Shifting and Two-Level Iterative Techniques to Speedup p-CMFD Acceleration in Transport Calculation

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1. Introduction

To reduce the computational burden in neutron transport calculation, various acceleration methods have been developed [1]. Among them, coarse mesh based acceleration methods [2-5] are widely used, because they are easily applied to the original transport calculation with any geometries. However, a drawback of coarse mesh based acceleration methods is that they exhibit slow convergence or divergent behavior for optically thick coarse mesh cells. This drawback limits the size of coarse mesh cells. In eigenvalue problems, it incurs non-negligible computational burden per iteration.

The present paper investigates the cause of such slow convergence or divergent behavior. To overcome the limitation, three techniques are introduced to speedup coarse mesh based acceleration: Coarse mesh shifting and two two-level iterative schemes. These techniques are applied to the partial current-based coarse mesh finite difference (p-CMFD) method [3-5], that is a more stable acceleration method than other coarse mesh based acceleration methods. A simple analysis of a numerical problem shows that the techniques enhance the convergence speed of p-CMFD, especially for optically thick coarse mesh cells.

2. Limitations of Coarse Mesh Based Acceleration

This section deals with the nonlinear acceleration methods currently used, the optimization of p-CMFD, and the limitation of coarse mesh based acceleration methods.

2.1 Coarse Mesh Based Acceleration

Coarse mesh based acceleration methods usually consist of two parts: a high-order calculation and a low-order calculation. The high-order calculation employs transport methods with fixed fission source. The low-order calculation uses the balance equation, in which the high-order calculation provides parameters over each coarse mesh cell. The low-order calculation gives the multiplication factor and the coarse mesh cell averaged scalar flux. The coarse mesh cell averaged scalar fluxes are modulated to be used in the fission source of the high-order equation for the next iteration.

To study the efficiency of such kinds of acceleration, Fourier convergence analysis was performed [6, 7] on coarse mesh rebalance (CMR), coarse mesh finite difference (CMFD) [2], and p-CMFD [3-5] for the eigenvalue problem with the step characteristic scheme. Figure 1 shows that p-CMFD is always stable and more stable than the other methods. p-CMFD and CMFD are efficient for optically thin coarse mesh cells but not efficient for optically thick coarse mesh cells.

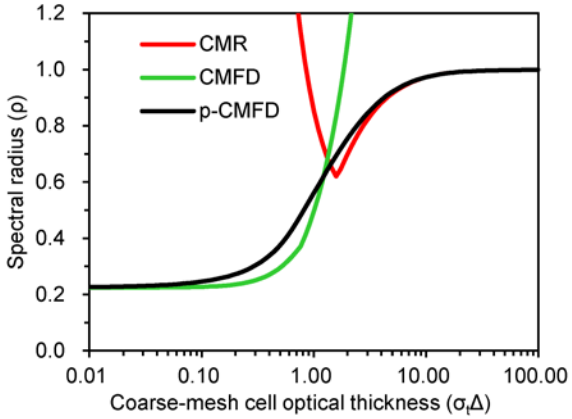

Fig. 1. Results of Fourier stability analysis of CMR, CMFD, and p-CMFD with SC and S_{16} for the eigenvalue problem (number of scattering iterations = 1, granularity = 20).

2.2 Optimization of Diffusion Coefficient

CMFD and p-CMFD use the diffusion coefficient which comes from the standard diffusion theory. (The results of spectral radius designated as p-CMFD throughout the figures in this paper are those of "standard" p-CMFD). In theory, the diffusion coefficient can be chosen arbitrarily. Refs. 8 and 9 adjusted the diffusion coefficient in CMFD to improve the convergence speed. We adjust the diffusion coefficient in p-CMFD to optimize the spectral radius. The golden section search method is used to find the optimized diffusion coefficient. Figure 2 shows that the optimized p-CMFD leads to a slightly reduced spectral radius.

2.3 Lower Bound of the Rate of Convergence

Even though of the optimization, p-CMFD shows slow convergence in optically thick coarse mesh cells. This is a common property for all coarse mesh based acceleration methods. It comes from the disagreement between the coarse-mesh level convergence and the fine-mesh level convergence. In the sense of error reduction, there are errors which cannot be reduced by coarse-mesh based low-order calculation (these errors are not zeros, but the average value over each coarse mesh cell is zero). Thus, the reduction rate of these errors depends only on the high-order transport calculation. These errors give the lower limit of the spectral radius for coarse mesh based accelerations. Figure 2 displays the theoretical lower bound of coarse mesh based acceleration methods and the spectral radius of fine mesh limit with the optimizing diffusion coefficient. Optimized p-CMFD is very close to the fine mesh limit for optically thin coarse mesh cells and to the lower bound for optically thick coarse mesh cells. Because the lower bound of the spectral radius of coarse mesh based acceleration methods is close to unity for optically thick coarse mesh cells, any coarse mesh based acceleration method will exhibit slow convergence for optically thick coarse mesh cells.

