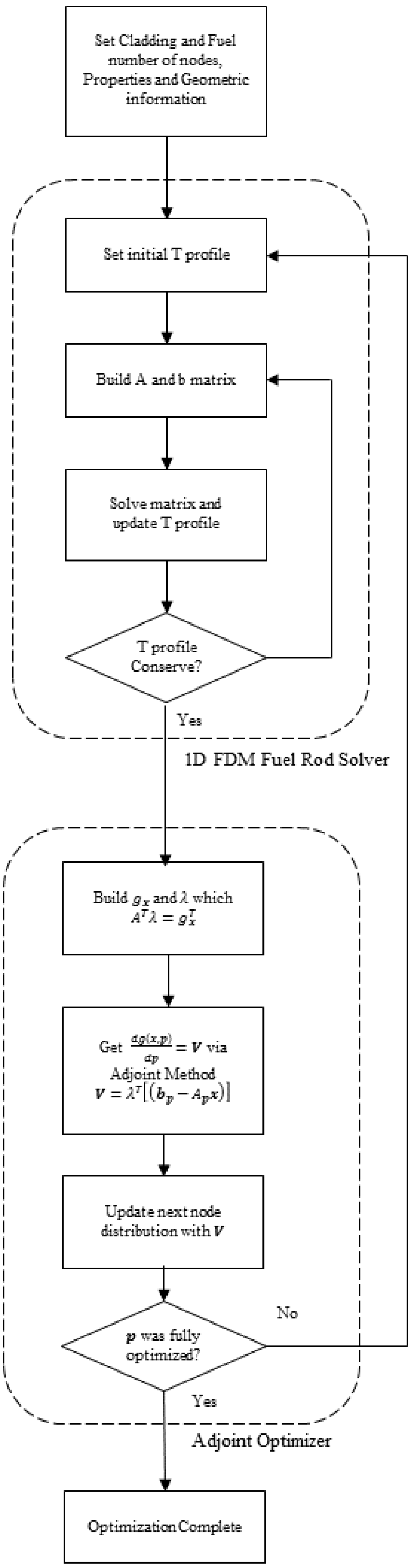

Figure 2 Calculation Procedure of the Adjoint-based optimization program

3. Results

3.1. FCT optimization Result

Pre-optimized radial node distribution which predicts the FCT at the steady state condition used in MARS-KS 1.4 input deck of APR-1400. A cold leg Large Break Loss of Coolant Accident (LBLOCA) was analyzed, and the time dependent FCT was checked for each case.

Figure 3 contains three data groups: The most computationally expensive calculation using 94 nodes for the fuel and 5 for the cladding, calculated result with 17 nodes (12 for the fuel, 5 for the cladding) using uniform mesh distribution, and a result with 17 nodes using previously suggested method, with optimized mesh distribution.

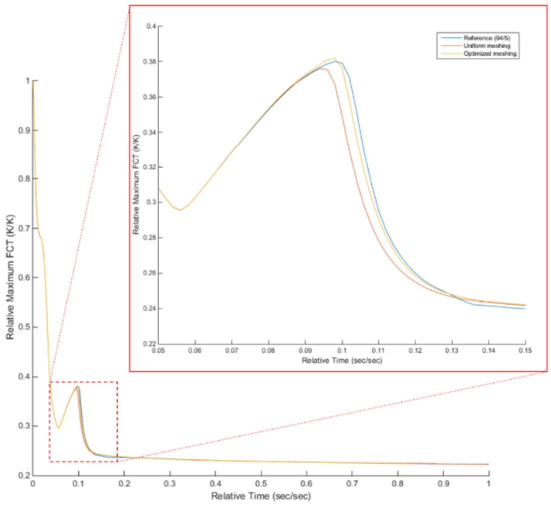

Figure 3. Difference between the MARS-KS APR-1400 LBLOCA result of radially 99 nodes / 17 uniformly distributed node / 17 optimized nodes

As shown in Figure 3, the reflood peaks of the maximum FCT are slightly different for various radial mesh configurations. The optimized meshing shows closer match with the most computationally expensive solution over the uniform meshing scheme. It is interesting to observe that even though the radial mesh is optimized from the steady state condition, still the optimized node provides better match even during a fast transient such LBLOCA.

3.2. PCT optimization Result

As in the previous section, the axial node optimization was applied to MARS-KS 1.4. The obtained PCT results from three different meshing conditions are shown: 50 axial nodes and 50 radial nodes, uniformly distributed mesh with 20 axial and 17 radial nodes, and optimized mesh with 20 axial and 17 radial nodes. The results are shown in Figure 4.

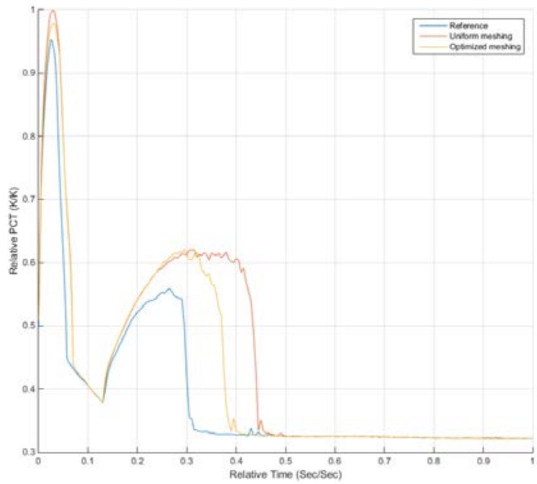

Figure 4. Difference between the MARS-KS APR-1400 LBLOCA result of radially 50, Axially 50 nodes / radially 17, axially 20 uniformly distributed nodes / radially 17, axially 20 optimized nodes

As it can be observed from Figure 4, the optimized node distribution, although it was optimized for the steady state, always show numerically better result than that of the uniform meshing even during a transient analysis.

4. Discussion

The developed adjoint-based mesh optimization method in the study is applied to MARS-KS, which is a nuclear system analysis code. Results show that the newly established method yields better results than that of the uniform meshing method from the numerical point of view. It is again stressed that the optimized mesh for the steady state can also give better numerical results even during a transient analysis.

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