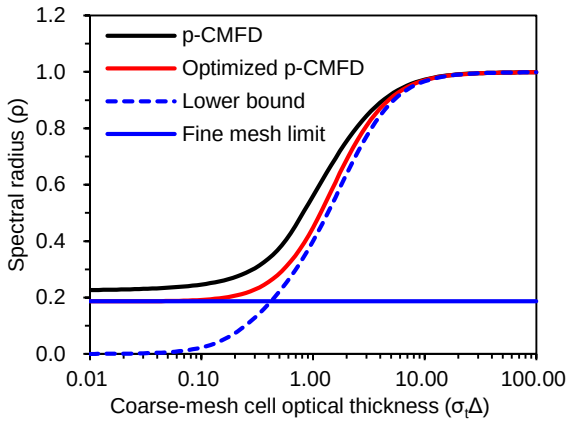

Fig. 2. Spectral radius of the optimized p-CMFD and the lower bound of coarse mesh based acceleration.

3. Techniques to Speedup p-CMFD

To overcome the limitations of coarse mesh based acceleration methods, we should reduce the corresponding errors which cannot be reduced by coarse mesh based calculation. This section describes three techniques to speedup p-CMFD. A one-dimensional test problem (1000 cm, homogeneous problem with vacuum boundary condition on both ends) is used to test the efficiency of the techniques. The computational conditions are as follows: Fine mesh size is 0.01 cm, granularity is 100, and S_{16} calculations with SC scheme are performed. We change the value of the total cross section to estimate the spectral radius for any coarse mesh optical thickness.

3.1 Coarse Mesh Shifting p-CMFD

The first technique is coarse mesh shifting. Figure 3 is the schematic of coarse mesh shifting. For each iteration, the lattice of coarse meshes is shifted. If we shift coarse meshes for each iteration, the above mentioned errors can be reduced in the next iteration because of the change of coarse meshes.

Figure 4 compares the spectral radius for various cases of coarse mesh shifting. In overall, shifting coarse meshes by 25 fine mesh cells per iteration gives the best spectral radius for the test problem. Note that the spectral radii of coarse mesh shifting are smaller than the lower bound for optically thick coarse mesh cells.

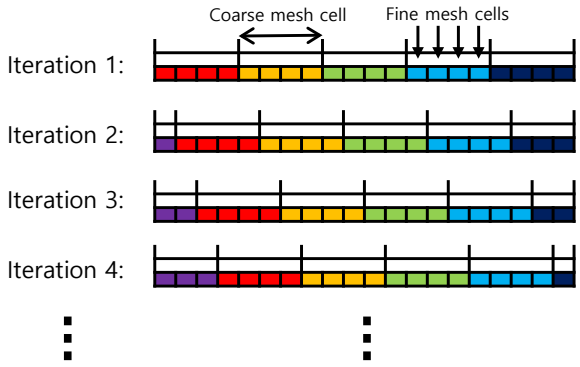

Fig. 3. A schematic of coarse mesh shifting.

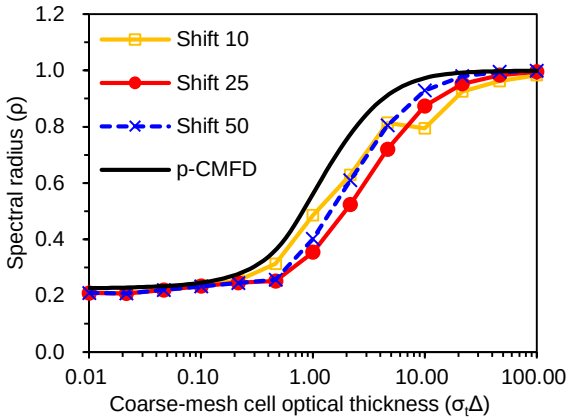

Fig. 4. Spectral radius of p-CMFD with coarse mesh shifting (Shift *q*: shift coarse meshes by *q* fine mesh cells per iteration).

3.2 p-FMFD Augmented Two-Level p-CMFD

The second technique is a two-level p-CMFD. Let us consider a partial current-based fine mesh finite difference acceleration with fixed fission source (denoted as p-FMFD) before or after p-CMFD eigenvalue calculation. Compared to the conventional p-CMFD acceleration, p-FMFD with fixed fission source incurs small additional computational burden. When the p-FMFD calculation is performed before the p-CMFD eigenvalue calculation, whole-core p-FMFD calculation is used for the stability. On the other hand, when the p-FMFD calculation is performed after the p-CMFD calculation, local p-FMFD calculations over coarse mesh cells are used. Each p-FMFD calculation over a coarse mesh cell is performed independently of p-FMFD calculations over other coarse mesh cells. To set up the boundary conditions of each p-FMFD, the incoming partial currents are necessary, and that is why we use p-CMFD framework, which provides the transport corrected partial currents.

Figure 5 compares the p-FMFD augmented two-level p-CMFD techniques with the coarse mesh shifting technique described in Section 3.1. Note that they show similar convergence rate. Even though we add p-FMFD procedures, it gives slow convergence for optically thick coarse mesh cells.

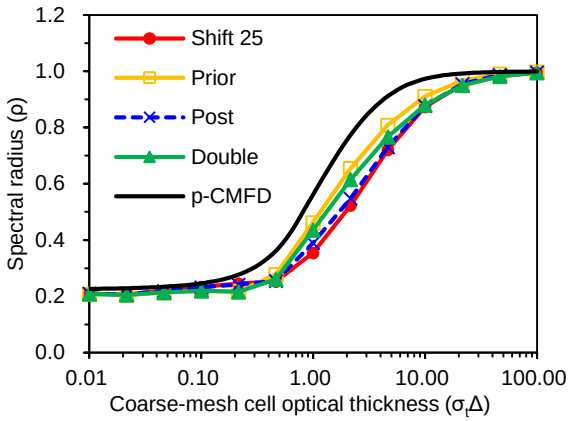

Fig. 5. Spectral radius of two-level p-CMFD augmented with p-FMFD (Prior: p-FMFD before p-CMFD calculation, Post: p-FMFD after p-CMFD calculation, Double: p-FMFD before and after p-CMFD calculation).

3.3 Local/Global Iteration in Two-Level p-CMFD

To reduce the spectral radius further, we introduce the local/global iterative framework to the low-order calculation in a two-level p-CMFD. Figure 6 is the flow chart of such a two-level p-CMFD. Modifying the p-FMFD doubly augmented two-level p-CMFD in Section 3.2, an inner iteration procedure is introduced between the whole-core p-CMFD eigenvalue calculation and the local p-FMFD calculations.

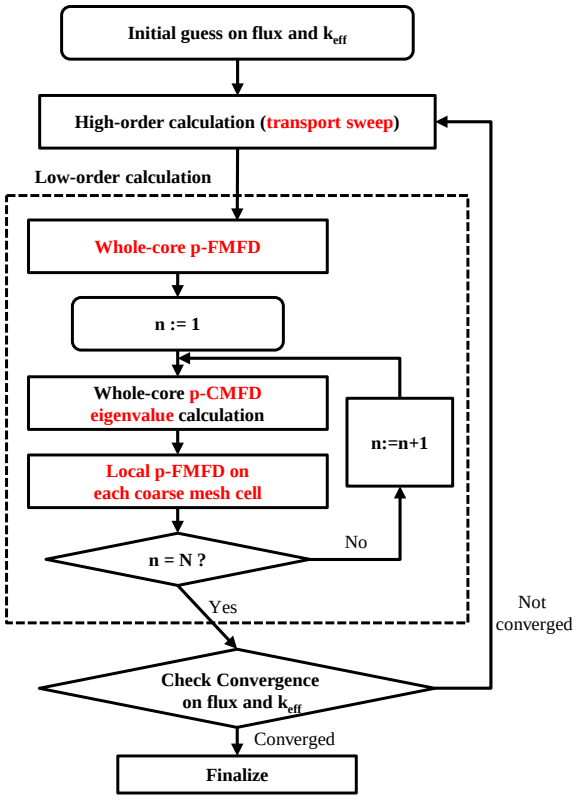

Fig. 6. Flow chart of local/global iterative framework in two-level p-CMFD.

Figure 7 is the result of local/global iterations in a two-level p-CMFD. As the number of local/global iterations increases, the spectral radius for optically thick coarse mesh cells reduces drastically approaching that of fine mesh limit.

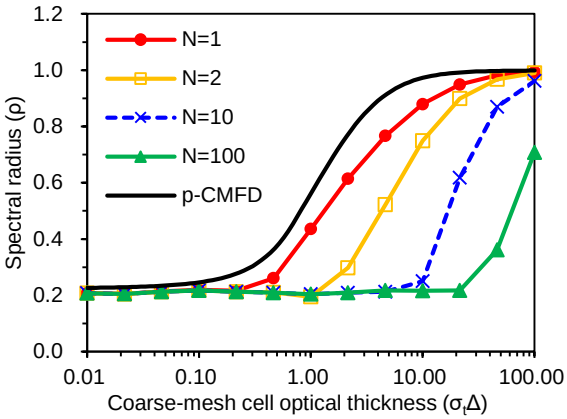

Fig. 7. Spectral radius of local/global iteration in a two-level p-CMFD (N: number of local/global iterations in a two-level p-CMFD).

4. Conclusions

This paper presents the speedup techniques for p-CMFD acceleration in neutron transport calculation. We introduce three techniques that are more effective than the optimized p-CMFD (with optimized diffusion coefficient) for optically thick coarse mesh cells. The spectral radii of the first two techniques are smaller than the lower bound of spectral radius of coarse mesh based acceleration methods. However, the first two techniques are still slow for optically thick coarse mesh cells.

To improve this situation, we also consider the local/global iterations of local p-FMFD and global p-CMFD. The results of this third technique show that we obtain fast convergence (even for large coarse-mesh cells) if we use a sufficient number of local/global iterations. Hence, it has the potential to overcome the limitation of coarse mesh based acceleration.

There are some problems still remaining for future work. For example, the number of local/global iterations should be controlled adaptively to reduce the unnecessary waste of computational time. The application and performance of the techniques to two-dimensional and three-dimensional realistic problems remain to be studied.

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